HIGHER ORDER CORRECTIONS TO THE DRELL-YAN PROCESS

T. MATSUURA

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PROEFSCHRIFT

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Contents

Chapter I

Introduction

1 The experiments

For already some twenty years one is interested in the physics of massive lepton pair production in hadronic collisions. The first observation of such a lepton pair was made by the Columbia-BNL group [1] in proton nucleus collisions. Many other fixed target experiments followed at increasing energies (reviews can be found in ref. [2]). A motivation behind this type of experiments was the search for new vector mesons. These efforts were rewarded by the discoveries of the J/ψ resonance (the c quark) [3] and the Y family (the b quark) [4]. Another reason for studying the massive lepton pair production, which is of more interest to us, is to investigate the structure of hadrons. In this case the hadrons are probed by a highly virtual, timelike photon, which is experimentally observed through its decay into a massive lepton pair. The mechanism, by which the virtual photon is produced, is now believed to be described by the so called QCD improved parton model. Therefore, due to the rather high statistics, nowadays fixed target experiments [5] can be used as a good testing ground for QCD.

When the CERN $p\bar{p}$ collider became operational in the beginning of this decade, one of the main issues was to find the intermediate vector bosons Z and W. These vector bosons can be observed through their decay products, viz. massive lepton pairs. The production mechanism of the Z and W in a $p\bar{p}$ collider is the same as for the virtual photon, mentioned earlier. Consequently, it can also be described in the framework of perturbative QCD. The vector bosons Z and W were indeed found by the UA1 and UA2 collaborations [6] and the production rates for the Z and W [7] are in rather good agreement with the first order calculations in the literature [8]. Recently, the FNAL $p\bar{p}$ collider has started working at a C.M. energy of 1.8 TeV. It is expected that the Tevatron will provide more precise measurements for the Z and W production rates.

It is clear that when experiments become more accurate, there is a need for better theoretical predictions. Until now the cross-section for the lepton pair production has been calculated up to order α_s [9]. In this thesis we will go beyond this order and will present the calculation of the dominant second order corrections to the massive lepton pair production.

2 The theory

From the first days of hadron-hadron colliders people have been searching for a model to describe the production of massive lepton pairs. An important step forward was made with the proposal of the parton model by R.P. Feynman [10] to describe processes involving hadrons. In this model a colliding hadron is considered to be built up of free, pointlike constituents, which are called partons. It was first applied to the deep inelastic lepton-hadron scattering (DIS) process [11]. In the parton model deep inelastic eP scattering is interpreted as the incoherent sum of electron parton scatterings. Using this picture one found the same results for the deep inelastic scattering process as obtained by J.D. Bjorken [12], who had derived them in a more formal way. Inspired by this success S.D. Drell and T.M. Yan [13] formulated their famous mechanism for the massive lepton pair production, using the parton picture. According to them the process underlying the massive lepton pair production is given by

$$\begin{array}{c}\text{parton from hadron 1} + \text{anti-parton from hadron 2}\\ \downarrow \\ \text{virtual photon} \\ \downarrow \\ \text{lepton pair}\end{array}$$

The cross-section for the lepton pair production can then be obtained by summing over all parton cross-sections multiplied by a combination of weight functions. This weight function, $f_a^H(x)$, which we will call the parton distribution function, gives the probability of finding in hadron H a parton 'a' carrying a momentum fraction x of its parent hadron. Due to the success of the Drell-Yan formalism, the production of massive lepton pairs in hadron-hadron colliders is often referred to as the Drell-Yan (DY) process.

One of the main predictions of the Drell-Yan model is that the lepton pair production should be scale invariant, by which we mean that the dynamics of the process is independent of the interaction scale. In the mid-seventies [14] this was verified to be consistent with the experimental data. However, the DY model clearly has some deficiencies. One of them is the rather large normalization constant, the so called K-factor, by which the theoretical cross-section has to be multiplied to reproduce the experimental data (K-factor $\sim$ 2-3) [14].

By then a theory for the strong interactions had been formulated in the form of QCD (for reviews see ref. [15]). In the framework of QCD the partons are identified with quarks and gluons. Further, the asymptotic freedom of QCD could be used to explain the success of the parton model. The Drell-Yan model is now interpreted as the lowest order QCD process contributing to the massive lepton pair production.

With the advent of QCD also higher order corrections in α_s could be calculated. A serious problem with this kind of calculations was the appearance of initial state collinear divergences. These singularities arise because both the gluons and the quarks are treated as massless particles. It is clear that physically sensible cross-sections should be free of divergences. The solution to this problem was suggested by H.D. Politzer [16], who, studying the O(α_s) corrections to both the deep inelastic scattering process and the lepton pair production, observed that

1. the DIS and DY process contained the same initial state collinear divergences and

2. that they could be factored out of the cross-section.

This observation allows us to absorb the initial state collinear divergences, in a process independent way, into the parton distribution functions. This procedure is now known as mass factorization and has been worked out in more detail by many people [17].

Thus one was able to determine the first order correction to the Drell-Yan process [9]. A striking feature was the very large size of these corrections. At low energies one found the first order contribution to be of the same order of magnitude as the Born cross-section. Although these large corrections are consistent with the data (K-factor $\sim$ 2-3), one might start doubting the reliability of perturbative QCD. One should include an estimate of the higher order corrections to check the convergence of the perturbation series.

Carefully studying the higher order contributions one finds that the bulk of the corrections are due to parton subprocesses with the maximum number of gluons in the final state. Terms, characteristic of these processes, are $C\ \delta(1-x)$ and $\ln^i(1-x)/(1-x)$, where x is the scaling variable. The δ-function stems from the virtual and soft gluon contributions. Actual calculations [9] show that the constant C is very large. The logarithmic terms appear in the hard gluon cross-section and are dominant in the limit $x \rightarrow 1$, which corresponds to the gluons becoming soft. Many people have tried to handle these terms to improve the prediction of perturbative QCD. This is often done by resumming the large corrections [18], which mostly results in the exponentiation of the first order term. Such an approach is based on extrapolation of the next to leading terms without an exact knowledge of the effects coming from the higher order corrections. Therefore, it is our opinion that the calculation of the second order corrections would be necessary to check this kind of resummation prescriptions.

3 Outline

In this thesis we will present the results for the dominant second order contributions to the Drell-Yan process. This means that we will be calculating the O(α_s^2) coefficients of the terms, $\delta(1-x)$ and $\ln^i(1-x)/(1-x)$, mentioned earlier.

In the next chapter we will describe the formalism needed to perform such a calculation. In chapter III the actual computation of the cross-section $d\sigma/dQ^2$ is presented. Furthermore, we will study exponentiation and also will discuss the implications of the O(α_s^2) corrections for the Z and W production at the CERN and FNAL p$\bar{\text{p}}$-colliders. Finally, in chapter IV we will derive the second order contributions to the differential cross-section $d^2\sigma/(dQ^2 dx_F)$ and examine briefly the NA10 experiment.

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Chapter II

The Drell-Yan formalism

1 Introduction

In this chapter we will present the formalism needed for the calculation of higher order corrections to the Drell-Yan process. It will become clear in the subsequent sections that, due to our choice of the mass factorization scheme, it will also be necessary to compute the QCD corrections to the deep inelastic scattering process. Before going into the details of the Drell-Yan formalism, we will start with defining some of the relevant quantities of the Drell-Yan and deep inelastic processes.

In fig. 1 we have depicted the relevant kinematics of the Drell-Yan (DY) process

$$\begin{aligned} H_1 + H_2 \rightarrow \quad & V + \text{"hadronic states"} \\ & | \\ & \rightarrow \ell_1 + \ell_2 \end{aligned} \tag{1.1}$$

where H_1 and H_2 are incoming hadrons and V represents a vector boson ($V = \gamma$, Z or W), which decays into a lepton pair (ℓ_1, ℓ_2).

For future use it will be convenient to introduce the so called Drell-Yan scaling variable τ,

$$\tau = \frac{Q^2}{S}, \qquad S = (P_1 + P_2)^2, \qquad Q^2 = q^2 \tag{1.2}$$

The reason for calling τ a scaling variable will become clear in the next section.

We are interested in calculating the colour averaged cross-section of the DY process, which can be written as

$$\frac{d\sigma^V}{dQ^2} = \tau\ \sigma_V(Q^2, M_V{}^2)\ W_V(\tau, Q^2) \tag{1.3}$$

In the above formula σ_V is the pointlike DY cross-section and $W_V(\tau, Q^2)$ will be referred to as the hadronic DY structure function. Our main goal will be to calculate the higher order corrections to $W_V(\tau, Q^2)$.

As stated earlier, the calculation of the hadronic DY structure function also involves the study of the deep inelastic scattering (DIS) process,

$$\ell_1 + H \rightarrow \ell_2 + \text{"hadronic states"} \tag{1.4}$$

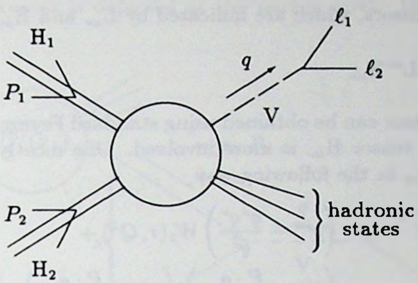

Fig. 1. The kinematics of the DY process

In this case ℓ_1 and ℓ_2 are leptons, which are scattered by a hadron H. In fig. 2 we show the kinematics of this process.

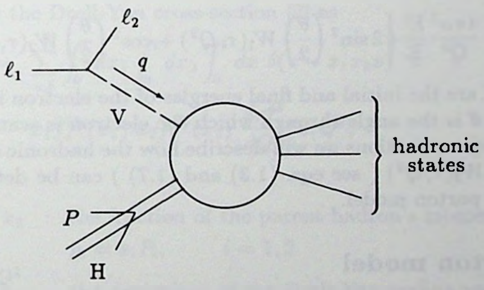

Fig. 2. The kinematics of the DIS process

Some variables, which are frequently used, are

$$\begin{aligned} Q^2 &= -q^2 \\ \nu &= \frac{P \cdot q}{\mathrm{M}} \\ \tau &= \frac{-q^2}{2P \cdot q} \end{aligned} \tag{1.5}$$

here M is the mass of the hadron and τ is called the Bjorken scaling variable*.

The cross-section of the DIS process is propotional to the product of the leptonic and hadronic tensors, which are indicated by $L_{\mu\nu}$ and $H_{\mu\nu}$ respectively.

$$\frac{d^2\sigma}{dQ^2 d\nu} \sim L^{\mu\nu} H_{\mu\nu} \tag{1.6}$$

The leptonic tensor can be obtained using standard Feynman rules. The calculation of the hadronic tensor $H_{\mu\nu}$ is more involved. One mostly tackles this problem by decomposing $H_{\mu\nu}$ in the following way

$$\begin{aligned} H^{\mu\nu}(\tau, Q^2) &= -\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) W_1(\tau, Q^2) + \\ &+ \left(P^\mu - \frac{P\cdot q}{q^2} q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2} q^\nu\right)\frac{1}{M^2} W_2(\tau, Q^2) + \\ &- \frac{1}{2} i\varepsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta \frac{1}{M^2} W_3(\tau, Q^2) \end{aligned} \tag{1.7}$$

The functions $W_i(\tau, Q^2)$ are called the hadronic DIS structure functions. The last structure function $W_3(\tau, Q^2)$ is absent in pure e.m. processes, because of parity conservation. For our purpose of mass factorization we will only need the structure function $W_2(\tau, Q^2)$. Using this decomposition of the hadronic tensor $H_{\mu\nu}$ the cross-section $d^2\sigma/(dQ^2 d\nu)$ can be expressed in terms of the hadronic structure functions $W_i(\tau, Q^2)$. For example, in the case of deep inelastic eP scattering one has

$$\frac{d^2\sigma}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\left\{2\sin^2\left(\frac{\theta}{2}\right) W_1(\tau, Q^2) + \cos^2\left(\frac{\theta}{2}\right) W_2(\tau, Q^2)\right\} \tag{1.8}$$

where E and E′ are the initial and final energies of the electron in the rest frame of the proton and θ is the angle through which the electron is scattered.

In the subsequent sections we will describe how the hadronic structure functions $W_V(\tau, Q^2)$ and $W_2(\tau, Q^2)$ (see eqs. (1.3) and (1.7)) can be determined using the QCD improved parton model.

2 The parton model

Processes involving hadrons can be described in an intuitively appealing way by the parton model [1]. According to this model a hadron is built up of constituents, called partons, which can be characterized by a set of distribution functions $f_a(x)$. Such a function $f_a(x)$ gives the probability of finding in the hadron a parton 'a', carrying a momentum fraction x of its parent hadron. Furthermore, one assumes that the partons within a hadron do not interact with each other. This means that a process involving hadrons can be considered as the incoherent sum of the parton subprocesses.

We will now apply the parton model to the Drell-Yan and deep inelastic scattering processes.

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*Notice that we use the symbol τ for both the Drell-Yan and Bjorken scaling variable.

2.1 The Drell-Yan process

In fig. 3 the Drell-Yan process is depicted at the parton level. Using the parton

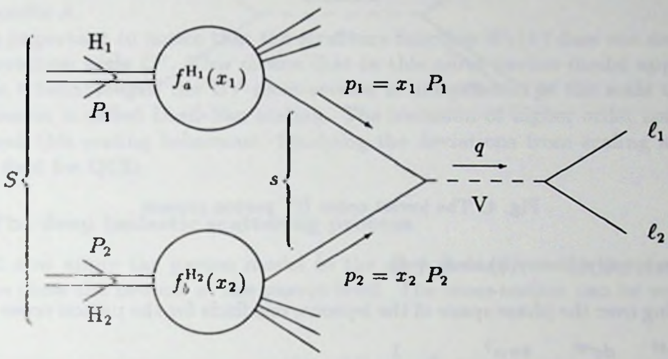

Fig. 3. The Drell-Yan process at the parton level

model we can write the Drell-Yan cross-section [2] as

$$\frac{d\sigma}{dQ^2}(\tau, Q^2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx\ \delta(\tau - x_1 x_2 x) \times$$
$$x_1 f_a^{H_1}(x_1)\ x_2 f_b^{H_2}(x_2)\ \frac{d\sigma^{ab}}{dQ^2}(x, Q^2) \tag{2.1}$$

with

x_1, x_2	:	the fraction of the parent hadron's momentum $p_i = x_i P_i, \qquad i = 1, 2$
$x = \frac{Q^2}{s}$	:	the equivalent of the Drell-Yan scaling variable τ at the parton level
$f_a^{H_1}(x_1), f_b^{H_2}(x_2)$	:	the distribution functions of the partons a and b in the hadrons H_1 and H_2, respectively
$\frac{d\sigma^{ab}}{dQ^2}(x, Q^2)$	:	the cross-section of the process: parton a + parton $b \to V \to \ell_1 + \ell_2$

Notice that we use capital letters for the hadronic variables and small letters for the partonic ones.

Let us work out eq. (2.1) for V=γ. Identifying the partons with quarks and gluons, the lowest order parton cross-section is given by the process (see fig. 4)

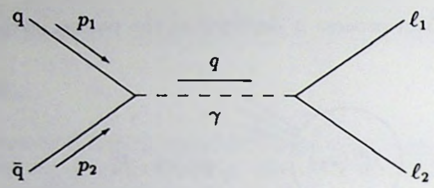

Fig. 4. The lowest order DY parton process

$$q(p_1) + \bar{q}(p_2) \rightarrow \gamma(q) \rightarrow \ell_1 + \ell_2 \tag{2.2}$$

Integrating over the phase space of the leptons, one finds for the parton cross-section

$$\frac{d\sigma^{ab}}{dQ^2} = \frac{d\sigma^{q\bar{q}}}{dQ^2} = \frac{4\pi\alpha^2}{3Q^4N}\,\tau\, e_q^2\, \frac{1}{x_1 x_2}\delta(1-x) \tag{2.3}$$

with

- α : the fine structure constant
- N : the number of colours
- e_q : the charge of the quark

Substituting eq. (2.3) into eq. (2.1) the Drell-Yan cross-section becomes

$$\frac{d\sigma^{V=\gamma}}{dQ^2} = \tau\, \sigma_\gamma(Q^2)\, W_\gamma(\tau) \tag{2.4}$$

with

$$\sigma_\gamma(Q^2) = \frac{4\pi\alpha^2}{3Q^4N} \tag{2.5}$$

and

$$\begin{aligned} W_\gamma(\tau) &= \sum_q e_q^2 \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx\, \delta(\tau - x_1 x_2 x) \\ &\quad \left\{ f_q(x_1) f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1) f_q(x_2) \right\}\, \delta(1-x) \end{aligned} \tag{2.6}$$

Of course, the above calculations can also be carried out for the Z and W vector bosons. One then finds

$$\frac{d\sigma^V}{dQ^2} = \tau\, \sigma_V(Q^2, M_V^2)\, W_V(\tau) \tag{2.7}$$

with

$$W_V(\tau) = \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx\ \delta(\tau - x_1 x_2 x)\ PD_V^{q\bar{q}}(x_1, x_2)\ \delta(1-x) \tag{2.8}$$

The function $PD_V^{q\bar{q}}(x_1, x_2)$ is a combination of parton distribution functions and σ_V is the pointlike cross-section. The exact expressions for $PD_V^{q\bar{q}}$ and σ_V can be found in Appendix A.

It is important to notice that the structure function $W_V(\tau)$ does not depend on the interaction scale Q^2. This means that in this naive parton model approximation the τ behaviour of the DY cross-section is independent of the scale Q^2. This phenomenon is called Drell-Yan scaling. The inclusion of higher order corrections will break this scaling behaviour. Studying the deviations from scaling is a good testing field for QCD.

2.2 The deep inelastic scattering process

We will now apply the parton model to the deep inelastic scattering process. In fig. 5 we show the process at the parton level. The cross-section can be written as

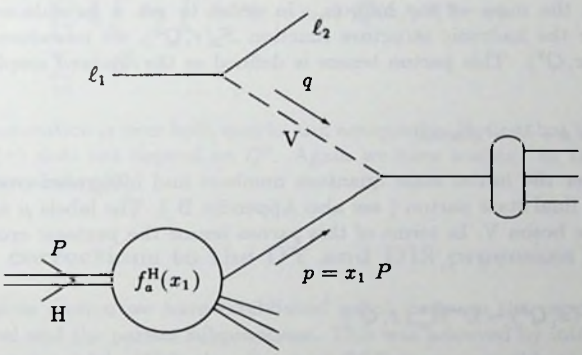

Fig. 5. The deep inelastic scattering process at the parton level

$$\frac{d^2\sigma}{dQ^2 d\nu}(\tau, Q^2) = \sum_a \int_0^1 dx_1 \int_0^1 dx\ \delta(\tau - x_1 x)\ x_1 f_a^H(x_1)\ \frac{d^2\sigma^a}{dQ^2 d\nu}(x, Q^2) \tag{2.9}$$

with

x_1 : the fraction of the parent hadron's momentum $p = x_1 P$

$x = \frac{-q^2}{2p \cdot q}$: the equivalent of the Bjorken scaling variable τ

at the parton level

$f_a^{\mathrm{H}}(x_1)$: the distribution function of the parton a in the hadron H

$\frac{d^2\sigma^a}{dQ^2 d\nu}(x, Q^2)$: the cross-section of the process:

parton $a + \ell_1 \rightarrow \ell_2 +$ parton

We will now derive a formula for the structure function $W_2(\tau, Q^2)$ in eq. (1.7). It is common practice to work with a redefinition of $W_2(\tau, Q^2)$, viz.:

$$\mathcal{F}_2(\tau, Q^2) = \frac{1}{\tau} \; \nu W_2(\tau, Q^2) \tag{2.10}$$

Notice that $\mathcal{F}_2(\tau, Q^2)$ can be obtained by applying the following projection operator to the hadronic tensor $\mathrm{H}^{\mu\nu}(\tau, Q^2)$ (see eq. (1.7))

$$\mathcal{F}_2(\tau, Q^2) = -\mathrm{M} \left\{ g^{\mu\nu} + 12\frac{\tau^2}{q^2} P^\mu P^\nu \right\} \mathrm{H}_{\mu\nu}(\tau, Q^2) \tag{2.11}$$

where M is the mass of the hadron. In order to get a formula equivalent to eq. (2.8) for the hadronic structure function $\mathcal{F}_2(\tau, Q^2)$, we introduce the parton tensor $\hat{\mathrm{H}}^a_{\mu\nu}(x, Q^2)$. This parton tensor is defined as the squared amplitude of the process

$$\text{parton } a + \mathrm{V} \rightarrow \text{parton} \tag{2.12}$$

averaged over the initial state quantum numbers and integrated over the phase space of the final state parton (see also Appendix B). The labels μ and ν belong to the vector boson V. In terms of this *parton* tensor the *partonic* cross-section is equal to

$$\frac{d^2\sigma^a}{dQ^2 d\nu}(x, Q^2) \sim \mathrm{L}^{\mu\nu} \hat{\mathrm{H}}^a_{\mu\nu}(x, Q^2) \tag{2.13}$$

where $\mathrm{L}^{\mu\nu}$ is the lepton tensor. This expression should be compared with eq. (1.6), where the *hadronic* cross-section was written as the product of the lepton tensor and the *hadronic* tensor $\mathrm{H}_{\mu\nu}$.

Analogous to the hadronic structure function $\mathcal{F}_2(\tau, Q^2)$ we introduce the parton structure function $\hat{\mathcal{F}}_2^a(x, Q^2)$, which is defined by

$$\hat{\mathcal{F}}_2^a(x, Q^2) = -\frac{1}{4\pi} \left\{ g^{\mu\nu} + 12\frac{x^2}{q^2} p^\mu p^\nu \right\} \hat{\mathrm{H}}^a_{\mu\nu}(x, Q^2) \tag{2.14}$$

In the above formula p^μ is the momentum of parton a and the factor $1/(4\pi)$ is needed to get the right normalization. It can be shown that the following equation holds

$$\mathcal{F}_2(\tau, Q^2) = \sum_a \int_0^1 dx_1 \int_0^1 dx \; \delta(\tau - x_1 x) \; f_a^{\mathrm{H}}(x_1) \; \hat{\mathcal{F}}_2^a(x, Q^2) \tag{2.15}$$

We will work out the above formula for V=γ. At the lowest order the parton tensor is given by the process

$$q(p) + \gamma(q) \rightarrow q(p') \tag{2.16}$$

The parton tensor is then equal to

$$\hat{H}^{q}_{\mu\nu}(x, Q^2) = \frac{2\pi}{p \cdot q} e_q^2 \, \delta(1-x) \left\{ 2p_\mu p_\nu + p_\mu q_\nu + p_\nu q_\mu - p \cdot q \; g_{\mu\nu} \right\} \tag{2.17}$$

Applying the projection operator given in eq. (2.14) to the above expression, we find

$$\hat{\mathcal{F}}^{q}_{2}(x) = e_q^2 \, \delta(1-x) \tag{2.18}$$

Finally, the hadronic structure function can be written as

$$\begin{aligned} \mathcal{F}_2(\tau) &= \sum_{q,\bar{q}} e_q^2 \int_0^1 dx_1 \int_0^1 dx \; \delta(\tau - x_1 x) \; f_q^{H}(x_1) \; \delta(1-x) \\ &= \sum_{q,\bar{q}} e_q^2 \; f_q^{H}(\tau) \end{aligned} \tag{2.19}$$

where the summation is over both quarks and antiquarks. Notice that the structure function $\mathcal{F}_2(\tau)$ does not depend on Q^2. Again we have scaling. In this case it is called Bjorken scaling [3].

3 QCD corrections to the DY and DIS processes

In the previous section we have established a link between the processes at the hadronic level and the parton subprocesses. This was achieved by introducing the so called parton model. With the advent of QCD it was possible to identify the partons with quarks and gluons. Thus, the parton subprocesses could be calculated using QCD Feynman rules. Furthermore, QCD turned out to be an asymptotic free theory, which means that the strong coupling constant α_s decreases with growing energies. This feature of QCD is very promising for two reasons. Firstly, it suggests a reconciliation of the incoherence assumption in the parton model with the fact that free quarks have not yet been observed. Secondly, it justifies the use of perturbative techniques for the calculation of processes at high energies. Of course, these two considerations are of importance to us, because we will rely heavily on both the parton model and perturbative QCD for our computations.

In this section we will discuss the calculation of higher order corrections to the Drell-Yan and deep inelastic scattering processes. In particular, we will discuss the divergences, which occur due to loop and phase space integrations.

To give an impression of the complexity of calculating higher order corrections to the DY and DIS processes, we list all the contributions up to order α_s^2.

Drell-Yan processes

α_s^0	: $q+\bar{q}\rightarrow V$	
α_s^1	: $q+\bar{q}\rightarrow V$	one loop correction
	$q+\bar{q}\rightarrow V+g$	
	$q+g\rightarrow V+q$	
α_s^2	: $q+\bar{q}\rightarrow V$	two loop correction
	$q+\bar{q}\rightarrow V+g$	one loop correction
	$q+\bar{q}\rightarrow V+g+g$	
	$q+\bar{q}\rightarrow V+q+\bar{q}$	
	$q+g\rightarrow V+q$	one loop correction
	$q+g\rightarrow V+q+g$	
	$q+q\rightarrow V+q+q$	
	$g+g\rightarrow V+q+\bar{q}$	

DIS processes

α_s^0	: $V+q\rightarrow q$	
α_s^1	: $V+q\rightarrow q$	one loop correction
	$V+q\rightarrow q+g$	
	$V+g\rightarrow q+\bar{q}$	
α_s^2	: $V+q\rightarrow q$	two loop correction
	$V+q\rightarrow q+g$	one loop correction
	$V+q\rightarrow q+g+g$	
	$V+q\rightarrow q+q+\bar{q}$	
	$V+g\rightarrow q+\bar{q}$	one loop correction
	$V+g\rightarrow q+\bar{q}+g$	

From these lists one can see that including higher order corrections involves the calculation of:

1. virtual corrections
2. bremsstrahlung processes
3. new parton subprocesses

Therefore the formulae for the hadronic structure functions $W_V(\tau,Q^2)$ (DY) and $\mathcal{F}_2(\tau,Q^2)$ (DIS) will become more complicated, viz.

$$W_V(\tau,Q^2)=\int_0^1 dx_1\int_0^1 dx_2\int_0^1 dx\ \delta(\tau-x_1x_2x)\times$$

$$\Big\{PD_V^{q\bar{q}}(x_1,x_2)\ \hat{W}^{q\bar{q}}(x,Q^2,\varepsilon) + PD_V^{qg}(x_1,x_2)\ \hat{W}^{qg}(x,Q^2,\varepsilon) +$$

$$+ PD_V^{qq}(x_1,x_2)\ \hat{W}^{qq}(x,Q^2,\varepsilon) + PD_V^{gg}(x_1,x_2)\ \hat{W}^{gg}(x,Q^2,\varepsilon)\Big\} \tag{3.1}$$

and

$$\begin{aligned}\mathcal{F}_2(\tau,Q^2) &= \sum_{q,\bar{q}} e_q^2 \int_0^1 dx_1 \int_0^1 dx\ \delta(\tau - x_1 x) \\ &\Big\{f_q(x_1)\ \hat{\mathcal{F}}_2^q(x,Q^2,\varepsilon) + \frac{1}{2} f_g(x_1)\ \hat{\mathcal{F}}_2^g(x,Q^2,\varepsilon)\Big\}\end{aligned} \tag{3.2}$$

See Appendices A and B for the definitions of $\hat{W}^{ab}$ and $\hat{\mathcal{F}}_2^a$.

Comparing eqs. (3.1) and (3.2) with their naive parton model equivalents (see eqs. (2.8) and (2.19)) we find several changes

- $\delta(1-x) \rightarrow \hat{W}^{q\bar{q}}(x,Q^2,\varepsilon)$ or $\hat{\mathcal{F}}_2^q(x,Q^2,\varepsilon)$
- the structure functions $W_V(\tau,Q^2)$ and $\mathcal{F}_2(\tau,Q^2)$ become Q^2 dependent : breakdown of scale invariance!
- the parton structure functions $\hat{W}$ and $\hat{\mathcal{F}}_2$ contain poles in ε.

In the remainder of this section we will be concerned with the last point of the above remarks. The calculation of higher order corrections involves loop and phase space integrals. These integrals do not always converge, therefore one has to introduce a regularization scheme to handle the divergent integrals. We have chosen for the so called dimensional regularization method [4]. In this scheme the space time dimension is continued from 4 to n, and the singularities manifest themselves as poles in $\varepsilon = n - 4$. To get a better understanding of these divergences, we classify them into three groups. Each of these three types will be discussed below. In this discussion the quarks are treated as massless particles.

Type I : Ultraviolet divergences :

This type of divergences is found in loop integrals and occurs when the loop momentum k goes to infinity, $|k| \rightarrow \infty$. It is dealt with by the coupling constant renormalization, which we will perform in the $\overline{\text{MS}}$ scheme [5].

Type II : Infrared divergences :

Infrared singularities appear in both loop and phase space integrals. They are found in the low energy region, where the integration momentum k goes to zero, $|k| \rightarrow 0$. According to the Bloch-Nordsieck theorem [6] infrared divergences cancel, when both the virtual and their corresponding bremsstrahlung diagrams are taken into account.

Type III : Collinear divergences :

These divergences can also be encountered in both loop and phase space integrals. Collinear divergences, also called mass singularities, arise when the momenta of two massless particles become parallel to each other. For example, the propagator $\{(p-k)^2\}^{-1}$ can be written as $\{2|p|\,|k|(1-\cos\theta)\}^{-1}$ if p and k are the momenta of two massless particles. When p and k become parallel, $\cos\theta$ goes to one. Therefore the propagator diverges and can cause singularities. For the treatment of the mass singularities one has to distinguish two possibilities.

1. Final state collinear divergences :
   In this case the integration momentum becomes parallel to the momentum of a massless particle in the final state, According to the KLN theorem [7], this type of singularities cancels, because we are considering a final state inclusive process, where one sums over all the degenerate (= experimentally undistinguishable) final states.
2. Initial state collinear divergences :
   Here the integration momentum is collinear with the momentum of a particle in the initial state. Initial state collinear divergences are left over in the expressions for the parton structure functions $\hat{W}$ and $\hat{\mathcal{F}}_2$.

From the above discussion it is clear that the ε poles in the parton structure functions $\hat{W}$ and $\hat{\mathcal{F}}_2$ in eqs. (3.1) and (3.2) are due to initial state collinear divergences. Of course, they have to be dealt with to obtain physically sensible results. How this can be achieved, will be the main issue in the next section, where we will discuss the mass factorization theorem.

4 Mass factorization

4.1 The mass factorization theorem

From the discussion of the divergences, found in higher order corrections, it is clear that we are confronted with a problem. Namely, the parton structure functions $\hat{W}(x,Q^2,\varepsilon)$ and $\hat{\mathcal{F}}_2(x,Q^2,\varepsilon)$ still contain initial state collinear divergences, which manifest themselves as poles in ε. As these parton structure functions are to be used to calculate cross-sections, which can be measured and therefore should be finite, the collinear singularities have to be taken care of. The basic ingredient in the treatment of the collinear divergences is the mass factorization theorem [8]. This theorem makes two statements. Firstly, it says that the collinear divergences can be factored out of the cross-section. Secondly, according to this theorem, the mass singularities encountered in QCD have a universal character, i.e. they are process independent.

This universality of the collinear divergences can be used to absorb them, in a process independent way, into the parton distribution functions. Besides making the cross-section free of mass singularities, this redefinition of the parton distribution functions also has another effect. The parton distribution functions become scale dependent. This means that the scaling behaviour of the hadronic structure functions $W_V(\tau,Q^2)$ and $\mathcal{F}_2(\tau,Q^2)$, discussed in section 2, will be lost.

4.2 Application of the mass factorization theorem to the DY and DIS processes

To discuss the mass factorization for the DY and DIS processes in more detail, let us start with recapitulating the situation. The hadronic structure functions can be written as

$$\begin{aligned} W(\tau, Q^2) &= \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx \; \delta(\tau - x_1 x_2 x) \\ & \quad f_a^{H_1}(x_1)\, f_b^{H_2}(x_2)\, \hat{W}_{ab}(x, Q^2/\mu^2, \varepsilon) \end{aligned} \tag{4.1}$$

and

$$\mathcal{F}_2(\tau, Q^2) = \sum_a \int_0^1 dx_1 \int_0^1 dx \; \delta(\tau - x_1 x)\, f_a^{H}(x_1)\, \hat{\mathcal{F}}_{2,a}(x, Q^2/\mu^2, \varepsilon) \tag{4.2}$$

In the above equations $f_a^H(x)$ is the scale independent parton distribution function and $\hat{W}_{ab}$ and $\hat{\mathcal{F}}_{2,a}$ denote the DY and DIS parton structure functions after renormalization and cancellation of the I.R. divergences. The functions $\hat{W}_{ab}$ and $\hat{\mathcal{F}}_{2,a}$ still contain initial state collinear divergences, which appear as poles in ε. Furthermore, they depend on an arbitrary mass scale μ, which is introduced at the moment of renormalization.

For further discussion it will be convenient to introduce the moments of the hadronic structure functions $W(\tau, Q^2)$ and $\mathcal{F}_2(\tau, Q^2)$, which are defined by the Mellin transforms

$$W^n(Q^2) = \int_0^1 d\tau \; \tau^{n-1}\, W(\tau, Q^2) \tag{4.3}$$

and

$$\mathcal{F}_2^n(Q^2) = \int_0^1 d\tau \; \tau^{n-1}\, \mathcal{F}_2(\tau, Q^2) \tag{4.4}$$

In this language of moments eqs. (4.1) and (4.2) have a simpler form, viz.

$$W^n(Q^2) = \sum_{a,b} f_a^{n,H_1}\, f_b^{n,H_2}\, \hat{W}_{ab}^n(\alpha_s(\mu^2), Q^2/\mu^2, \varepsilon) \tag{4.5}$$

and

$$\mathcal{F}_2^n(Q^2) = \sum_a f_a^{n,H}\, \hat{\mathcal{F}}_{2,a}^n(\alpha_s(\mu^2), Q^2/\mu^2, \varepsilon) \tag{4.6}$$

where $f_a^{n,H}$, $\hat{W}_{ab}^n$ and $\hat{\mathcal{F}}_{2,a}^n$ are defined in a similar way as $W^n(Q^2)$ and $\mathcal{F}_2^n(Q^2)$.

According to the mass factorization theorem [8] a universal function A_{cd}^n should exist, which extracts the collinear divergences from the parton structure functions $\hat{W}_{ab}^n$ and $\hat{\mathcal{F}}_{2,a}^n$ at a mass factorization scale M. This means that one can write

$$\begin{aligned} \hat{W}_{ab}^n(\alpha_s(\mu^2), Q^2/\mu^2, \varepsilon) &= \\ & \sum_{c,d} C_{cd}^{n,\mathrm{DY}}(\alpha_s(M^2), Q^2/M^2) \\ & \quad A_{ca}^n(\alpha_s(M^2), M^2/\mu^2, \varepsilon)\, A_{db}^n(\alpha_s(M^2), M^2/\mu^2, \varepsilon) \end{aligned} \tag{4.7}$$

and

$$\hat{\mathcal{F}}^n_{2,a}(\alpha_s(\mu^2), Q^2/\mu^2, \varepsilon) =$$
$$\sum_b C^{n,\mathrm{DI}}_b(\alpha_s(M^2), Q^2/M^2)\; A^n_{ba}(\alpha_s(M^2), M^2/\mu^2, \varepsilon) \tag{4.8}$$

Some remarks should be made about the above equations. The most important feature is that the functions $C^{n,\mathrm{DY}}$ and $C^{n,\mathrm{DI}}$ are free of collinear divergences. Eventually, they will appear in the physical cross-sections. Furthermore, one notices that in eq. (4.7) two factors A^n appear, whereas in eq. (4.8) only one A^n is present. This is due to the fact that the DY process has two partons in the initial state contrary to the DIS process, where there is only one. Finally, comparing the l.h.s. of eqs. (4.7) and (4.8) with the r.h.s., the scale of the coupling constant changes from μ^2 into M^2. The coupling constants at the two different mass scales are related to each other through the differential equation

$$M\frac{d}{dM}\alpha_s(M^2) = \beta(\alpha_s(M^2)) \tag{4.9}$$

In fact we have introduced a running coupling constant. The exact definition of the β-function can be found in Appendix C.

Having isolated the mass singularities, the next step is to absorb them into the parton distribution functions. We will do this in the so called renormalization group improved way. For this purpose we will derive the renormalization group equations (RGE) for the functions C^n and A^n in case of the DIS process (see also Appendix C). For the DY process we will only give the final results.

As the mass factorization scale M has no physical meaning and can be chosen in an arbitrary way (just as the renormalization scale μ), the parton structure functions should not depend on it. Therefore one has

$$M\frac{d}{dM}\hat{\mathcal{F}}^n_{2,a}(\alpha_s(\mu^2), Q^2/\mu^2, \varepsilon) = 0 \tag{4.10}$$

Substituting eq.(4.8) into the above formula, we find

$$\left\{M\frac{\partial}{\partial M} + \beta(\alpha_s)\frac{\partial}{\partial \alpha_s}\right\}$$
$$\left\{\sum_b C^{n,\mathrm{DI}}_b(\alpha_s(M^2), Q^2/M^2)\; A^n_{ba}(\alpha_s(M^2), M^2/\mu^2, \varepsilon)\right\} = 0 \tag{4.11}$$

where the β-function is the same as in eq. (4.9). Introducing the anomalous dimension γ^n_{bc} as

$$M\frac{d}{dM}A^n_{ba} = -\sum_c \gamma^n_{bc}\; A^n_{ca} \tag{4.12}$$

the following RGE's can be derived for $C^{n,\mathrm{DI}}_b$ and A^n_{ba}

$$\sum_c\left\{\left(M\frac{\partial}{\partial M} + \beta(\alpha_s)\frac{\partial}{\partial \alpha_s}\right)\; \delta_{bc} + \gamma^n_{bc}\right\} A^n_{ca}(\alpha_s(M^2), M^2/\mu^2, \varepsilon) = 0 \tag{4.13}$$

and

$$\sum_b \left\{ \left(M\frac{\partial}{\partial M} + \beta(\alpha_s)\frac{\partial}{\partial \alpha_s} \right) \delta_{bc} - \gamma^n_{bc} \right\} C^{n,\mathrm{DI}}_b(\alpha_s(M^2), Q^2/M^2) = 0 \tag{4.14}$$

The easiest way to obtain the desired result is to choose the mass factorization scale M^2 equal to Q^2. In this case eq. (4.8) changes into

$$\hat{\mathcal{F}}^n_{2,a}(\alpha_s(\mu^2), Q^2/\mu^2, \varepsilon) = \sum_b C^{n,\mathrm{DI}}_b(\alpha_s(Q^2), 1)\ A^n_{ba}(\alpha_s(Q^2), Q^2/\mu^2, \varepsilon) \tag{4.15}$$

The advantage of this choice is that $C^{n,DI}_b$ now only depends on Q^2 through the running coupling constant and the Q^2 dependence of A^n_{ba} is determined by the RGE (eq. (4.13)). Solving the RGE one finds

$$A^n_{ba}(\alpha_s(Q^2), Q^2/\mu^2, \varepsilon) = \sum_c \Gamma_{bc}(t, t_0)\ A^n_{ca}(\alpha_s(Q^2_0), Q^2_0/\mu^2, \varepsilon) \tag{4.16}$$

with

$$\Gamma_{bc}(t, t_0) = \left\{ \mathrm{T}\exp\left(- \int_{t_0}^{t} \gamma(\alpha_s(t'))dt' \right) \right\}_{bc} \tag{4.17}$$

In the above formula we have introduce $t = \frac{1}{2}\ln(Q^2/\mu^2)$ and $t_0 = \frac{1}{2}\ln(Q^2_0/\mu^2)$, the Q^2_0 is fixed by the experiment. Further, $\mathrm{T}\exp\cdots$ stands for the t-ordered exponent, which has to be introduced, because the matrices $\gamma(\alpha_s(t'))$ do not commute. Combining eqs. (4.6), (4.15) and (4.16) we have

$$\begin{aligned} \mathcal{F}^n_2(Q^2) \ &= \ \sum_b C^{n,\mathrm{DI}}_b(\alpha_s(Q^2), 1) \times \\ &\qquad \left\{ \sum_c \Gamma_{bc}(t, t_0)\ \sum_a A^n_{ca}(\alpha_s(Q^2_0), Q^2_0/\mu^2, \varepsilon)\ f^n_a \right\} \end{aligned} \tag{4.18}$$

Defining new <u>scale dependent</u> parton distribution functions by

$$f^n_b(Q^2) = \sum_c \Gamma_{bc}(t, t_0) \underbrace{\sum_a A^n_{ca}(\alpha_s(Q^2_0), Q^2_0/\mu^2, \varepsilon)\ f^n_a}_{f^n_c(Q^2_0)} \tag{4.19}$$

the collinear divergences are absorbed into $f^n_c(Q^2_0)$. It is clear from this definition that once the parton distributions are measured at $Q^2 = Q^2_0$, they are determined by the differential equation

$$\frac{d}{dt} f^n_a(Q^2) = -\sum_b \gamma^n_{ab}\ f^n_b(Q^2) \tag{4.20}$$

This equation is mostly referred to as the Altarelli-Parisi equation [9].

Finally we find for the DIS process

$$\mathcal{F}^n_2(Q^2) = \sum_a C^{n,\mathrm{DI}}_a(\alpha_s(Q^2), 1)\ f^n_a(Q^2) \tag{4.21}$$

In a similar way one can derive for the Drell-Yan process

$$W^n(Q^2) = \sum_{a,b} C^{n,\mathrm{DY}}_{ab}(\alpha_s(Q^2),1)\ f^n_a(Q^2)\ f^n_b(Q^2) \tag{4.22}$$

Comparing the above equations with eqs. (4.5) and (4.6) one finds two important differences. Firstly, the collinear divergences have disappeared. Secondly, the parton distribution functions now depend on the interaction scale Q^2.

For completeness we mention that the above results can also be obtained in the context of the operator product expansion method [10], which is a more formal approach to the DIS process. In that case the functions C^n and A^n are called the Wilson coefficient and the operator matrix element, respectively.

4.3 Choosing a mass factorization prescription: the DIS scheme

Although we have described the treatment of collinear divergences in general terms, we have not yet given a detailed mass factorization prescription. For example, in eqs. (4.7) and (4.8) one still has the freedom to shift finite terms from the C^n's to the A^n's and vice versa.

In the mass factorization scheme that we have chosen, all the perturbative corrections to the DIS structure function $\mathcal{F}_2(\tau,Q^2)$ are absorbed into the parton distribution functions. This implies that to all orders in α_s, the deep inelastic structure function $\mathcal{F}_2$ can be written as (see also eq. (2.19))

$$\mathcal{F}_2(\tau,Q^2) = \sum_{\mathrm{q},\bar{\mathrm{q}}} e_\mathrm{q}^2\ f_\mathrm{q}(\tau,Q^2) \tag{4.23}$$

This mass factorization prescription is known as the DIS scheme [11]. The choice of this scheme seems quite natural, because a large amount of the information about the parton distribution functions comes from the measurements of the deep inelastic structure function $\mathcal{F}_2(\tau,Q^2)$.

As we will be mainly interested in the $\mathrm{q}\bar{\mathrm{q}}$ DY subprocess, we will discuss the application of the DIS mass factorization scheme to this particular subprocess. Restricting ourselves to the $\mathrm{q}\bar{\mathrm{q}}$ process, the hadronic DY structure function is given by

$$\begin{aligned} W_\gamma(\tau,Q^2) &= \sum_\mathrm{q} e_\mathrm{q}^2 \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx\ \delta(\tau - x_1x_2x) \\ &\quad \Big\{ f_\mathrm{q}(x_1)\ f_{\bar{\mathrm{q}}}(x_2) + f_{\bar{\mathrm{q}}}(x_1)\ f_\mathrm{q}(x_2) \Big\}\ \hat{W}^{\mathrm{q}\bar{\mathrm{q}}}(x,Q^2,\varepsilon) \end{aligned} \tag{4.24}$$

The DIS structure function needed for the mass factorization comes from the subprocess qV

$$\mathcal{F}_2(\tau,Q^2) = \sum_{\mathrm{q},\bar{\mathrm{q}}} e_\mathrm{q}^2 \int_0^1 dx_1 \int_0^1 dx\ \delta(\tau - x_1 x)\ f_\mathrm{q}(x_1)\ \hat{\mathcal{F}}_2^\mathrm{q}(x,Q^2,\varepsilon) \tag{4.25}$$

According to the mass factorization theorem the collinear divergences in $\hat{W}^{\mathrm{q}\bar{\mathrm{q}}}$ and $\hat{\mathcal{F}}_2^\mathrm{q}$ are related to each other in the following way (see eqs. (4.7) and (4.8))

$$\begin{aligned} \hat{W}^{\mathrm{q}\bar{\mathrm{q}}}(z,Q^2,\varepsilon) &= \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx\ \delta(z - x_1x_2x) \\ &\quad \hat{\mathcal{F}}_2^\mathrm{q}(x_1,Q^2,\varepsilon)\ \hat{\mathcal{F}}_2^\mathrm{q}(x_2,Q^2,\varepsilon)\ \Delta^{\mathrm{q}\bar{\mathrm{q}}}(x,Q^2) \end{aligned} \tag{4.26}$$

Notice that $\Delta^{q\bar{q}}(x, Q^2)$, which we will call the Drell-Yan correction term, is free of collinear divergences.

Following the DIS mass factorization scheme we define the scale dependent quark distribution functions as

$$f_q(\tau, Q^2) = \int_0^1 dx_1 \int_0^1 dx \; \delta(\tau - x_1 x) \; f_q(x_1) \; \hat{\mathcal{F}}_2^q(x, Q^2, \varepsilon) \tag{4.27}$$

Using this definition and combining eqs. (4.24) and (4.25), the hadronic structure function $W_\gamma(\tau, Q^2)$ can be written as

$$\begin{aligned} W_\gamma(\tau, Q^2) &= \sum_q e_q^2 \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx \; \delta(\tau - x_1 x_2 x) \\ & \left\{ f_q(x_1, Q^2) \; f_{\bar{q}}(x_2, Q^2) + f_{\bar{q}}(x_1, Q^2) \; f_q(x_2, Q^2) \right\} \Delta^{q\bar{q}}(x, Q^2) \end{aligned} \tag{4.28}$$

The above equations have been derived for V = γ, but, of course, similar formulae can be found for the Z and W vector bosons.

5 The first order corrections to the DY and DIS processes

To illustrate the mass factorization theorem, we will calculate the first order corrections to the Drell-Yan and deep inelastic scattering processes in the DIS scheme [12,13,14]. Performing these calculations we will explicitly find the universal splitting functions P^{qq} and P^{qg} at order α_s, which are directly related to the anomalous dimensions γ_{ab}^n introduced in the previous section. Furthermore, we will discuss the treatment of the soft gluon contributions, which will be the main subject of the next chapter.

5.1 The deep inelastic scattering process

In section 3 we have seen that at order α_s two types of parton processes contribute to the hadronic structure function $\mathcal{F}_2(\tau, Q^2)$, viz. the qV and the gV subprocesses.

Let us first consider the qV subprocess. The order α_s contribution from this process, which we will denote by $\hat{\mathcal{F}}_2^{(1),q}$, can be divided into two parts. Firstly, there is the interference of

$$q(p_1) + V(q) \to q(p_2) \text{ with one loop corrections} \tag{5.1}$$

with the lowest order process. Secondly, we have the one gluon bremsstrahlung process

$$q(p_1) + V(q) \to q(p_2) + g(k) \tag{5.2}$$

For the virtual gluon part one has to calculate the QCD corrections to the vector boson vertex. These corrections change the vertex $i\gamma^\mu$ into $i\gamma^\mu \; F(q^2)$, where the function $F(q^2)$ is the so called quark form factor. The first order part of the quark form factor, $F^{(1)}(q^2)$, is given by

$$F^{(1)}(q^2) = -g_b^2 s_n C_F (-q^2)^{\frac{1}{2}\varepsilon} \frac{8}{\varepsilon^2}\left(1+\frac{1}{4}\varepsilon+\frac{1}{4}\varepsilon^2\right)\frac{\Gamma(1-\frac{1}{2}\varepsilon)\Gamma^2(1+\frac{1}{2}\varepsilon)}{\Gamma(2+\varepsilon)} \tag{5.3}$$

with

$$s_n = \frac{1}{(4\pi)^2}\exp\left\{-\frac{1}{2}\varepsilon\ln(4\pi)\right\} \quad \text{and} \quad C_F = \frac{N^2-1}{2N} \tag{5.4}$$

where N is the number of colours. Furthermore, the $\Gamma(z)$ is the well known gamma-function. Using the definitions in Appendix B one finds that the virtual gluon contribution to $\hat{\mathcal{F}}_2^{(1),q}$ is given by

$$\hat{\mathcal{F}}_2^{(1),V}(x,Q^2,\varepsilon) = 2\,\delta(1-x)\,\mathrm{Re}F^{(1)}(q^2) \overset{q^2\leq 0}{=} 2\,\delta(1-x)\,F^{(1)}(q^2) \tag{5.5}$$

From the one gluon bremsstrahlung process we get (see Appendix B)

$$\hat{\mathcal{F}}_2^{(1),S+H}(x,Q^2,\varepsilon) = \\ -\frac{1}{4\pi}\frac{1}{(1+\frac{1}{2}\varepsilon)}\int dPS_2^{\mathrm{DI}}\left\{g^{\mu\nu}+12\frac{x^2}{q^2}\left(1+\frac{1}{3}\varepsilon\right)p_1^\mu p_1^\nu\right\}\langle M_\mu^q M_\nu^{q\dagger}\rangle_{av} \tag{5.6}$$

where dPS_2^{DI} is defined in Appendix D and M_μ^q is the amplitude of this process. The brackets $\langle\cdots\rangle_{av}$ indicate averaging over initial state quantum numbers and summation over the final state ones. Carrying out the phase space integration we find

$$\begin{aligned}\hat{\mathcal{F}}_2^{(1),S+H}(x,Q^2,\varepsilon) &= g_b^2 s_n C_F (1-x)^{-1+\frac{1}{2}\varepsilon}\,x^{-\frac{1}{2}\varepsilon}\,(-q^2)^{\frac{1}{2}\varepsilon}\,\frac{1}{\varepsilon}\frac{\Gamma(1+\frac{1}{2}\varepsilon)}{\Gamma(2+\varepsilon)} \\ &\times\left\{4(1+x^2)+(7-2x)\varepsilon+\frac{1}{2}\varepsilon^2\right\}\end{aligned} \tag{5.7}$$

The above formula has to be treated carefully. When the gluon becomes soft ($|k|\to 0$), the Bjorken variable x goes to one and the $(1-x)^{-1+\varepsilon/2}$ term gives rise to I.R. singularities. To make the I.R. divergence explicit, we introduce an infrared cutoff δ [12,14]. This δ separates eq. (5.7) into a soft gluon ($x > 1-\delta$) part and a hard gluon ($x \leq \delta$) part. This separation can be achieved using the identity

$$(1-x)^{-1+a\varepsilon} = \frac{1}{a\varepsilon}\delta^{a\varepsilon}\,\delta(1-x) + \theta(1-\delta-x)\,(1-x)^{-1+a\varepsilon} \tag{5.8}$$

Therefore, one has

$$\hat{\mathcal{F}}_2^{(1),S+H} \equiv \hat{\mathcal{F}}_2^{(1),S} + \hat{\mathcal{F}}_2^{(1),H} \tag{5.9}$$

with

$$\hat{\mathcal{F}}_2^{(1),S} = \delta(1-x)\,g_b^2 s_n C_F(-q^2)^{\frac{1}{2}\varepsilon}\,\delta^{\frac{1}{2}\varepsilon}\,\frac{16}{\varepsilon^2}\frac{\Gamma(1+\frac{1}{2}\varepsilon)}{\Gamma(2+\varepsilon)}\left\{1+\frac{5}{8}\varepsilon+\frac{1}{16}\varepsilon^2\right\} \tag{5.10}$$

and

$$\hat{\mathcal{F}}_2^{(1),H} = \theta(1-\delta-x)\,\hat{\mathcal{F}}_2^{(1),S+H} \tag{5.11}$$

Finally, the order α_s contribution from the qV process is equal to

$$\hat{\mathcal{F}}_2^{(1),q}(x,Q^2,\varepsilon) = \hat{\mathcal{F}}_2^{(1),V} + \hat{\mathcal{F}}_2^{(1),S} + \hat{\mathcal{F}}_2^{(1),H} = $$
$$= \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{Q^2}{\mu^2}\right)^{\frac{1}{2}\varepsilon}\left\{\frac{1}{\varepsilon}P_0^{qq}(x) + f_0^q(x)\right\} \tag{5.12}$$

with

$$P_0^{qq}(x) = C_F\Big\{6\ \delta(1-x) + 4(1+x^2)\ \mathcal{D}_0(x)\Big\} \tag{5.13}$$

and

$$f_0^q(x) = \delta(1-x)\ C_F\Big[-4\zeta(2) - 9\Big] + $$
$$+C_F\Big[4\mathcal{D}_1(x) - 3\mathcal{D}_0(x) - 2(1+x)\ln(1-x) +$$
$$-2\frac{(1+x^2)}{(1-x)}\ln x + 4x + 6\Big] \tag{5.14}$$

In the above expressions we have used the shorthand notation

$$\mathcal{D}_i(x) = \delta(1-x)\frac{\ln^{i+1}\delta}{(i+1)} + \theta(1-\delta-x)\frac{\ln^i(1-x)}{1-x} \tag{5.15}$$

In eq. (5.12) we have have replaced g_b^2 by α_s using

$$g_b^2 s_n \exp\left(\frac{1}{2}\varepsilon\gamma_E\right) \longrightarrow \left(\frac{\alpha_s}{4\pi}\right)\left(\mu^2\right)^{-\frac{1}{2}\varepsilon} \tag{5.16}$$

where μ is an arbitrary mass scale and γ_E is the Euler constant. The function $P_0^{qq}(x)$ is the order α_s part of the splitting function $P^{qq}(x)$, which is related to the anomalous dimension γ_{qq}^n (see eq. (4.12)) through the Mellin transform

$$\gamma_{qq}^n = -\int_0^1 dx\ x^{n-1}\ P^{qq}(x) \tag{5.17}$$

Notice that in eq. (5.12) the I.R. poles $1/\varepsilon^2$ have cancelled between $\hat{\mathcal{F}}_2^{(1),V}$ and $\hat{\mathcal{F}}_2^{(1),S}$. The remaining pole, with residue $P_0^{qq}(x)$, is due to the initial state collinear divergences.

The contribution $\hat{\mathcal{F}}_2^{(1),g}$ from the gV subprocess

$$\mathrm{g}(k) + \mathrm{V}(q) \rightarrow \mathrm{q}(p_1) + \bar{\mathrm{q}}(p_2) \tag{5.18}$$

is given by

$$\hat{\mathcal{F}}_2^{(1),g}(x,Q^2,\varepsilon) =$$
$$-\frac{1}{4\pi}\frac{1}{(1+\frac{1}{2}\varepsilon)}\int dPS_2^{\mathrm{DI}}\left\{g^{\mu\nu} + 12\frac{x^2}{q^2}\left(1+\frac{1}{3}\varepsilon\right)k^\mu k^\nu\right\}\langle M_\mu^g M_\nu^{g\dagger}\rangle_{av} \tag{5.19}$$

Performing the phase space integration we find $\hat{\mathcal{F}}_2^{(1),g}$ to be

$$\begin{aligned}\hat{\mathcal{F}}_2^{(1),g}(x,Q^2,\varepsilon) &= g_b^2 s_n (1-x)^{\frac{1}{2}\varepsilon}\, x^{-\frac{1}{2}\varepsilon}\,(-q^2)^{\frac{1}{2}\varepsilon}\,\frac{1}{\varepsilon}\frac{1}{(1+\frac{1}{2}\varepsilon)}\frac{\Gamma(1+\frac{1}{2}\varepsilon)}{\Gamma(2+\varepsilon)} \\ &\quad\times\left\{4(1-2x+2x^2)+6\varepsilon+4\varepsilon^2+\varepsilon^3\right\} = \\ &= \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{Q^2}{\mu^2}\right)^{\frac{1}{2}\varepsilon}\left\{\frac{2}{\varepsilon}P_0^{qg}+f_0^g(x)\right\}\end{aligned} \tag{5.20}$$

with

$$P_0^{qg}(x) = 2(1-2x+2x^2) \tag{5.21}$$

and

$$f_0^g(x) = 2(1-2x+2x^2)\ln\left(\frac{1-x}{x}\right) + 12x(1-x) \tag{5.22}$$

Of course, the splitting function P_0^{qg} is related to the anomalous dimension γ_{qg}^n through a similar formula as eq. (5.17). Notice that in eq. (5.20) the residue P_0^{qg} appears with a factor 2, this is due to the fact that both the quark and the anti-quark can become collinear to the gluon.

Using the parton model the hadronic structure function $\mathcal{F}_2(\tau,Q^2)$ can be written as

$$\begin{aligned}\mathcal{F}_2(\tau,Q^2) &= \sum_{q,\bar{q}} e_q^2 \int_0^1 dx_1 \int_0^1 dx\; \delta(\tau - x_1 x) \\ &\quad \left[f_q(x_1)\left\{\delta(1-x)+\hat{\mathcal{F}}_2^{(1),q}(x,Q^2,\varepsilon)\right\} + \right. \\ &\quad \left. +\frac{1}{2} f_g(x_1)\,\hat{\mathcal{F}}_2^{(1),g}(x,Q^2,\varepsilon)\right]\end{aligned} \tag{5.23}$$

On the other hand, in the DIS mass factorization scheme the scale dependent quark distributions functions are defined by (see eq. (4.23)) the relation

$$\mathcal{F}_2(\tau,Q^2) = \sum_{q,\bar{q}} e_q^2\, f_q(\tau,Q^2) \tag{5.24}$$

Comparing the above two formuale for $\mathcal{F}_2(\tau,Q^2)$ we find for the scale dependent quark distribution functions

$$\begin{aligned}f_q(\tau,Q^2) &= \int_0^1 dx_1 \int_0^1 dx\; \delta(\tau - x_1 x)\left[f_q(x_1)\left\{\delta(1-x)+\hat{\mathcal{F}}_2^{(1),q}(x,Q^2,\varepsilon)\right\} + \right. \\ &\quad \left. +\frac{1}{2} f_g(x_1)\,\hat{\mathcal{F}}_2^{(1),g}(x,Q^2,\varepsilon)\right]\end{aligned} \tag{5.25}$$

This expression for the scale dependent parton distribution functions will be used for the mass factorization of the Drell-Yan cross-section.

5.2 The Drell-Yan process

As in the case of the DIS process, there are two parton subprocesses, which we have to consider, viz. the q$\bar{q}$ and the qg processes.

The q$\bar{q}$ subprocess is the counterpart of the qV process and also consists of a virtual gluon part

$$q(p_1) + \bar{q}(p_2) \to V(q) \quad \text{with one loop corrections} \tag{5.26}$$

and a one gluon bremsstrahlung piece

$$q(p_1) + \bar{q}(p_2) \to V(q) + g(k) \tag{5.27}$$

The virtual gluon part can be handled analogously to the DIS case. Using the normalization given in Appendix A we find

$$\hat{W}^{(1),V}(x, Q^2, \varepsilon) = 2\ \delta(1-x)\ \mathrm{Re}F^{(1)}(q^2) \tag{5.28}$$

Contrary to the DIS process the q^2 is now positive, therefore one has to use the analytic continuation

$$(-q^2)^{a\varepsilon} = (q^2)^{a\varepsilon} \left\{ \frac{\Gamma(1+a\varepsilon)\Gamma(1-a\varepsilon)}{\Gamma(1+2a\varepsilon)\Gamma(1-2a\varepsilon)} - i\pi \frac{1}{\Gamma(1+a\varepsilon)\Gamma(1-a\varepsilon)} \right\} \tag{5.29}$$

to obtain the correct result.

The contribution of the one gluon bremsstrahlung process is given by (see Appendix A)

$$\hat{W}^{(1),S+H}(x, Q^2, \varepsilon) = -\frac{N}{2\pi(1+\frac{1}{2}\varepsilon)} \int dPS_2^{\mathrm{DY}}\ g^{\mu\nu}\ \langle M_\mu^{q\bar{q}} M_\nu^{q\bar{q}\dagger} \rangle_{av} \tag{5.30}$$

here dPS_2^{DY} is the DY 2-particle phase space integral (see Appendix D) and $M_\mu^{q\bar{q}}$ is the matrix element of this process. Again the brackets $\langle \cdots \rangle_{av}$ stand for averaging over the initial state quantum numbers and summation over the final state ones.

As for the qV process there is the possibility of the gluon becoming soft. Following the prescription, given in the previous subsection, we separate the one gluon bremsstrahlung contribution into a soft and a hard part

$$\hat{W}^{(1),S+H} \equiv \hat{W}^{(1),S} + \hat{W}^{(1),H} \tag{5.31}$$

Performing the phase space integration and using eq. (5.8), we get

$$\hat{W}^{(1),S} = \delta(1-x)\ g_b^2 s_n C_F (Q^2)^{\frac{1}{2}\varepsilon}\ \delta^\varepsilon\ \frac{16}{\varepsilon^2} \frac{\Gamma(1+\frac{1}{2}\varepsilon)}{\Gamma(1+\varepsilon)} \tag{5.32}$$

and

$$\begin{aligned} \hat{W}^{(1),H} &= \theta(1-\delta-x)\ g_b^2 s_n C_F (1-x)^{-1+\varepsilon}\ x^{-\frac{1}{2}\varepsilon}\ (Q^2)^{\frac{1}{2}\varepsilon}\ \frac{1}{\varepsilon} \frac{\Gamma(1+\frac{1}{2}\varepsilon)}{\Gamma(2+\varepsilon)} \\ &\times \left\{ 8(1+x^2) + 8(1+x^2)\varepsilon + 4(1-x)^2\varepsilon^2 \right\} \end{aligned} \tag{5.33}$$

Combining the obtained results, the $O(\alpha_s)$ contribution from the $q\bar{q}$ subprocess, $\hat{W}^{(1),q\bar{q}}$, is found to be equal to

$$\hat{W}^{(1),q\bar{q}}(x,Q^2,\varepsilon) = \hat{W}^{(1),V} + \hat{W}^{(1),S} + \hat{W}^{(1),H} = \\ = \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{Q^2}{\mu^2}\right)^{\frac{1}{2}\varepsilon}\left\{\frac{2}{\varepsilon}P_0^{qq}(x) + w_0^{q\bar{q}}(x)\right\} \tag{5.34}$$

with

$$w_0^{q\bar{q}}(x) = \delta(1-x)\,C_F\left[8\zeta(2) - 16\right] + \\ +C_F\left[16\mathcal{D}_1(x) - 8(1+x)\ln(1-x) - 4\frac{(1+x^2)}{(1-x)}\ln x\right] \tag{5.35}$$

As expected from the mass factorization theorem, the residue of the collinear pole is equal to that of the qV process. Of course, here we have an extra factor 2, because the DY process has two initial state partons instead of only one in the DIS case.

The last parton subprocess that has to be calculated is the qg process

$$q(p_1) + g(k) \to V(q) + q(p_2) \tag{5.36}$$

The parton structure function of this subprocess is indicated by $\hat{W}^{(1),qg}$. Using the definition in Appendix A we get

$$\hat{W}^{(1),qg}(x,Q^2,\varepsilon) = g_b^2 s_n (1-x)^{\varepsilon}\, x^{-\frac{1}{2}\varepsilon}\,(Q^2)^{\frac{1}{2}\varepsilon}\,\frac{1}{\varepsilon}\frac{\Gamma(1+\frac{1}{2}\varepsilon)}{\Gamma(2+\varepsilon)} \\ \times\left\{2(1-2x+2x^2) + \frac{1}{2}(7-6x+5x^2)\varepsilon + \frac{1}{2}(1+x)^2\varepsilon^2\right\} \\ = \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{Q^2}{\mu^2}\right)^{\frac{1}{2}\varepsilon}\left\{\frac{1}{\varepsilon}P_0^{qg}(x) + w_0^{qg}(x)\right\} \tag{5.37}$$

with

$$w_0^{qg}(x) = (1-2x+2x^2)\ln\left(\frac{(1-x)^2}{x}\right) + \frac{1}{2}(3+2x-3x^2) \tag{5.38}$$

In accordance with the mass factorization theorem, one finds that the residues of the collinear poles are the same for the gV and the qg processes.

5.3 Mass factorization at order α_s

In the previous subsections we have presented the parton structure functions $\hat{\mathcal{F}}_2$ and $\hat{W}$ at order α_s. We have seen that the pole structure of the DIS and DY processes behaves as expected from the mass factorization theorem.

The next step is to determine the Drell-Yan cross-section in the DIS mass factorization scheme. According to the parton model the first order Drell-Yan cross-section is equal to

$$\frac{d\sigma^V}{dQ^2} = \tau\,\sigma_V(Q^2, M_V^2)\,W_V(\tau, Q^2) \tag{5.39}$$

with

$$
\begin{aligned}
W_V(\tau, Q^2) &= \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx\ \delta(\tau - x_1 x_2 x) \\
&\Big[PD_V^{q\bar{q}}(x_1, x_2) \Big\{ \delta(1-x) + \hat{W}^{(1),q\bar{q}}(x, Q^2, \varepsilon) \Big\} + \\
&+ PD_V^{qg}(x_1, x_2)\ \hat{W}^{(1),qg}(x, Q^2, \varepsilon) \Big] \qquad (5.40)
\end{aligned}
$$

At this level the PD_V's are combinations of scale independent parton distribution functions*. However, this will change, because the initial state collinear divergences, appearing in $\hat{W}^{(1),q\bar{q}}$ and $\hat{W}^{(1),qg}$, have to be absorbed into the parton distribution functions, which then become scale dependent. In the DIS mass factorization scheme the absoption of the collinear poles takes place as defined in eq. (5.25). We then find

$$
\begin{aligned}
W_V(\tau, Q^2) &= \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx\ \delta(\tau - x_1 x_2 x) \\
&\Big[PD_V^{q\bar{q}}(x_1, x_2, Q^2) \Big\{ \delta(1-x) + \Delta_0^{q\bar{q}}(x, Q^2) \Big\} + \\
&+ PD_V^{qg}(x_1, x_2, Q^2)\ \Delta_0^{qg}(x, Q^2) \Big] \qquad (5.41)
\end{aligned}
$$

where $PD_V(x_1, x_2, Q^2)$ can be obtained by replacing in $PD_V(x_1, x_2)$ the scale independent parton distributions by the scale dependent ones. Furthermore, the first order Drell-Yan correction terms $\Delta_0^{q\bar{q}}$ and Δ_0^{qg} are given by

$$
\begin{aligned}
\Delta_0^{q\bar{q}}(x, Q^2) &= \hat{W}^{(1),q\bar{q}}(x, Q^2, \varepsilon) - 2\, \hat{\mathcal{F}}_2^{(1),q}(x, Q^2, \varepsilon) = \\
&= \left(\frac{\alpha_s}{4\pi}\right) \Big\{ w_0^{q\bar{q}}(x) - 2 f_0^{q}(x) \Big\} = \\
&= \left(\frac{\alpha_s}{4\pi}\right) C_F \Big\{ \delta(1-x) \Big[16\zeta(2) + 2 \Big] + 8\mathcal{D}_1(x) + 6\mathcal{D}_0(x) + \\
&\qquad - 4(1+x)\ln(1-x) - 8x - 12 \Big\} \qquad (5.42)
\end{aligned}
$$

and

$$
\begin{aligned}
\Delta_0^{qg}(x, Q^2) &= \hat{W}^{(1),qg}(x, Q^2, \varepsilon) - \frac{1}{2} \hat{\mathcal{F}}_2^{(1),g}(x, Q^2, \varepsilon) = \\
&= \left(\frac{\alpha_s}{4\pi}\right) \Big\{ w_0^{qg}(x) - \frac{1}{2} f_0^{g}(x) \Big\} = \\
&= \left(\frac{\alpha_s}{4\pi}\right) \Big\{ (1 - 2x + 2x^2)\ln(1-x) + \frac{9}{2}x^2 - 5x + \frac{3}{2} \Big\} \qquad (5.43)
\end{aligned}
$$

As predicted by the mass factorization theorem the Drell-Yan correction terms are free of collinear divergences.

---

*For the exact expressions for the PD_V's see Appendix A.

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Chapter III

The second order corrections to the Drell-Yan cross-section

1 What can be learned from the first order corrections?

Before embarking on a journey through a jungle of hideous two loop and 3-particle phase space integrations, let us investigate the first order corrections given in section II.5.

A useful quantity for the study of higher order corrections to the Drell-Yan cross-section is the theoretical K-factor, which is defined by

$$K_{th} = \sum_{n=0}^{\infty} K^{(n)} \tag{1.1}$$

where $K^{(n)}$ is the $O(\alpha_s^n)$ contribution to the K-factor. It is given by

$$K^{(n)} = \frac{W_V^{(n)}(\tau, Q^2)}{W_V^{(0)}(\tau, Q^2)} \tag{1.2}$$

here $W_V^{(n)}(\tau, Q^2)$ is the order α_s^n term of the hadronic structure function $W_V(\tau, Q^2)$.

To give an impression of the size of the first order correction, we have plotted in fig. 1 the K-factor for the NA10 experiment* ($\sqrt{S}$ =19.1GeV) [1] and the Z production at the CERN p$\bar{\text{p}}$-collider ($\sqrt{S}$ =630GeV). As is known from the literature [2] and also can be seen in fig. 1 the $O(\alpha_s)$ correction is rather large, especially at low energies. To find the source of these large corrections we divide the first order K-factor, $K^{(1)}$, into three parts, viz. $K^{(1)}_{qg}$, $K^{(1)}_{soft}$ and $K^{(1)}_{reg}$. The $K^{(1)}_{qg}$ part is due to the qg subprocess (see eq. (II.5.43)). The other two terms belong to the Drell-Yan correction term $\Delta_0^{q\bar{q}}$ (see eq. (II.5.42)), which we split up in the following way

$$\Delta_0^{q\bar{q}}(x, Q^2) = \Delta_{soft}^{q\bar{q}}(x, Q^2) + \Delta_{reg}^{q\bar{q}}(x, Q^2) \tag{1.3}$$

with

*In the NA10 experiment a pion beam is put on a tungsten target at a C.M. energy of 19.1GeV

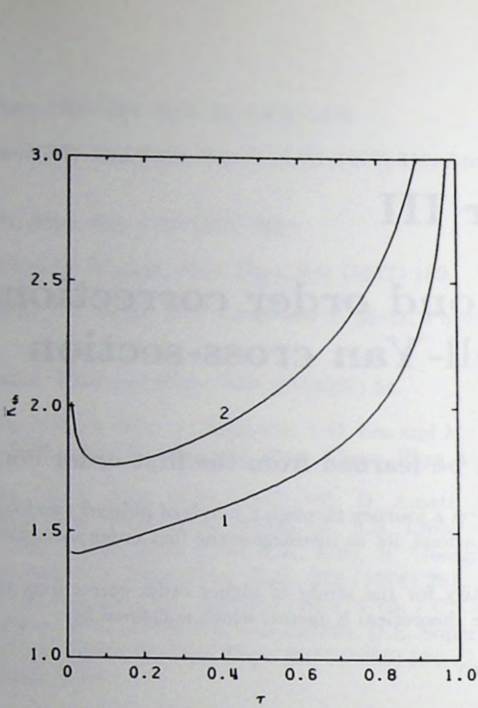

Fig. 1. The first order K-factor:
line 1 : $K^{(0)} + K^{(1)}$ for CERN $p\bar{p}$ collider
line 2 : $K^{(0)} + K^{(1)}$ for NA10 experiment

$$\Delta^{q\bar{q}}_{soft}(x,Q^2) = \left(\frac{\alpha_s}{4\pi}\right) C_F \Big\{\delta(1-x)\ \Big[16\zeta(2)+2\Big] + 8\mathcal{D}_1(x) + 6\mathcal{D}_0(x)\Big\} \tag{1.4}$$

and

$$\Delta^{q\bar{q}}_{reg}(x,Q^2) = \left(\frac{\alpha_s}{4\pi}\right) C_F \Big\{ -4(1+x)\ln(1-x) - 8x - 12\Big\} \tag{1.5}$$

In fig. 2 we show $K^{(1)}_{q\bar{q}}$, $K^{(1)}_{soft}$ and $K^{(1)}_{reg}$ for the Z production at the CERN $p\bar{p}$ collider. From this figure we infer that the large corrections are due to the $\delta(1-x)$ term and the distributions $\mathcal{D}_i(x)$ appearing in eq. (1.4). The calculations in section II.5 show that these contributions can be isolated be taking the limit $x \to 1$ (the gluon becomes soft). Therefore we will refer to them as the soft gluon contributions.

Extrapolating this first order result to higher orders in α_s, we assume that the $O(\alpha_s^2)$ correction is also dominated by the soft gluon contributions. In the subsequent part of this chapter we will therefore be mainly interested in the second order soft gluon contributions.

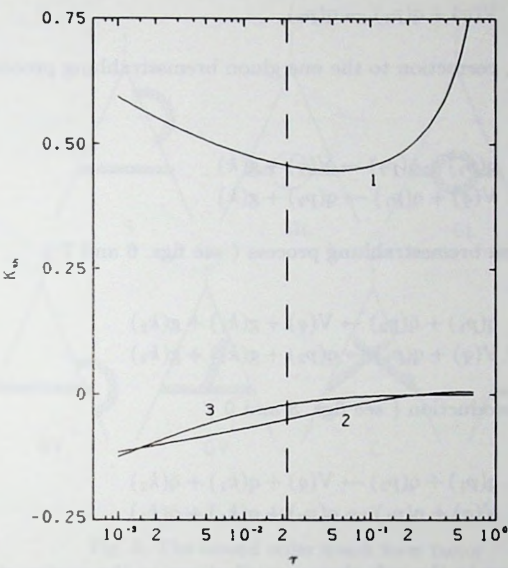

Fig. 2. $K^{(1)}_{soft}$ [1], $K^{(1)}_{reg}$ [2] and $K^{(1)}_{q g}$ [3] for the Z production at the CERN $p\bar{p}$ collider

2 Notations and conventions

As a complete calculation of the second order corrections to the Drell-Yan process would be rather cumbersome and very time consuming, we have set ourselves a less ambitious goal, namely, the calculation of the dominant $O(\alpha_s^2)$ contributions. We have already discussed in the previous section that we expect the large corrections to come from the soft gluon contributions. As in the case of the first order corrections the $O(\alpha_s^2)$ soft gluon contributions are due to the $q\bar{q}$ subprocess. Therefore we will restrict ourselves to the calculation of the second order part of the parton structure function $\hat{W}^{q\bar{q}}(x, Q^2, \varepsilon)$ (see eq. (II.3.1)). Of course, because we have chosen for the DIS mass factorization scheme, also the corresponding DIS parton structure function $\hat{\mathcal{F}}_2^q(x, Q^2, \varepsilon)$ has to be calculated up to $O(\alpha_s^2)$ (see eq. (II.3.2)).

To determine the second order contributions to $\hat{W}^{q\bar{q}}(x, Q^2, \varepsilon)$ and $\hat{\mathcal{F}}_2^q(x, Q^2, \varepsilon)$ the following four processes have to be taken into account

1. The $O(\alpha_s^2)$ quark form factor (see fig. 3), which is given by the order α_s^2 correction to the processes

$$\begin{aligned} \text{DY} &: \; q(p_1) + \bar{q}(p_2) \to V(q) \\ \text{DI} &: \; V(q) + q(p_1) \to q(p_2) \end{aligned}$$

2. The order α_s correction to the one gluon bremsstrahlung process (see figs. 4 and 5)

$$\begin{aligned} \text{DY} &: \; q(p_1) + \bar{q}(p_2) \to V(q) + g(k) \\ \text{DI} &: \; V(q) + q(p_1) \to q(p_2) + g(k) \end{aligned}$$

3. The two gluon bremsstrahlung process (see figs. 6 and 7)

$$\begin{aligned} \text{DY} &: \; q(p_1) + \bar{q}(p_2) \to V(q) + g(k_1) + g(k_2) \\ \text{DI} &: \; V(q) + q(p_1) \to q(p_2) + g(k_1) + g(k_2) \end{aligned}$$

4. Quark pair production (see figs. 8 and 9)

$$\begin{aligned} \text{DY} &: \; q(p_1) + \bar{q}(p_2) \to V(q) + q(k_1) + \bar{q}(k_2) \\ \text{DI} &: \; V(q) + q(p_1) \to q(p_2) + q(k_1) + \bar{q}(k_2) \end{aligned}$$

Restricting ourselves to the soft gluon contributions to the parton structure functions $\hat{W}^{q\bar{q}}(x, Q^2, \varepsilon)$ and $\hat{\mathcal{F}}_2^q(x, Q^2, \varepsilon)$ simplifies the calculation, because in this approximation one may neglect all the terms which vanish in the soft limit $x \to 1$. In particular, one may omit the $p^\mu p^\nu$ projection in the definition of $\hat{\mathcal{F}}_2^q(x, Q^2, \varepsilon)$ (see eq. (B.3)). In fact, if one is only interested in the soft gluon contributions, the definitions for $\hat{W}^{q\bar{q}}(x, Q^2, \varepsilon)$ and $\hat{\mathcal{F}}_2^q(x, Q^2, \varepsilon)$ become quite similar (see eqs. (A.12) and (B.3)). In the soft limit $x \to 1$, the contribution $\sigma^{\text{DY/DI}}$ from a particular subprocess to $\hat{W}^{q\bar{q}}$ or $\hat{\mathcal{F}}_2^q$ can be written as

$$\sigma^{\text{DY/DI}} = C \int dPS_m^{\text{DY/DI}} \, |A|^2 \tag{2.1}$$

with

$$C = -\frac{1}{8\pi} \frac{1}{N} \frac{1}{\left(1 + \frac{1}{2}\varepsilon\right)}$$

$\int dPS_m^{\text{DY/DI}}$: m-particle phase space integral (see Appendix D).

$|A|^2 = S \sum A^\mu A_\mu^\dagger$
where the summation is over the polarizations and colours,
A_μ is the amplitude of the process
and S is a statistical factor.

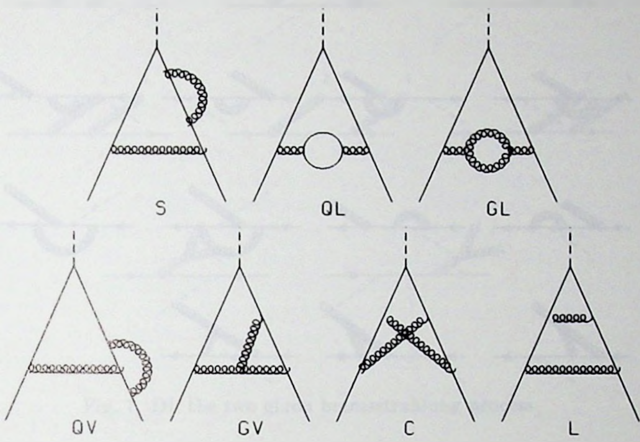

Fig. 3. The second order quark form factor

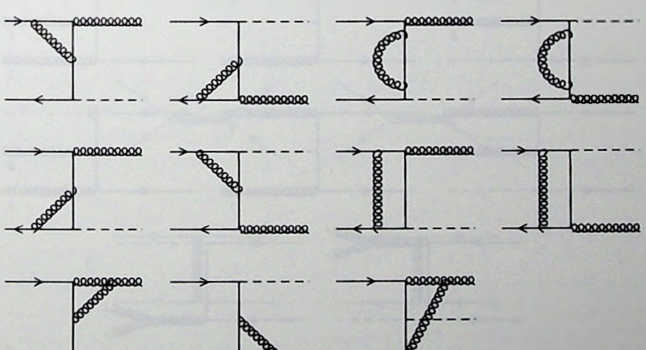

Fig. 4. DY: order α_s correction to the one gluon bremsstrahlung process

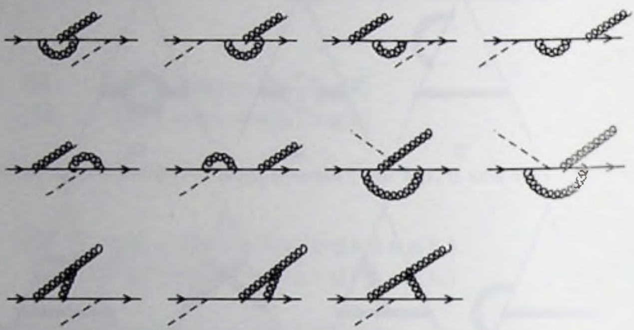

Fig. 5. DI: order α_s correction to the one gluon bremsstrahlung process

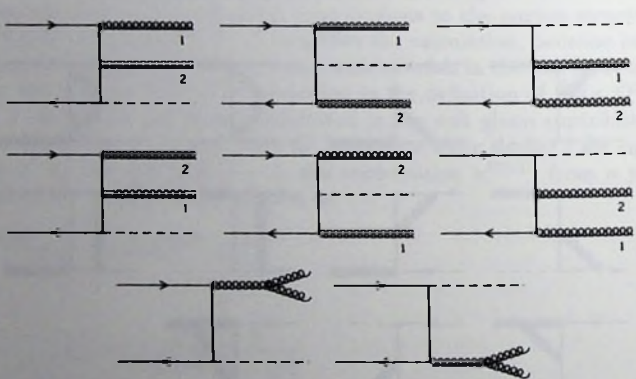

Fig. 6. DY: the two gluon bremsstrahlung process

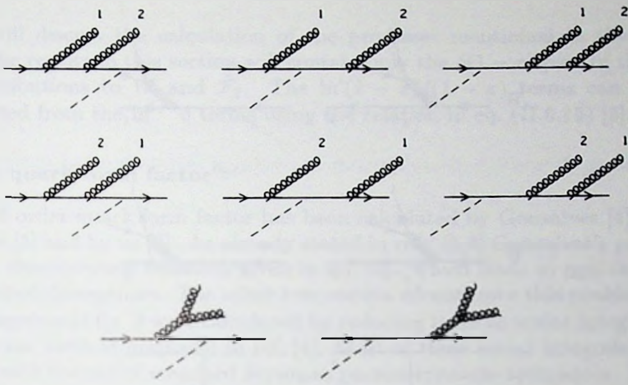

Fig. 7. DI: the two gluon bremsstrahlung process

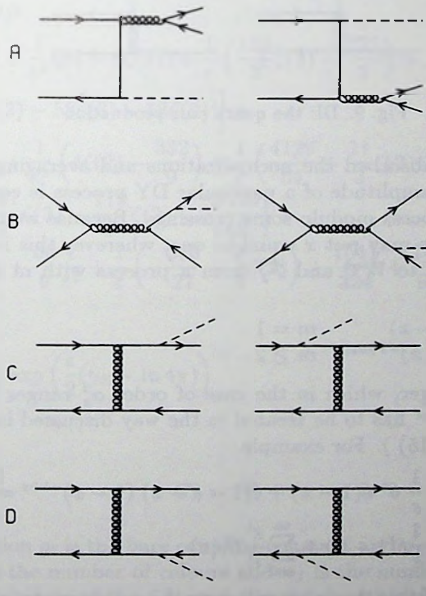

Fig. 8. DY: the quark pair production

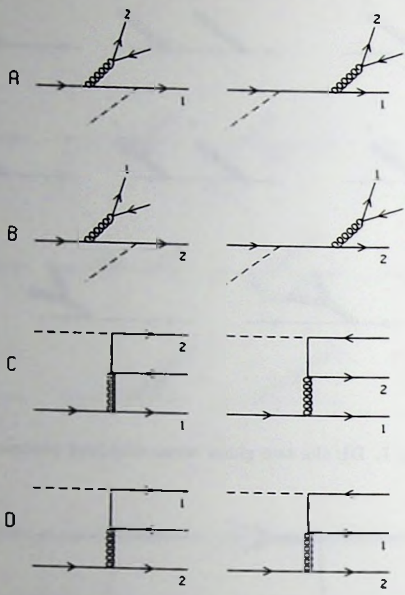

Fig. 9. DI: the quark pair production

Notice that we have absorbed the normalizations and averagings in the constant C. Furthermore, the amplitude of a particular DY process is equal to that of the corresponding DIS process modulo some crossings. Because we only want the soft gluon contributions we may put x equal to one, wherever this is possible. In this limit the contribution to $\hat{W}^{q\bar{q}}$ and $\hat{\mathcal{F}}_2^q$ from a process with m outgoing particles behaves as

$$\sigma^{DY/DI} \sim \begin{cases} \delta(1-x) & m=1 \\ (1-x)^{-1+a_m\varepsilon} & m \geq 2 \end{cases} \tag{2.2}$$

here a_m is a half integer, which in the case of order α_s^2 ranges between $\frac{1}{2}$ and 2. The term $(1-x)^{-1+a_m\varepsilon}$ has to be treated in the way discussed in section II.5 (see eqs. (II.5.8) and (II.5.15)). For example

$$\begin{aligned} (1-x)^{-1+\varepsilon} &= \frac{1}{\varepsilon}\,\delta^\varepsilon\,\delta(1-x) + \theta(1-\delta-x)\,(1-x)^{-1+\varepsilon} = \\ &= \frac{1}{\varepsilon}\delta(1-x) + \sum_{i=0}^{\infty}\frac{\varepsilon^i}{i!}\mathcal{D}_i(x) \end{aligned} \tag{2.3}$$

In the remainder of this thesis we will drop the superscripts 'q$\bar{\text{q}}$' and 'q', if there is no cause for confusion.

3 The second order soft gluon contributions

Here we will discuss the calculation of the processes mentioned in the previous section. The results in this section will contain only the $\delta(1-x)$ part of the second order contributions to $\hat{W}$ and $\hat{\mathcal{F}}_2$. The $\ln^i(1-x)/(1-x)$ terms can be easily reconstructed from the $\ln^{i+1}\delta$ terms using the relation in eq. (II.5.15) [3].

3.1 The quark form factor

The second order quark form factor has been calculated by Gonsalves [4], Kramer and Lampe [5] and by us [3]. As already stated in refs. [5,6] Gonsalves's result does not satisfy the evolution equation given in ref. [6] , which leads to non-cancelation of the infrared divergences. The other two results do not have this problem.

The diagrams in fig. 3 were calculated by reducing them to scalar integrals using the projection method discussed in ref. [4]. Most of these scalar integrals could be computed with the aid of standard Feynman parametrization techniques. But some of them, notably those due to the diagrams C and L, had to be handled by applying the Cutkosky rules together with dispersion relations [7]. In Appendix E we present the contributions to the second order quark form factor from the diagrams given in fig. 3. Putting these together, the unrenormalized $O(\alpha_s^2)$ quark form factor is given by

$$\begin{aligned} F^{(2)}(q^2) = g_n^4\,(-q^2)^\varepsilon & \\ \Big\{ C_F^2 \Big[& \frac{32}{\varepsilon^4} - \frac{48}{\varepsilon^3} + \frac{1}{\varepsilon^2}(82 - 8\zeta(2)) + \frac{1}{\varepsilon}\left(\frac{128}{3}\zeta(3) - \frac{221}{2}\right) + \\ & + \frac{1151}{8} + \frac{17}{2}\zeta(2) - 58\zeta(3) - 13\zeta(2)^2 \Big] + \\ + C_A C_F \Big[& \frac{44}{3}\frac{1}{\varepsilon^3} + \frac{1}{\varepsilon^2}\left(4\zeta(2) - \frac{332}{9}\right) + \frac{1}{\varepsilon}\left(\frac{4129}{54} + \frac{11}{3}\zeta(2) - 26\zeta(3)\right) + \\ & + \frac{44}{5}\zeta(2)^2 + \frac{467}{9}\zeta(3) - \frac{119}{9}\zeta(2) - \frac{89173}{648} \Big] + \\ + n_f C_F \Big[& -\frac{8}{3}\frac{1}{\varepsilon^3} + \frac{56}{9}\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(-\frac{353}{27} - \frac{2}{3}\zeta(2)\right) + \frac{7541}{324} + \frac{14}{9}\zeta(2) - \frac{26}{9}\zeta(3) \Big] \Big\} \end{aligned} \tag{3.1}$$

with

$$g_n^2 = g_b^2 \frac{1}{(4\pi)^2} \exp\left(\frac{\varepsilon}{2}(\gamma_E - \ln 4\pi)\right) \tag{3.2}$$

and

$$C_F = \frac{N^2-1}{2N} \qquad\qquad C_A = N \tag{3.3}$$

In the above equation g_b is the bare coupling constant and γ_E is the Euler constant. Furthermore, N is the number of colours and n_f is the number of flavours. Notice that the first three terms of the C_F^2 part can obtained by exponentiating the first order quark form factor $F^{(1)}(q^2)$ (see eq. (II.5.3)).

The above formula can be immediately applied in the case of the DIS process, but for the Drell-Yan process ($q^2 > 0$) one first has to perform analytic continuation (see eq. (II.5.29)). The contribution from the quark form factor to the second order $\hat{W}$ and $\hat{\mathcal{F}}_2$ is given by

$$\sigma_A = \delta(1-x)\left\{|F^{(1)}(q^2)|^2 + 2Re\left(F^{(2)}(q^2)\right)\right\} \tag{3.4}$$

3.2 The first order correction to the one gluon bremsstrahlung process

We will denote the contribution from this process (see figs. 4 and 5) by

$$\sigma_B = C\int dPS_2\left\{2\ Re\left(A^0_g A^{1\dagger}_g\right)\right\} \tag{3.5}$$

where A^0_g and A^1_g are the Born and $O(\alpha_s)$ corrected amplitudes, respectively.

To calculate σ_B we first used an n-dimensional version of the Passarino-Veltman reduction scheme [8] to handle the virtual corrections. A list of useful one loop integrals is given in Appendix F. Then we carried out the 2-particle phase space integration, putting the variable x equal to one, where possible. We find

$$\begin{aligned}\sigma_B^{DY} &= \delta(1-x)\ g_n^4\ (Q^2)^\epsilon \\ &\left\{C_F^2\ \delta^\epsilon\left[-\frac{256}{\varepsilon^4} + \frac{192}{\varepsilon^3} + \frac{1}{\varepsilon^2}(320\zeta(2) - 256) + \right.\right.\\ &+\frac{1}{\varepsilon}\left(256 - 240\zeta(2) - \frac{448}{3}\zeta(3)\right) - 256 + 320\zeta(2) + 112\zeta(3) - \left.\frac{536}{5}\zeta(2)^2\right] + \\ &\left.+C_A C_F\ \delta^{2\epsilon}\left[-\frac{32}{\varepsilon^4} + 56\zeta(2)\frac{1}{\varepsilon^2} - \frac{224}{3}\zeta(3)\frac{1}{\varepsilon} + \frac{21}{5}\zeta(2)^2\right]\right\}\end{aligned} \tag{3.6}$$

and

$$\begin{aligned}\sigma_B^{DI} &= \delta(1-x)\ g_n^4\ (-q^2)^\epsilon \\ &\left\{C_F^2\left[\delta^{\frac{1}{2}\epsilon}\left\{-\frac{256}{\varepsilon^4} + \frac{288}{\varepsilon^3} + \frac{1}{\varepsilon^2}(128\zeta(2) - 440) + \right.\right.\right.\\ &+\frac{1}{\varepsilon}\left(548 - 144\zeta(2) - \frac{448}{3}\zeta(3)\right) - 660 + 220\zeta(2) + 168\zeta(3) + \left.\frac{64}{5}\zeta(2)^2\right\} + \\ &+\delta^\epsilon\left.\left\{\frac{1}{\varepsilon^2}(32\zeta(2) - 20) + \frac{1}{\varepsilon}(40 - 80\zeta(3)) - 61 + 25\zeta(2) + \frac{48}{5}\zeta(2)^2\right\}\right] + \\ &+C_A C_F\ \delta^\epsilon\left[-\frac{32}{\varepsilon^4} + \frac{24}{\varepsilon^3} + \frac{1}{\varepsilon^2}(24\zeta(2) - 32) + \right.\\ &\left.\left.+\frac{1}{\varepsilon}\left(40 - 30\zeta(2) + \frac{64}{3}\zeta(3)\right) - 47 + 40\zeta(2) + 14\zeta(3) - \frac{91}{5}\zeta(2)^2\right]\right\}\end{aligned} \tag{3.7}$$

The σ_B's exhibit two interesting features. Firstly, the $\delta^{\epsilon/2}$ term of the Abelian (C_F^2) part of σ_B^{DI} can be written as $2F^{(1)}\hat{\mathcal{F}}_2^{(1),S}$, where $F^{(1)}$ is the $O(\alpha_s)$ quark form factor (eq. (II.5.3)) and $\hat{\mathcal{F}}_2^{(1),S}$ is the first order soft contribution to $\hat{\mathcal{F}}_2$ (eq. (II.5.10)). A similar relation holds for the δ^ϵ term of the Abelian part of σ_B^{DY}, in this case it is equal to $2F^{(1)}\hat{W}^{(1),S}$ (see eq. (II.5.32) for $\hat{W}^{(1),S}$).

Secondly, we have here a nice illustration of the KLN theorem [9]. Namely, both σ_B^{DY} and σ_B^{DI} contain a pole $1/\varepsilon^4$ in the $C_A C_F$ part. This is a singularity due to final state collinear divergences. We will see that, when all the degenerate final states, i.e. all the four subprocesses, are added, it will disappear, as dictated by the KLN theorem. In fact, it will be cancelled by a similar pole in the two gluon bremsstrahlung process.

3.3 The two gluon bremsstrahlung process

For the two gluon bremsstrahlung process (see figs. 6 and 7) one has to compute

$$\sigma_C = C \int dPS_3 \, |A_{gg}|^2 \tag{3.8}$$

where A_{gg} is the tree level amplitude of this process.

After squaring the amplitude* we used the symmetry properties of the process and performed partial fractioning to obtain a minimal set of 3-particle phase space integrals. The computation of these integrals was done in two different frames. The first one is the C.M. frame of the incoming particles. It is presented in refs. [7,10] and a convenient parametrization is given in Appendix D. The second one is the C.M. frame of the outgoing gluons, which is described in ref. [11] (see also Appendix D). The latter turned out to be very well suited for the calculation of our integrals, especially of the DY integrals. Therefore, most of them were performed in the second frame, but many were also calculated in the first one as a check.

In Appendix G we have listed all the DY integrals and the most difficult ones in the DIS case. Using these integrals we have

$$\begin{aligned}
\sigma_C^{DY} &= \delta(1-x)\, g_n^4\, (Q^2)^\varepsilon\, \delta^{2\varepsilon} \\
&\left\{ C_F^2 \left[\frac{128}{\varepsilon^4} - 224\zeta(2)\frac{1}{\varepsilon^2} + \frac{992}{3}\zeta(3)\frac{1}{\varepsilon} - \frac{36}{5}\zeta(2)^2 \right] + \right. \\
&+ C_A C_F \left[\frac{32}{\varepsilon^4} - \frac{88}{3}\frac{1}{\varepsilon^3} + \frac{1}{\varepsilon^2}\left(\frac{268}{9} - 64\zeta(2)\right) + \frac{1}{\varepsilon}\left(\frac{308}{3}\zeta(3) + \frac{154}{3}\zeta(2) - \frac{808}{27}\right) + \right. \\
&\left.\left. + \frac{2428}{81} - \frac{469}{9}\zeta(2) - \frac{682}{9}\zeta(3) - \frac{1}{5}\zeta(2)^2 \right] \right\}
\end{aligned} \tag{3.9}$$

and

$$\begin{aligned}
\sigma_C^{DI} &= \delta(1-x)\, g_n^4\, (-q^2)^\varepsilon\, \delta^{\varepsilon} \\
&\left\{ C_F^2 \left[\frac{128}{\varepsilon^4} - \frac{96}{\varepsilon^3} + \frac{1}{\varepsilon^2}(150 - 160\zeta(2)) + \frac{1}{\varepsilon}\left(\frac{464}{3}\zeta(3) + 120\zeta(2) - \frac{417}{2}\right) + \right. \right. \\
&\left. + \frac{2275}{8} - \frac{375}{2}\zeta(2) - 128\zeta(3) + \frac{244}{5}\zeta(2)^2 \right] + \\
&+ C_A C_F \left[\frac{32}{\varepsilon^4} - \frac{160}{3}\frac{1}{\varepsilon^3} + \frac{1}{\varepsilon^2}\left(\frac{754}{9} - 32\zeta(2)\right) + \frac{1}{\varepsilon}\left(\frac{68}{3}\zeta(3) + \frac{182}{3}\zeta(2) - \frac{3175}{27}\right) + \right. \\
&\left.\left. + \frac{51337}{324} - \frac{1741}{18}\zeta(2) - \frac{451}{9}\zeta(3) + \frac{41}{5}\zeta(2)^2 \right] \right\}
\end{aligned} \tag{3.10}$$

Notice that the $1/\varepsilon^4$ pole in the $C_A C_F$ part of σ_C cancels its equivalent in σ_B.

*When squaring the amplitude one has to take into account a statistical factor 1/2, due to identical particles in the final state.

3.4 Quark pair production

The calculation of this process is quite analogous to that of the two gluon bremsstrahlung process. We define

$$\sigma_D = C \int dPS_3 \, |A_{q\bar{q}}|^2 \tag{3.11}$$

where A_{qq} is the Born amplitude of the quark pair production (see figs. 8 and 9). Notice that not all the diagrams in figs. 8 and 9 contribute in the soft limit $x \to 1$. In the case of the DY process only the diagrams indicated by A have to be taken into account. For the DIS process the diagrams denoted by A and B have to be calculated. In the latter case one has a statistical factor 1/2 due to identical quarks in the final state.

A new feature is the appearence of an integral with a "massive" propagator in the DIS case. This causes the calculation of the angular part of the phase space integral to be very complicated. We tried to avoid this problem by looking for a more suitable integration frame, but we did not succeed in it. Therefore, we had to calculate this integral using brute force methods (see Appendix G for more details). The computation of σ_D gives

$$\sigma_D^{DY} = \delta(1-x)\, g_n^4\, (Q^2)^\epsilon\, \delta^{2\epsilon}\, n_f C_F \left\{ \frac{16}{3}\frac{1}{\varepsilon^3} - \frac{40}{9}\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{112}{27} - \frac{28}{3}\zeta(2)\right) + \frac{124}{9}\zeta(3) + \frac{70}{9}\zeta(2) - \frac{328}{81} \right\} \tag{3.12}$$

and

$$\begin{aligned}\sigma_D^{DI} = {} & \delta(1-x)\, g_n^4\, (-q^2)^\epsilon\, \delta^\epsilon \\ & \left\{ \left(C_F - \frac{1}{2}C_A\right) C_F \left[\frac{1}{\varepsilon}(13 - 12\zeta(2) + 8\zeta(3)) + \right.\right. \\ & \left. -\frac{175}{4} + 12\zeta(2) + 42\zeta(3) - \frac{66}{5}\zeta(2)^2\right] + \\ & \left. + n_f C_F \left[\frac{16}{3}\frac{1}{\varepsilon^3} - \frac{76}{9}\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{373}{27} - \frac{20}{3}\zeta(2)\right) - \frac{7081}{324} + \frac{95}{9}\zeta(2) + \frac{64}{5}\zeta(3)\right]\right\}\end{aligned} \tag{3.13}$$

The $n_f C_F$ part of this process has also been calculated exactly [3]. The results can be found in Appendix H.

4 The Drell-Yan correction term $\Delta(x, Q^2)$

In the previous section we have presented the results of the processes contributing to $\hat{W}$ and $\hat{\mathcal{F}}_2$ at order α_s^2. These will now be combined to form the Drell-Yan correction term $\Delta(x, Q^2)$.

The second order soft and virtual contributions to $\hat{W}$ and $\hat{\mathcal{F}}_2$ are given by

$$\hat{W}^{(2),S+V} = \sigma_A^{DY} + \sigma_B^{DY} + \sigma_C^{DY} + \sigma_D^{DY} \tag{4.1}$$

and

$$\hat{\mathcal{F}}_2^{(2),S+V} = \sigma_A^{DI} + \sigma_B^{DI} + \sigma_C^{DI} + \sigma_D^{DI} \tag{4.2}$$

Adding the results of section 3 we find that $\hat{W}^{S+V}$ and $\hat{\mathcal{F}}_2^{S+V}$ can be written as

$$\hat{W}^{S+V} = \delta(1-x) + \hat{W}^{(1),S+V} + \hat{W}^{(2),S+V} = \delta(1-x)|F|^2 B_{DY} \tag{4.3}$$

and

$$\hat{\mathcal{F}}_2^{S+V} = \delta(1-x) + \hat{\mathcal{F}}_2^{(1),S+V} + \hat{\mathcal{F}}_2^{(2),S+V} = \delta(1-x)|F|^2 B_{DI} \tag{4.4}$$

where F is the quark form factor, which determines completely the virtual corrections to $\hat{W}$ and $\hat{\mathcal{F}}_2$. The B's contain all the information about the soft behaviour of the parton structure functions. They can be written as

$$B_{DY}(Q^2,\varepsilon,\delta) = 1 + g_n^2\,(Q^2)^{\frac{1}{2}\varepsilon}\,\delta^\varepsilon\,B_{DY}^{(1)}(\varepsilon) + g_n^4\,(Q^2)^\varepsilon\,\delta^{2\varepsilon}\,B_{DY}^{(2)}(\varepsilon) \tag{4.5}$$

and

$$B_{DI}(Q^2,\varepsilon,\delta) = 1 + g_n^2\,(-q^2)^{\frac{1}{2}\varepsilon}\,\delta^{\frac{1}{2}\varepsilon}\,B_{DI}^{(1)}(\varepsilon) + g_n^4\,(-q^2)^\varepsilon\,\delta^\varepsilon\,B_{DI}^{(2)}(\varepsilon) \tag{4.6}$$

The first order parts of the above equations are equal to

$$g_n^2\,(Q^2)^{\frac{1}{2}\varepsilon}\,\delta^\varepsilon\,B_{DY}^{(1)}(\varepsilon) = \hat{W}^{(1),S} \tag{4.7}$$

and

$$g_n^2\,(-q^2)^{\frac{1}{2}\varepsilon}\,\delta^{\frac{1}{2}\varepsilon}\,B_{DI}^{(1)}(\varepsilon) = \hat{\mathcal{F}}_2^{(1),S} \tag{4.8}$$

The expressions of $\hat{\mathcal{F}}_2^{(1),S}$ and $\hat{W}^{(1),S}$ can be found in eqs. (II.5.10) and (II.5.32), respectively. The second order part of the B's is given by

$$\begin{aligned}
B_{DY}^{(2)}(\varepsilon) = {}& \\
& C_F^2\left\{\frac{128}{\varepsilon^4} - 224\zeta(2)\frac{1}{\varepsilon^2} + \frac{992}{3}\zeta(3)\frac{1}{\varepsilon} - \frac{36}{5}\zeta(2)^2\right\} + \\
& + C_A C_F\left\{-\frac{88}{3}\frac{1}{\varepsilon^3} + \frac{1}{\varepsilon^2}\left(\frac{268}{9} - 8\zeta(2)\right) + \frac{1}{\varepsilon}\left(28\zeta(3) + \frac{154}{3}\zeta(2) - \frac{808}{27}\right) + \right. \\
& \left. + \frac{2428}{81} - \frac{469}{9}\zeta(2) - \frac{682}{9}\zeta(3) + 4\zeta(2)^2\right\} + \\
& + n_f C_F\left\{\frac{16}{3}\frac{1}{\varepsilon^3} - \frac{40}{9}\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{112}{27} - \frac{28}{3}\zeta(2)\right) + \frac{124}{9}\zeta(3) + \frac{70}{9}\zeta(2) - \frac{328}{81}\right\}
\end{aligned} \tag{4.9}$$

and

$$\begin{aligned}
B_{DI}^{(2)}(\varepsilon) = {}& \\
& C_F^2\left\{\frac{128}{\varepsilon^4} - \frac{96}{\varepsilon^3} + \frac{1}{\varepsilon^2}(130 - 128\zeta(2)) + \frac{1}{\varepsilon}\left(\frac{248}{3}\zeta(3) + 108\zeta(2) - \frac{311}{2}\right) + \right. \\
& \left. + \frac{1437}{8} - \frac{301}{2}\zeta(2) - 86\zeta(3) + \frac{226}{5}\zeta(2)^2\right\} + \\
& + C_A C_F\left\{-\frac{88}{3}\frac{1}{\varepsilon^3} + \frac{1}{\varepsilon^2}\left(\frac{466}{9} - 8\zeta(2)\right) + \frac{1}{\varepsilon}\left(40\zeta(3) + \frac{110}{3}\zeta(2) - \frac{4541}{54}\right) + \right. \\
& \left. + \frac{86393}{648} - \frac{1129}{18}\zeta(2) - \frac{514}{9}\zeta(3) - \frac{17}{5}\zeta(2)^2\right\} + \\
& + n_f C_F\left\{\frac{16}{3}\frac{1}{\varepsilon^3} - \frac{76}{9}\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{373}{27} - \frac{20}{3}\zeta(2)\right) - \frac{7081}{324} + \frac{95}{9}\zeta(2) + \frac{64}{9}\zeta(3)\right\}
\end{aligned} \tag{4.10}$$

Note that the first two terms of the Abelian (C_F^2) part of the $B^{(2)}$'s exponentiate. As the leading poles of the C_F^2 part of the quark form factor could also be obtained by exponentiation, the Abelian part of both $\hat{W}^{S+V}$ and $\hat{\mathcal{F}}_2^{S+V}$ is a product of two exponents (see eqs. (4.3) and (4.4)) and therefore exponentiates too.

Until now we have looked at the unrenormalized $\hat{W}$ and $\hat{\mathcal{F}}_2$. To renormalize the results given in eqs. (4.1) and (4.2), we will use the $\overline{\mathrm{MS}}$ scheme. This comes down to replacing g_n^2 by

$$g_n^2 \longrightarrow \frac{g_R^2}{(4\pi)^2}\left(\mu^2\right)^{-\frac{1}{2}\varepsilon} Z_g^2 \equiv \left(\frac{\alpha_s}{4\pi}\right)\left(\mu^2\right)^{-\frac{1}{2}\varepsilon} Z_g^2 \tag{4.11}$$

where g_R is the renormalized coupling constant. The renormalization constant of the coupling constant, Z_g, is given by

$$Z_g = 1 + \left(\frac{\alpha_s}{4\pi}\right)\frac{1}{\varepsilon}\left(\frac{11}{3}C_A - \frac{2}{3}n_f\right) \tag{4.12}$$

After renormalization we have

$$\begin{aligned}
&\hat{W}(x,Q^2,\varepsilon) = \\
&\quad \delta(1-x) + \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{Q^2}{\mu^2}\right)^{\frac{1}{2}\varepsilon}\left[2P_0(x)\frac{1}{\varepsilon} + w_0(x)\right] + \\
&\quad + \left(\frac{\alpha_s}{4\pi}\right)^2\Big[\Big\{2(P_0\otimes P_0)(x) + 2\beta_0 P_0(x)\Big\}\frac{1}{\varepsilon^2} + \\
&\quad + \Big\{P_1(x) + 2(P_0\otimes w_0)(x)\Big\}\frac{1}{\varepsilon} + 2(P_0\otimes P_0)(x)\frac{1}{\varepsilon}\ln\left(\frac{Q^2}{\mu^2}\right) + \\
&\quad + \Big\{(P_0\otimes P_0)(x) - \frac{1}{2}\beta_0 P_0(x)\Big\}\ln^2\left(\frac{Q^2}{\mu^2}\right) + \\
&\quad + \Big\{P_1(x) + 2(P_0\otimes w_0)(x) - \beta_0 w_0(x)\Big\}\ln\left(\frac{Q^2}{\mu^2}\right) + w_1(x)\Big]
\end{aligned} \tag{4.13}$$

and

$$\begin{aligned}
&\hat{\mathcal{F}}_2(x,Q^2,\varepsilon) = \\
&\quad \delta(1-x) + \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{Q^2}{\mu^2}\right)^{\frac{1}{2}\varepsilon}\left[P_0(x)\frac{1}{\varepsilon} + f_0(x)\right] + \\
&\quad + \left(\frac{\alpha_s}{4\pi}\right)^2\Big[\Big\{\frac{1}{2}(P_0\otimes P_0)(x) + \beta_0 P_0(x)\Big\}\frac{1}{\varepsilon^2} + \\
&\quad + \Big\{\frac{1}{2}P_1(x) + (P_0\otimes f_0)(x)\Big\}\frac{1}{\varepsilon} + \frac{1}{2}(P_0\otimes P_0)(x)\frac{1}{\varepsilon}\ln\left(\frac{Q^2}{\mu^2}\right) + \\
&\quad + \Big\{\frac{1}{4}(P_0\otimes P_0)(x) - \frac{1}{4}\beta_0 P_0(x)\Big\}\ln^2\left(\frac{Q^2}{\mu^2}\right) + \\
&\quad + \Big\{\frac{1}{2}P_1(x) + (P_0\otimes f_0)(x) - \beta_0 f_0(x)\Big\}\ln\left(\frac{Q^2}{\mu^2}\right) + f_1(x)\Big]
\end{aligned} \tag{4.14}$$

More explicit expressions of $\hat{W}$ and $\hat{\mathcal{F}}_2$ can be found in Appendix J. The convolution symbol $\otimes$ is defined by

$$(f \otimes g)(x) = \int_0^1 dx_1 \int_0^1 dx_2 \; \delta(x - x_1 x_2) \; f(x_1) \; g(x_2) \tag{4.15}$$

The functions $w_0(x)$ and $f_0(x)$ have been introduced in section II.5, in eqs. (J.9) and (J.10) they can be found expanded up to order ε. The $P_i(x)$ are the $O(\alpha_s^{i+1})$ parts of the splitting function $P^{qq}(x)$, which we already encountered in section II.5

$$P^{qq}(x) = \left(\frac{\alpha_s}{4\pi}\right) P_0(x) + \left(\frac{\alpha_s}{4\pi}\right)^2 P_1(x) + \cdots \tag{4.16}$$

The functions $P_0(x)$ and $P_1(x)$ have also been calculated in the literature [12] (see Appendix J). Further, β_0 is the lowest order coefficient of the β-function

$$\beta(\alpha_s) = 2\;\alpha_s \left\{ -\beta_0 \left(\frac{\alpha_s}{4\pi}\right) + \cdots \right\} \tag{4.17}$$

The exact expression of β_0 can be found in Appendix J.

Eqs. (4.13) and (4.14) can also be derived using the renormalization group (see also section IV.3). In ref. [3] we used this fact to predict $w_1(x)$ and $f_1(x)$ in the limit $x \to 1$. However, the coefficient of the $\delta(1-x)$ part of $w_1(x)$ and $f_1(x)$ cannot be determined without an explicit calculation. We have now carried out the computation needed to fix this coefficient. In the limit $x \to 1$ the expressions for $w_1(x)$ and $f_1(x)$ are

$$\begin{aligned}
w_1(x) &\overset{x\to 1}{=} \\
&\delta(1-x)\Big\{ C_F^2 \Big[\frac{8}{5}\zeta(2)^2 - 60\zeta(3) - 70\zeta(2) + \frac{511}{4}\Big] + \\
&\qquad + C_A C_F \Big[-\frac{12}{5}\zeta(2)^2 + 28\zeta(3) + \frac{592}{9}\zeta(2) - \frac{1535}{12}\Big] + \\
&\qquad + n_f C_F \Big[8\zeta(3) - \frac{112}{9}\zeta(2) + \frac{127}{6}\Big]\Big\} + \\
&+ C_F^2 \Big[128\mathcal{D}_3(x) - (256 + 128\zeta(2))\,\mathcal{D}_1(x) + 256\zeta(3)\mathcal{D}_0(x)\Big] + \\
&+ C_A C_F \Big[-\frac{176}{3}\mathcal{D}_2(x) + \Big(\frac{1072}{9} - 32\zeta(2)\Big)\mathcal{D}_1(x) + \\
&+ \Big(56\zeta(3) + \frac{176}{3}\zeta(2) - \frac{1616}{27}\Big)\mathcal{D}_0(x)\Big] + \\
&+ n_f C_F \Big[\frac{32}{3}\mathcal{D}_2(x) - \frac{160}{9}\mathcal{D}_1(x) + \Big(\frac{224}{27} - \frac{32}{3}\zeta(2)\Big)\mathcal{D}_0(x)\Big]
\end{aligned} \tag{4.18}$$

and

$$\begin{aligned}
f_1(x) &\overset{x\to 1}{=} \\
&\delta(1-x)\Big\{ C_F^2 \Big[6\zeta(2)^2 - 78\zeta(3) + 69\zeta(2) + \frac{331}{8}\Big] + \\
&\qquad + C_A C_F \Big[\frac{71}{5}\zeta(2)^2 + \frac{140}{3}\zeta(3) - \frac{251}{3}\zeta(2) - \frac{5465}{72}\Big] + \\
&\qquad + n_f C_F \Big[\frac{4}{3}\zeta(3) + \frac{38}{3}\zeta(2) + \frac{457}{36}\Big]\Big\} +
\end{aligned}$$

$$
\begin{aligned}
&+C_F^2\Big[8\mathcal{D}_3(x) - 18\mathcal{D}_2(x) - (27+32\zeta(2))\,\mathcal{D}_1(x) + \\
&+\left(-8\zeta(3)+36\zeta(2)+\frac{51}{2}\right)\mathcal{D}_0(x)\Big] + \\
&+C_AC_F\Big[-\frac{22}{3}\mathcal{D}_2(x)+\left(\frac{367}{9}-8\zeta(2)\right)\mathcal{D}_1(x)+ \\
&+\left(40\zeta(3)+\frac{44}{3}\zeta(2)-\frac{3155}{54}\right)\mathcal{D}_0(x)\Big] + \\
&+n_fC_F\Big[\frac{4}{3}\mathcal{D}_2(x)-\frac{58}{9}\mathcal{D}_1(x)+\left(\frac{247}{27}-\frac{8}{3}\zeta(2)\right)\mathcal{D}_0(x)\Big] \qquad (4.19)
\end{aligned}
$$

We now have all the ingredients to determine the Drell-Yan correction term $\Delta(x, Q^2)$. As already discussed in section II.4, the $q\bar{q}$ part of the Drell-Yan correction term is defined by

$$
\begin{aligned}
\hat{W}(z, Q^2, \varepsilon) &= \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx\; \delta(z - x_1 x_2 x) \\
&\quad \hat{\mathcal{F}}_2(x_1, Q^2, \varepsilon)\; \hat{\mathcal{F}}_2(x_2, Q^2, \varepsilon)\; \Delta(x, Q^2) \qquad (4.20)
\end{aligned}
$$

From eqs. (4.13), (4.14) and (4.20) one can derive that $\Delta(x, Q^2)$ is equal to

$$
\Delta(x,Q^2) = \delta(1-x) + \left(\frac{\alpha_s(Q^2)}{4\pi}\right)\Delta_0(x) + \left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2 \Delta_1(x) \qquad (4.21)
$$

with

$$
\Delta_0(x) = w_0(x) - 2f_0(x) \qquad (4.22)
$$

$$
\Delta_1(x) = w_1(x) - 2f_1(x) - (f_0\otimes f_0)(x) - 2(f_0\otimes\Delta_0)(x) \qquad (4.23)
$$

The running coupling constant $\alpha_s(Q^2)$ is defined by

$$
\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1+\beta_0\frac{\alpha_s(\mu^2)}{4\pi}\ln\left(\frac{Q^2}{\mu^2}\right)} = \frac{4\pi}{\beta_0\ln\left(\frac{Q^2}{\Lambda^2}\right)} \qquad (4.24)
$$

where Λ is a parameter which has to be determined experimentally. Performing the mass factorization we find

$$
\begin{aligned}
&\Delta(x,Q^2) = \\
&\delta(1-x)\Big\{1+\left(\frac{\alpha_s(Q^2)}{4\pi}\right)C_F\Big[2+16\zeta(2)\Big] + \\
&\quad +\left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2\Big[C_F^2\left(\frac{548}{5}\zeta(2)^2+120\zeta(3)-3\zeta(2)\right) + \\
&\quad +C_AC_F\left(\frac{215}{9}+\frac{2098}{9}\zeta(2)-\frac{196}{3}\zeta(3)-\frac{154}{5}\zeta(2)^2\right) + \\
&\quad +n_fC_F\left(\frac{16}{3}\zeta(3)-\frac{340}{9}\zeta(2)-\frac{38}{9}\right)\Big]\Big\} +
\end{aligned}
$$

$$+\left(\frac{\alpha_s(Q^2)}{4\pi}\right) C_F \left\{8\mathcal{D}_1(x) + 6\mathcal{D}_0(x) - 4(1+x)\ln(1-x) - 8x - 12\right\} +$$
$$+\left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2 \left\{C_F^2 \left[32\mathcal{D}_3(x) + 72\mathcal{D}_2(x) + (64\zeta(2)+52)\mathcal{D}_1(x) + \right.\right.$$
$$\left. + (112\zeta(3) + 24\zeta(2) + 15)\mathcal{D}_0(x)\right] +$$
$$+C_A C_F \left[-44\mathcal{D}_2(x) + \left(\frac{338}{9} - 16\zeta(2)\right)\mathcal{D}_1(x) + \right.$$
$$\left. + \left(57 + \frac{88}{3}\zeta(2) - 24\zeta(3)\right)\mathcal{D}_0(x)\right] +$$
$$\left. +n_f C_F \left[8\mathcal{D}_2(x) - \frac{44}{9}\mathcal{D}_1(x) - \left(10 + \frac{16}{3}\zeta(2)\right)\mathcal{D}_0(x)\right]\right\} \tag{4.25}$$

5 A closer look at the Drell-Yan correction term

5.1 The second order K-factor

Having calculated the second order contribution to the Drell-Yan correction term, one is of course interested in the size of the $O(\alpha_s^2)$ correction. To this end we will compute the second order K-factor (see eq. (1.2)). The relevant quantity for the calculation of the K-factor is the hadronic structure function $W_V(\tau, Q^2)$. It is equal to (see eq. (II.4.28))

$$W_V(\tau, Q^2) = \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx\ \delta(\tau - x_1 x_2 x)\ PD_V(x_1, x_2, Q^2)\ \Delta(x, Q^2) \tag{5.1}$$

where the PD_V's can be found in Appendix A and $\Delta(x, Q^2)$ is given in eq. (4.25).

For numerical calculations it is very useful to introduce the parton flux $\Phi_V(\xi, Q^2)$

$$\Phi_V(\xi, Q^2) = \int_0^1 dx_1 \int_0^1 dx_2\ PD_V(x_1, x_2, Q^2)\ \delta(\xi - x_1 x_2) \tag{5.2}$$

Notice that $\Phi_V(\tau, Q^2)$ is nothing but the lowest order Drell-Yan cross-section in the leading log approximation ($= W_V^{(0)}(\tau, Q^2)$). In terms of this parton flux the hadronic Drell-Yan structure function can be written as

$$W_V(\tau, Q^2) = \int_0^1 dx \int_0^1 d\xi\ \Phi_V(\xi, Q^2)\ \Delta(x, Q^2)\ \delta(\tau - x\xi) \tag{5.3}$$

Putting the expressions for $PD_V(x_1, x_2, Q^2)$ and $\Delta(x, Q^2)$ in the computer, one has to be careful with the treatment of the distributions $\mathcal{D}_i(x)$. One should use the following relation*

$$\int_0^1 dx \int_0^1 d\xi\ \Phi_V(\xi, Q^2)\ \mathcal{D}_i(x)\ \delta(\tau - x\xi) =$$
$$= \Phi_V(\tau, Q^2)\frac{1}{(1+i)}\ln^{i+1}(1-\tau) +$$
$$+ \int_\tau^1 dx\ \left\{\frac{1}{x}\Phi_V(\tau/x, Q^2) - \Phi_V(\tau, Q^2)\right\}\frac{\ln^i(1-x)}{1-x} \tag{5.4}$$

*Notice that the convolution integral does not depend on the I.R. cutoff δ.

In fig. 10 we have plotted $(K^{(0)} + K^{(1)})$ and $(K^{(0)} + K^{(1)} + K^{(2)})$ for the NA10 experiment. As can be seen both the first and second order corrections are very large. In fig. 11 we show the same quantities for Z production at $\sqrt{S}$ =630 GeV. We find that the corrections are smaller due to the decreasing running coupling constant, but they are still of considerable size.

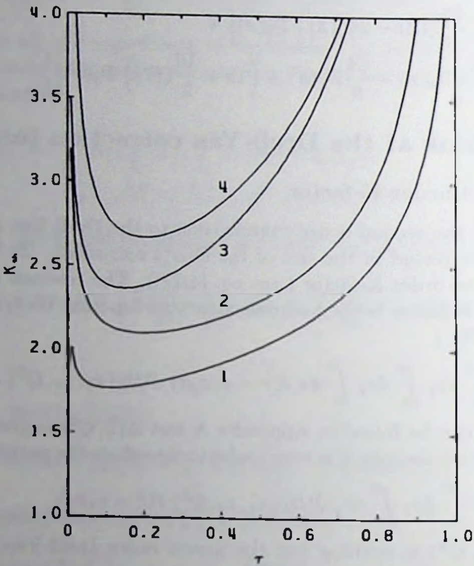

Fig. 10. The K-factor for NA10 experiment:
line 1 : $(K^{(0)} + K^{(1)})$ line 2 : $(K^{(0)} + K^{(1)})_{PI}$ (cf. eq. (5.10))
line 3 : $(K^{(0)} + K^{(1)} + K^{(2)})$ line 4 : $(K^{(0)} + K^{(1)} + K^{(2)})_{PI}$

5.2 Resumming the large corrections

As already discussed in section 1 the large corrections are due to the soft gluons, which give rise to the $\delta(1-x)$ and $\mathcal{D}_i(x)$ terms. It would be ideal if in some way the soft gluon contributions could be determined up to all orders of α_s.

In section 4 we found that the soft and virtual contributions to the Abelian part of $\hat{W}$ and $\hat{\mathcal{F}}_2$ exponentiate. This results in the exponentiation of the large corrections in the Drell-Yan correction term $\Delta(x, Q^2)$, which was already suggested by many people [13,14].

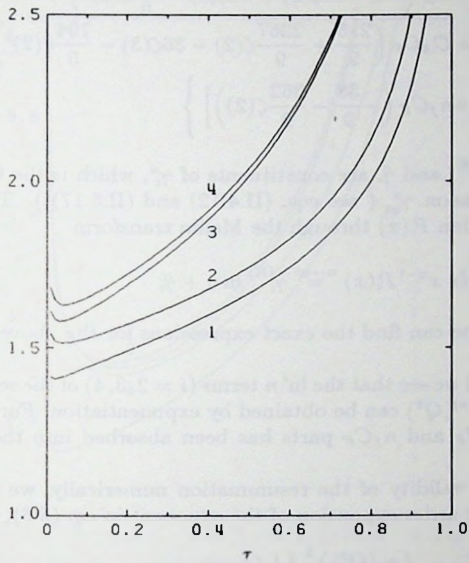

Fig. 11. The K-factor for the Z production at $\sqrt{S}$=630 GeV
line 1 : $(K^{(0)} + K^{(1)})$ line 2 : $(K^{(0)} + K^{(1)})_{PI}$
line 3 : $(K^{(0)} + K^{(1)} + K^{(2)})$ line 4 : $(K^{(0)} + K^{(1)} + K^{(2)})_{PI}$

A convenient way to describe this feature is to work with the Mellin transform of the Drell-Yan correction term

$$\Delta^{(n)}(Q^2) = \int_0^1 dx\; x^{n-1}\; \Delta(x, Q^2) \tag{5.5}$$

In ref. [3] we noticed that the whole $O(\alpha_s)$ term of $\Delta^{(n)}(Q^2)$ could be exponentiated. In the limit $n \to \infty$, which corresponds to the soft limit $x \to 1$, we find*

$$\Delta^{(n)}(Q^2) \overset{n\to\infty}{=}$$

$$\exp\left\{\left(\frac{\alpha_s(Q^2/n)}{4\pi}\right)\left[\frac{1}{2}\gamma_0^{(K)}\ln^2 n + \bar{\gamma}_0 \ln n + C_F\left(2 + 20\zeta(2)\right)\right]\right\} \times$$

$$\times\left\{1 + \left(\frac{\alpha_s(Q^2/n)}{4\pi}\right)^2\left[\frac{1}{2}\left(\gamma_1^{(K)} - \beta_0\bar{\gamma}_0\right)\ln^2 n +\right.\right.$$

*Notice that we have absorbed the Euler constant γ_E into the $\ln n$. Therefore, one should read $\ln n + \gamma_E$ for every $\ln n$ (see Appendix I).

$$+\left(\tilde{\gamma}_1+\beta_0 C_F(-8\zeta(2)-16)\right)\ln n+$$
$$+C_F^2\left(-2-17\zeta(2)+72\zeta(3)-\frac{76}{5}\zeta(2)^2\right)+$$
$$+C_AC_F\left(\frac{215}{9}+\frac{2267}{9}\zeta(2)-36\zeta(3)-\frac{194}{5}\zeta(2)^2\right)+$$
$$+n_fC_F\left(-\frac{38}{9}-\frac{362}{9}\zeta(2)\right)\Bigg]\Bigg\} \tag{5.6}$$

The constants $\gamma_i^{(K)}$ and $\tilde{\gamma}_i$ are constituents of γ_i^n, which is the $O(\alpha_s^{i+1})$ part of the anomalous dimension γ_{qq}^n (see eqs. (II.4.12) and (II.5.17)). The γ_i^n is related to the splitting funtion $P_i(x)$ through the Mellin transform

$$\gamma_i^n=-\int_0^1 dx\; x^{n-1}P_i(x) \overset{n\to\infty}{=} \gamma_i^{(K)}\ln n+\tilde{\gamma}_i \tag{5.7}$$

In Appendix J one can find the exact expressions for the above anomalous dimensions.

From eq. (5.6) we see that the $\ln^i n$ terms ($i=2,3,4$) of the second order Abelian (C_F^2) part of $\Delta^{(n)}(Q^2)$ can be obtained by exponentiation. Furthermore, the $\ln^3 n$ term of the C_AC_F and n_fC_F parts has been absorbed into the running coupling constant.

To check the validity of the resummation numerically, we look at the second order term in the series expansion of the exponent in eq. (5.6), which equals

$$\mathrm{E}_2^{(n)}(Q^2) \overset{n\to\infty}{=} \left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2\left\{\frac{1}{2}\left[\frac{1}{2}\gamma_0^{(K)}\ln^2 n+\tilde{\gamma}_0\ln n+C_F\left(2+20\zeta(2)\right)\right]^2+\right.$$
$$\left.+\frac{1}{2}\beta_0\gamma_0^{(K)}\ln^3 n+\beta_0\tilde{\gamma}_0\ln^2 n+\beta_0C_F(2+20\zeta(2))\ln n\right\} \tag{5.8}$$

For numerical calculations we prefer to work in the x-language, therefore we take the inverse Mellin transform of the above expression. We then find

$$\mathrm{E}_2(x,Q^2) \overset{x\to 1}{=} \left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2\times$$
$$\left\{\delta(1-x)\left[C_F^2\left(2+14\zeta(2)+48\zeta(3)+\frac{624}{5}\zeta(2)^2\right)+\right.\right.$$
$$\left.+C_AC_F\left(22\zeta(2)-\frac{88}{3}\zeta(3)\right)+n_fC_F\left(-4\zeta(2)+\frac{16}{3}\zeta(3)\right)\right]+$$
$$+C_F^2\left[32\mathcal{D}_3(x)+72\mathcal{D}_2(x)+(52+64\zeta(2))\,\mathcal{D}_1(x)+\right.$$
$$\left.+(12+48\zeta(2)+64\zeta(3))\,\mathcal{D}_0(x)\right]+$$
$$+C_AC_F\left[-44\mathcal{D}_2(x)-44\mathcal{D}_1(x)-\left(\frac{22}{3}+\frac{88}{3}\zeta(2)\right)\mathcal{D}_0(x)\right]+$$
$$\left.+n_fC_F\left[8\mathcal{D}_2(x)+8\mathcal{D}_1(x)+\left(\frac{4}{3}+\frac{16}{3}\zeta(2)\right)\mathcal{D}_0(x)\right]\right\} \tag{5.9}$$

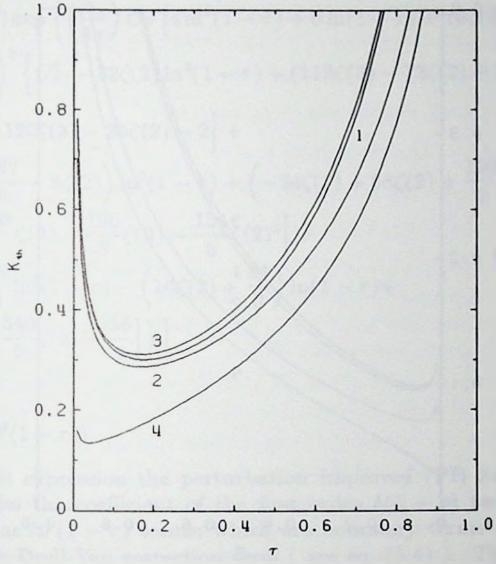

Fig. 12. The NA10 experiment:
Using the exact expression: line 1 : $K^{(2)}_{Ab}$ line 2 : $K^{(2)}_{NAb}$
Using eq. (5.9) : line 3 : $K^{(2)}_{Ab}$ line 4 : $K^{(2)}_{NAb}$

Comparing $E_2(x,Q^2)$ with the second order part of eq. (4.25), we observe that the Abelian (C_F^2) contribution is predicted very well by the exponentiation formula. Namely, the coefficients of $\mathcal{D}_i(x)$ (i=1,2,3) are the same in both expressions and although the coefficients of $\mathcal{D}_0(x)$ and $\delta(1-x)$ are different in the two cases, numerically they are very close. However, the non-Abelian parts do not match that well. Here, only the leading term, $\mathcal{D}_2(x)$, is given correctly by exponentiation. These observations can be illustrated more clearly by computing the theoretical K-factor defined in eq. (1.2). For this purpose we divide the second order K-factor, $K^{(2)}$, into an Abelian ($K^{(2)}_{Ab}$) and a non-Abelian ($K^{(2)}_{NAb}$) part. In figs. 12 and 13 we have plotted $K^{(2)}_{Ab}$ and $K^{(2)}_{NAb}$ for both the exact calculation (see eq. (4.25)) and the prediction by the exponentiation formula (see eq. (5.9)) at two different energies.

One can see from these figures that exponentiation works very well for the C_F^2 part of the second order correction. But, the C_AC_F and the n_fC_F contributions are underestimated by the resummation formula, especially for lower values of τ. In the

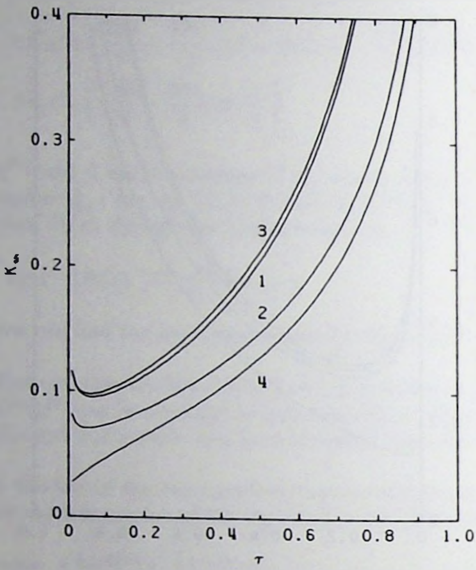

Fig. 13. Z production at $\sqrt{S}$=630 GeV:
Using the exact expression: line 1 : $K_{Ab}^{(2)}$ line 2 : $K_{NAb}^{(2)}$
Using eq. (5.9) : line 3 : $K_{Ab}^{(2)}$ line 4 : $K_{NAb}^{(2)}$

region of small τ values, which is of particular interest to the Z and W production, this underestimation is mainly due to the bad prediction by the exponentiation formula for the non-Abelian coefficient of the $\delta(1-x)$ term. The calculation of this coefficient is one of the major achievements of this thesis.

Although the exponentiation formula is not able to reproduce exactly the second order corrections, it is worthwhile to work it out in more detail. In order to improve the perturbative expansion of $\Delta(x, Q^2)$, one should take the inverse Mellin transform of the exponentiated form of $\Delta^{(n)}(Q^2)$ (eq. (5.6)). This is done in ref. [15], where the inverse Mellin transform has been carried out numerically. As this is rather laborious and seems to lead to an infrared cutoff dependent result, we have looked for an alternative method.

Our main concern was, that it should at least contain the exponentiation of the first order $\delta(1-x)$ term, because at small τ values, which are of experimental interest, this term dominates the correction. In ref. [3] we have proposed the following formula

$$W(\tau,Q^2) = R^{(1)}_{exact}(\tau,Q^2) + R^{(2)}_{app}(\tau,Q^2)+$$
$$+W^{(0)}(\tau,Q^2)\exp\left\{\left(\frac{\bar{\alpha}_s}{4\pi}\right)C_F\left[4\ln^2(1-\tau)+6\ln(1-\tau)+16\zeta(2)+2\right]\right\}\times$$
$$\times\left[1+\left(\frac{\bar{\alpha}_s}{4\pi}\right)^2\left\{C_F^2\left[-32\zeta(2)\ln^2(1-\tau)+(112\zeta(3)-72\zeta(2)+3)\ln(1-\tau)+\right.\right.\right.$$
$$\left.-\frac{92}{5}\zeta(2)^2+120\zeta(3)-35\zeta(2)-2\right]+$$
$$+C_AC_F\left[\left(\frac{367}{9}-8\zeta(2)\right)\ln^2(1-\tau)+\left(-24\zeta(3)+88\zeta(2)+\frac{193}{3}\right)\ln(1-\tau)+\right.$$
$$\left.+\frac{215}{9}+\frac{2098}{9}\zeta(2)-\frac{196}{3}\zeta(3)-\frac{154}{5}\zeta(2)^2\right]+$$
$$+n_fC_F\left[-\frac{58}{9}\ln^2(1-\tau)-\left(16\zeta(2)+\frac{34}{3}\right)\ln(1-\tau)+\right.$$
$$\left.\left.\left.+\frac{16}{3}\zeta(3)-\frac{340}{9}\zeta(2)-\frac{38}{9}\right]\right\}\right] \tag{5.10}$$

with

$$\bar{\alpha}_s = \alpha_s(Q^2(1-\tau)) \tag{5.11}$$

We will call this expression the perturbation improved (PI) hadronic structure function. Besides the coefficient of the first order $\delta(1-x)$ term, we have also exponentiated the $\ln^i(1-\tau)$ terms, which are boundary terms due to the $\mathcal{D}_i(x)$ appearing in the Drell-Yan correction term (see eq. (5.4)). These distributions also contribute to the $R^{(i)}(\tau,Q^2)$ in eq. (5.10). The function $R^{(1)}_{exact}$ also receives contributions from the non-singular terms in eq. (4.25), whereas $R^{(2)}_{app}$ is completely determined by the distributions $\mathcal{D}_i(x)$, since the regular part of $\Delta_1(x)$ (eq. (4.21)) has not been calculated yet. Contrary to eq. (5.6) we have resummed only the terms propotional to $W^{(0)}(\tau,Q^2)$, which correspond to the non-$\ln n$ terms. All the corrections due to the $\ln n$ terms are absorbed in the $R^{(i)}(\tau,Q^2)$. This means that our resummation formula will become less accurate for large τ values.

In figs. 10 and 11 we have also included the perturbation improved (PI) K-factors, which are denoted by $(K^{(0)}+K^{(1)})_{PI}$ and $(K^{(0)}+K^{(1)}+K^{(2)})_{PI}$. As expected we see that $(K^{(0)}+K^{(1)})_{PI}$ underestimates the $O(\alpha_s^2)$ correction. For small τ values this is caused by the inability of the exponentiation formula (eq. (5.6)) to predict correctly the non-Abelian coefficients of the $\delta(1-x)$ term. For larger τ values it is aggravated by the fact that we did not resum the $R^{(i)}(\tau,Q^2)$.

5.3 The validity of the soft gluon approximation

The $O(\alpha_s^2)$ Drell-Yan correction term, which we presented in eq. (4.25) is an approximate formula for two reasons. Firstly, when calculating the second order corrections to the $q\bar{q}$ process, we took the soft limit $x \to 1$, where possible. In this way we have left out all the non singular terms contributing to $\Delta(x,Q^2)$. Secondly, we did not consider the processes qg, qq [16] and gg, which all give $O(\alpha_s^2)$ contributions. It is important to know how this will affect the validity of our predictions. For the NA10 experiment ($\sqrt{S}$=19.1GeV) we will discuss the effects of the approximation in the next chapter , therefore we will now only consider Z and W production.

To get an impression of the size of the $O(\alpha_s^2)$ terms that we have left out, we study the following contributions to the theoretical K-factor

1. $K^{(1)}_{soft}$: $O(\alpha_s)$, $q\bar{q}$, in the soft limit $x \to 1$
2. $K^{(1)}_{reg}$: $O(\alpha_s)$, $q\bar{q}$, the non-singular terms
3. $K^{(1)}_{qg}$: $O(\alpha_s)$, qg contribution
4. $K^{(2)}_{soft}$: $O(\alpha_s^2)$, $q\bar{q}$, in the soft limit $x \to 1$
5. $K^{(2)}_{n_f}$: $O(\alpha_s^2)$, $q\bar{q}$, $n_f C_F$ part, in the soft limit $x \to 1$
6. $K^{(2)}_{n_f,reg}$: $O(\alpha_s^2)$, $q\bar{q}$, $n_f C_F$ part, the non-singular terms

In section 1 we have discussed how the first order $q\bar{q}$ contribution to the Drell-Yan cross-section is divided into a soft and regular part. The non-singular terms of the $n_f C_F$ part can be found in Appendix H.

In fig. 14 we have calculated the above K-factors at $\tau = {M_Z}^2/S$ for $\sqrt{S}$ ranging from 100 GeV to 10 TeV. We see that $K^{(1)}_{reg}$ and $K^{(2)}_{n_f,reg}$ are small compared to $K^{(1)}_{soft}$ and $K^{(2)}_{n_f}$ over the whole $\sqrt{S}$ range. From this we infer that we have a good approximation for the $q\bar{q}$ contributions up to $O(\alpha_s^2)$. We also see that for lower energies the correction at $O(\alpha_s)$ is dominated by the $q\bar{q}$ subprocess ($K^{(1)}_{soft}+K^{(1)}_{reg} \gg K^{(1)}_{qg}$). The reason for this is the absence of a $\delta(1-x)$ term in the qg subprocess. We expect that this dominance will also hold for $O(\alpha_s^2)$, because the subprocesses qg, qq and gg all have in common that they lack a $\delta(1-x)$ term. Thus we are confident that our calculation will give a good approximation of the $O(\alpha_s^2)$ correction for not too high energies (say up to the Tevatron energy). However, for higher energies we observe that $K^{(1)}_{qg}$ keeps on increasing, whereas the $q\bar{q}$ contributions to the K-factor level off to a constant value. This is due to the behaviour of the gluon distribution function, which becomes large for small values of τ. For this reason the subprocesses involving gluons will become more and more important at higher energies. We believe that at $\sqrt{S}$=1800GeV the contribution from the neglected subprocesses will not yet be that dramatic. However, it is clear that a complete $O(\alpha_s^2)$ calculation will be necessary, if we want reliable predictions for the Z and W production rates at higher energies.

6 Z and W production at the CERN and FNAL $p\bar{p}$ colliders

The production of the Z and W via the Drell-Yan mechanism has been studied rather extensively for order α_s [17,18]. Having calculated the $O(\alpha_s^2)$ contribution from the $q\bar{q}$ subprocess in the soft limit, we are interested how this will change the Z and W production rates.

For our numerical calculations we will use the parton distribution function DOI (see ref. [19]) and the following values for the electro-weak parameters

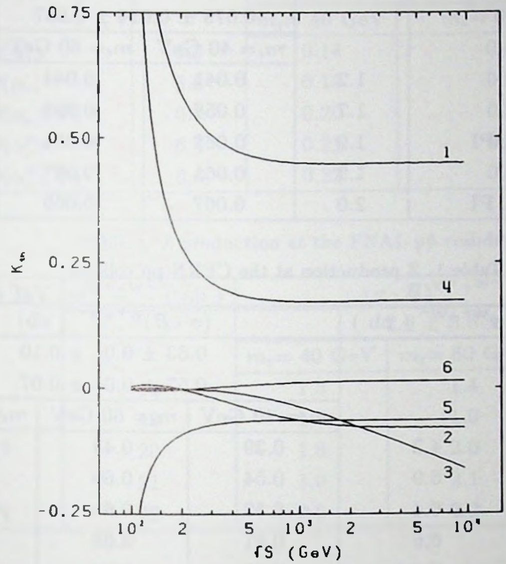

Fig. 14. The contributions to the K-factor for the Z production:
line 1: $K^{(1)}_{soft}$, line 2: $K^{(1)}_{reg}$, line 3: $K^{(1)}_{qg}$,
line 4: $K^{(2)}_{soft}$, line 5: $K^{(2)}_{n_f}$, line 6: $K^{(2)}_{n_f,reg}$

$$M_W = 81 \text{ GeV} \quad M_Z = 92 \text{ GeV}$$
$$\sin^2\theta_W = 0.23 \quad \sin^2\theta_C = 0.05$$

Furthermore, we have put $n_f = 6$ and $\Lambda_{\overline{MS}} = 0.2$ GeV in the running coupling constant (eq. (4.24)).

In tables 1 - 4 the Z and W production rates σ^Z and σ^W (see eqs. (A.25) and (A.26)) are presented for the CERN and FNAL p$\bar{\text{p}}$ colliders in five approximations. In those indicated by PI, we have used the perturbation improved expressions given in eq. (5.10). From these tables it is clear that the $O(\alpha_s^2)$ corrections are of considerable size (~ 10 %). Further, one should notice that the perturbation improved first order results underestimate the second order corrections as discussed in the previous section.

To compare with the experimental data one has to calculate the quantities $\sigma^Z B(Z \to e^+e^-)$ and $\sigma^W B(W \to e\nu)$, where the B's are the branching ratios. As these branching ratios depend on the mass of the top quark, m_t, the total cross-

$\sqrt{S}$= 630 GeV	σ^Z (nb)	$(\sigma \cdot B)^Z$ (nb)	
UA1		0.074 ± 0.014 ± 0.011	
UA2		0.073 ± 0.014 ± 0.007	
		m_t= 40 GeV	m_t= 60 GeV
$O(\alpha_s^0)$	1.2	0.041	0.043
$O(\alpha_s)$	1.7	0.058	0.059
$O(\alpha_s)$, PI	1.9	0.062	0.064
$O(\alpha_s^2)$	1.9	0.065	0.067
$O(\alpha_s^2)$, PI	2.0	0.067	0.069

Table 1. Z production at the CERN p$\bar{\text{p}}$ collider

$\sqrt{S}$= 630 GeV	$\sigma^{W^+ + W^-}$ (nb)	$(\sigma \cdot B)^{W^+ + W^-}$ (nb)		
UA1		0.63 ± 0.04 ± 0.10		
UA2		0.57 ± 0.04 ± 0.07		
		m_t= 40 GeV	m_t= 60 GeV	m_t= 80 GeV
$O(\alpha_s^0)$	4.2	0.39	0.43	0.47
$O(\alpha_s)$	5.9	0.54	0.60	0.65
$O(\alpha_s)$, PI	6.4	0.58	0.65	0.71
$O(\alpha_s^2)$	6.6	0.61	0.68	0.74
$O(\alpha_s^2)$, PI	6.9	0.63	0.70	0.77

Table 2. W production at the CERN p$\bar{\text{p}}$ collider

section $(\sigma \cdot B)$ are given for m_t=40, 60 and 80 GeV*. For the CERN p$\bar{\text{p}}$ collider we have included the experimental cross-sections found by the UA1 and UA2 collaborations [20]. In the case of the Tevatron, until now only the results for the W have been presented by the CDF collaboration [21]. We find that both the first and second order results are in good agreement with the experimental data for the three top masses, which we have chosen. Consequently, it is, at this moment, not possible to 'see' the second order effects experimentally. Hopefully, this will change when more data is accumulated at the Tevatron.

To study the m_t dependence of the Z and W cross-sections in more detail, $(\sigma \cdot B)^Z$ and $(\sigma \cdot B)^W$ are plotted as functions of the top mass in figs. 15 - 18 for the CERN and FNAL p$\bar{\text{p}}$ colliders. For the computation of these figures we have used the perturbation improved (PI), $O(\alpha_s)$ and $O(\alpha_s^2)$ Drell-Yan correction term. We find that within the statistical and systematic errors both approximations are

*The branching ratio $B(Z \rightarrow e^+e^-)$ does not change anymore for $m_t > M_Z/2$, therefore $(\sigma \cdot B)^Z$ is only given for m_t=40 and 60 GeV

$\sqrt{S}$= 1.8 TeV	σ^{Z} (nb)	$(\sigma \cdot B)^{Z}$ (nb)	
		no experimental data yet available	
		m_t= 40 GeV	m_t= 60 GeV
$O(\alpha_s^0)$	4.3	0.14	0.15
$O(\alpha_s)$	5.6	0.19	0.19
$O(\alpha_s)$, PI	6.1	0.20	0.21
$O(\alpha_s^2)$	6.4	0.21	0.22
$O(\alpha_s^2)$, PI	6.6	0.22	0.23

Table 3. Z production at the FNAL $p\bar{p}$ collider

$\sqrt{S}$= 1.8 TeV	$\sigma^{W^{+}+W^{-}}$ (nb)	$(\sigma \cdot B)^{W^{+}+W^{-}}$ (nb)		
CDF		2.6 ± 0.6 ± 0.5		
		m_t= 40 GeV	m_t= 60 GeV	m_t= 80 GeV
$O(\alpha_s^0)$	14	1.3	1.4	1.6
$O(\alpha_s)$	18	1.7	1.9	2.0
$O(\alpha_s)$, PI	20	1.8	2.0	2.2
$O(\alpha_s^2)$	21	1.9	2.1	2.3
$O(\alpha_s^2)$, PI	22	2.0	2.2	2.4

Table 4. W production at the FNAL $p\bar{p}$ collider

consistent with the experimental data. It is also clear that we need more precise measurements of the Z and W production rates to draw any definite conclusions about the top mass. Moreover, one should keep in mind that there are also theoretical uncertainties, such as the choice of the factorization scale, the value of the QCD parameter Λ and the parametrization of the parton distribution functions. To minimize the effect of these theoretical uncertainties the ratio R is introduced

$$R = \frac{\sigma^{W} B(W \to e\nu)}{\sigma^{Z} B(Z \to e^{+}e^{-})} \tag{6.1}$$

We find that R is hardly affected by higher order corrections, the curves for the Born, $O(\alpha_s)$ and $O(\alpha_s^2)$ approximations nearly coincide. This can be easily understood. The first and second order corrections are dominated by the $\delta(1-x)$ term appearing in the $q\bar{q}$ subprocess. Because this contribution is just an overall normalization factor for both σ_Z and σ_W, it cancels between numerator and denominator while calculating R in eq. (6.1). In fig. 19 the perturbation improved, $O(\alpha_s)$ and $O(\alpha_s^2)$ corrected R's are given for $\sqrt{S}$=630GeV together with the experimental R, obtained by the UA1 and UA2 collaborations. It is not possible to put any bounds on the top mass from this figure either. We have calculated the R in the same approximations for $\sqrt{S}$=1.8TeV, this can be found in fig. 20.

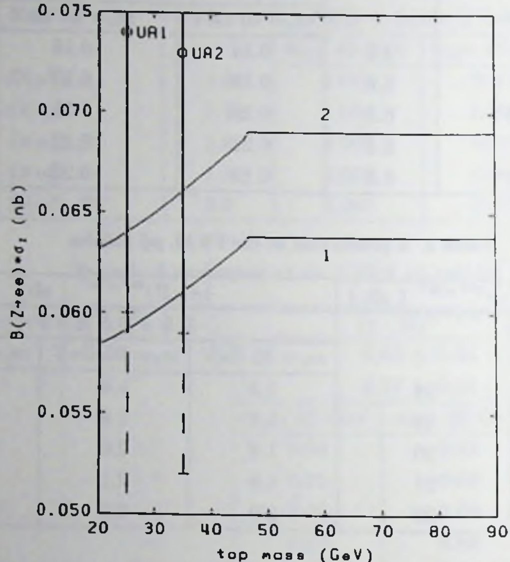

Fig. 15. Z production at $\sqrt{S}$=630 GeV
line 1: $O(\alpha_s)$ with PI, line 2: $O(\alpha_s^2)$ with PI

Summarizing, we find that theoretically the second order corrections give significant contributions ($\sim$ 10%), but due to both experimental and theoretical uncertainties these effects cannot yet be observed experimentally.

7 Summary

In this chapter we have presented the calculation of the second order virtual and soft contributions to the Drell-Yan cross section from the $q\bar{q}$ subprocess. Although this does not constitute the complete $O(\alpha_s^2)$ correction, we argued that it will give fairly reliable predictions for the $O(\alpha_s^2)$ corrected Z and W production rates at the CERN and FNAL $p\bar{p}$-colliders.

Having performed an explicit second order computation, we were able to check the resummation formula derived in ref. [14]. We find that the Abelian (C_F^2) part of the $O(\alpha_s^2)$ soft and virtual corrections can be obtained by exponentiating the complete first order term. However, it also turned out that mere exponentiation

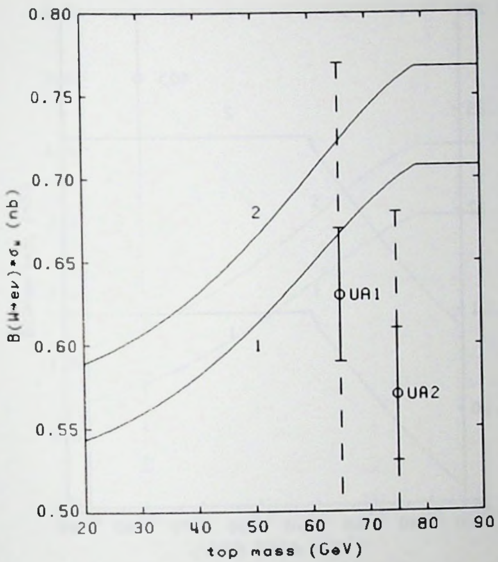

Fig. 16. W production at $\sqrt{S}$=630 GeV
line 1: $O(\alpha_s)$ with PI, line 2: $O(\alpha_s^2)$ with PI

underestimates the second order corrections due to the rather large contribution from the non-Abelian terms.

Furthermore, we have constructed a simple, perturbation improved formula for the Drell-Yan cross-section, which was used to predict the Z and W production rates for the CERN and FNAL $p\bar{p}$-colliders. Using these predictions we have tried to put bounds on the top mass, but due to experimental and theoretical uncertainties this turned out to be impossible.

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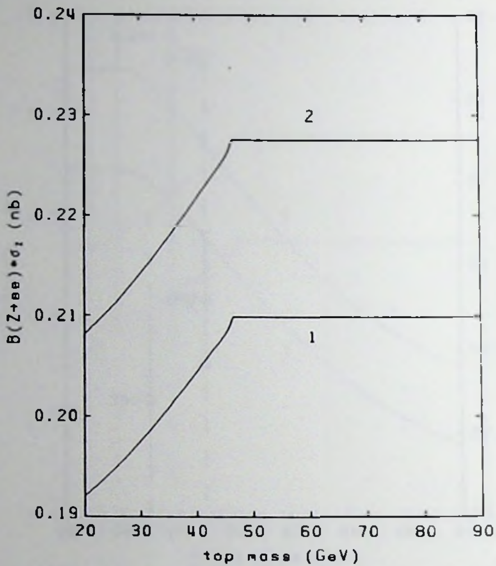

Fig. 17. Z production at $\sqrt{S}=1.8$ TeV
line 1: $O(\alpha_s)$ with PI, line 2: $O(\alpha_s^2)$ with PI

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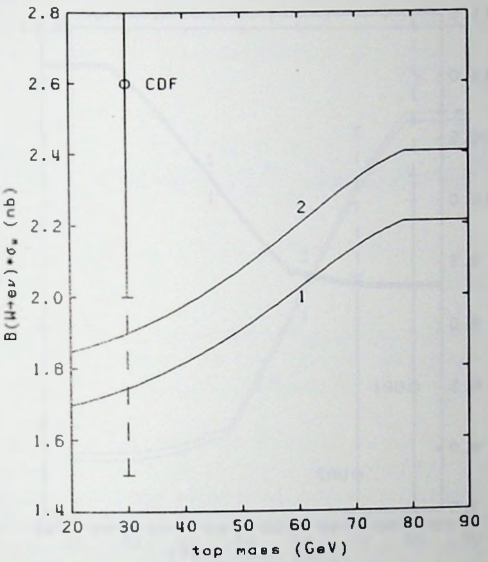

Fig. 18. W production at $\sqrt{S}=1.8$ TeV
line 1: $O(\alpha_s)$ with PI, line 2: $O(\alpha_s^2)$ with PI

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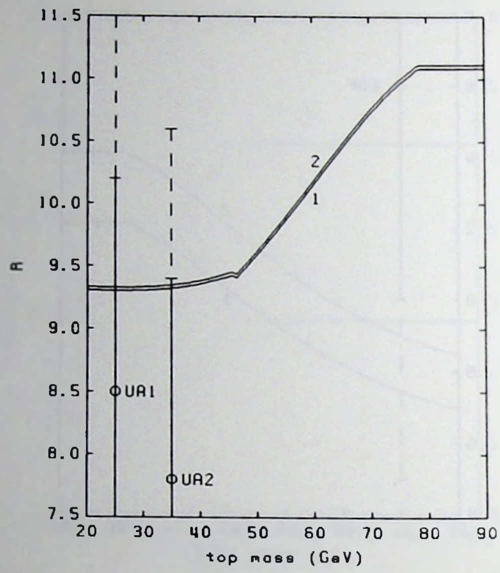

Fig. 19. The ratio R at $\sqrt{S}$=630 GeV
line 1: $O(\alpha_s)$ with PI, line 2: $O(\alpha_s^2)$ with PI

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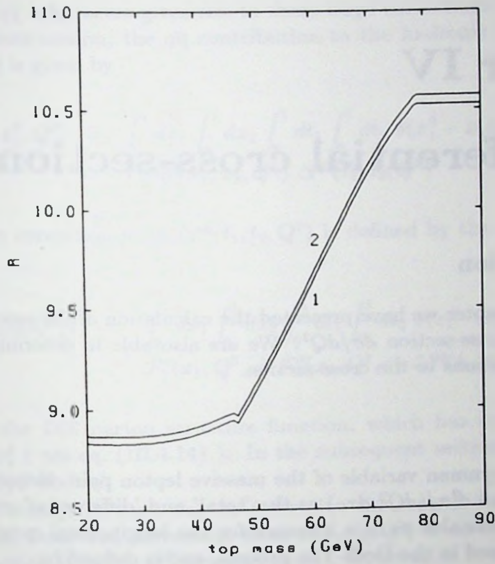

Fig. 20. The ratio R at $\sqrt{S}$=1.8 TeV
line 1: $O(\alpha_s)$ with PI, line 2: $O(\alpha_s^2)$ with PI

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C. Albajar et al., Phys. Lett. B198 (1987) 271; E. Locci, Proc. Int. Europhys. Conf. on High Energy Physics, Uppsala, 1987, ed. O. Botner, p. 8;
UA2 collaboration:
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Chapter IV

The differential cross-section

1 Introduction

In the previous chapter we have presented the calculation of the second order contribution to the cross-section $d\sigma/dQ^2$. We are also able to determine the second order soft contributions to the cross-section

$$\frac{d^2\sigma}{dQ^2 dx_F} \tag{1.1}$$

where x_F is the Feynman variable of the massive lepton pair. Henceforth, we will refer to $d\sigma/dQ^2$ and $d^2\sigma/(dQ^2\ dx_F)$ as the 'total' and 'differential' cross-sections.

The Feynman variable x_F is a measure for the longitudinal momentum of the lepton pair, produced in the Drell-Yan process, and is defined by

$$x_F = 2\frac{Q_L}{\sqrt{S}} \tag{1.2}$$

where Q_L is the longitudinal momentum of the lepton pair. As before we write the differential cross-section as the product of the pointlike cross-section σ_V and a hadronic structure function $W_V(x_1^0, x_2^0, Q^2)$

$$\frac{d^2\sigma^V}{dQ^2 dx_F} = \frac{\tau}{x_1^0 + x_2^0}\ \sigma_V(Q^2, M_V^2)\ W_V(x_1^0, x_2^0, Q^2) \tag{1.3}$$

The variables x_1^0 and x_2^0 are defined by

$$\begin{cases} \tau &= x_1^0 x_2^0 \\ x_F &= x_1^0 - x_2^0 \end{cases} \tag{1.4}$$

or

$$\begin{cases} x_1^0 &= \frac{1}{2}\left(x_F + \sqrt{x_F^2 + 4\tau}\right) \\ x_2^0 &= \frac{1}{2}\left(-x_F + \sqrt{x_F^2 + 4\tau}\right) \end{cases} \tag{1.5}$$

The hadronic structure function $W_V(x_1^0, x_2^0, Q^2)$ has been calculated up to order α_s in ref. [1]. Just as in the case of the total cross-section one finds that the bulk of the correction is due to the soft gluon contributions.

Again the $q\bar{q}$ subprocess gives rise to these large corrections. Quit analogously to the total cross-section, the $q\bar{q}$ contribution to the hadronic structure function $W_V(x_1^0, x_2^0, Q^2)$ is given by

$$W_V^{q\bar{q}}(x_1^0, x_2^0, Q^2) = \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dt_1 \int_0^1 dt_2 \; \delta(x_1^0 - x_1 t_1)\; \delta(x_2^0 - x_2 t_2) \; PD_V^{q\bar{q}}(x_1, x_2, Q^2)\; \Delta^{q\bar{q}}(t_1, t_2, Q^2) \tag{1.6}$$

The Drell-Yan correction term $\Delta^{q\bar{q}}(t_1, t_2, Q^2)$ is defined by the mass factorization formula

$$\hat{W}^{q\bar{q}}(z_1, z_2, Q^2, \varepsilon) = \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dt_1 \int_0^1 dt_2 \; \delta(z_1 - x_1 t_1)\; \delta(z_2 - x_2 t_2) \; \hat{\mathcal{F}}_2^q(x_1, Q^2, \varepsilon)\; \hat{\mathcal{F}}_2^q(x_2, Q^2, \varepsilon)\; \Delta^{q\bar{q}}(t_1, t_2, Q^2) \tag{1.7}$$

where $\hat{\mathcal{F}}_2^q$ is the DIS parton structure function, which has now been calculated up to order α_s^2 (see eq. (III.4.14)). In the subsequent sections we will drop the superscripts 'q$\bar{q}$' and 'q'.

The most straightforward way to determine the second order contribution to the differential Drell-Yan correction term $\Delta(t_1, t_2, Q^2)$ is to calculate the parton structure function $\hat{W}(t_1, t_2, Q^2, \varepsilon)$ up to $O(\alpha_s^2)$. However, it is clear from the computations presented in the previous chapter, that this would be quite a task. Fortunately, if one is only interested in the soft gluon contributions (= the behaviour of $\Delta(t_1, t_2, Q^2)$ in the limit $t_1, t_2 \to 1$), there are less laborious methods.

In section 3 we will present two methods. The first method [2] uses the fact that according to the mass factorization theorem, $\hat{W}(t_1, t_2, Q^2, \varepsilon)$ can be divided into two parts (the Wilson coefficient and the operator matrix element), which both satisfy a renormalization group equation (RGE). Using the RGE's we are able to detemine the pole structure of the parton structure function $\hat{W}(t_1, t_2, Q^2, \varepsilon)$, which in its turn fixes the soft gluon contributions. We will call this the renormalization group (RG) method. It can also be applied to the total cross-section, and has been used to check the pole structures of $\hat{W}(x, Q^2, \varepsilon)$ and $\hat{\mathcal{F}}_2(x, Q^2, \varepsilon)$ (see eqs. (III.4.13) and (III.4.14)).

The second method [2,3], which we will refer to as the Mellin transform method, exploits the resummation formula, which we found for the Mellin transform of the total Drell-Yan correction term $\Delta(x, Q^2)$ (see eq. (III.5.6)). We will show that a similar formula can be obtained for the Mellin transform of the differential DY correction term $\Delta(t_1, t_2, Q^2)$. Once such a resummation formula is found, $\Delta(t_1, t_2, Q^2)$ can be determined in the limit $t_1, t_2 \to 1$.

Before going into the details of the above mentioned methods, we will first discuss the soft gluon contributions to the first order differential cross-section.

2 Some first order results for the differential cross-section

To the renormalization group and Mellin transform methods the knowledge of the first order soft contributions is essential. Therefore, we will now give the relevant $O(\alpha_s)$ results for the differential cross-section. For the calculational details the reader is referred to ref. [1].

The lowest order contribution is given by

$$\hat{W}^{(0)}(t_1,t_2) = \delta(1-t_1)\ \delta(1-t_2) \tag{2.1}$$

The variables t_1 and t_2 are defined by

$$t_i = \frac{x_i^0}{x_i} \qquad (i = 1,2) \tag{2.2}$$

where x_i^0 is given in eq. (1.5) and x_i is the momentum fraction carried by the parton belonging to hadron H_i. Notice that in this approximation we have scaling. Namely, the hadronic structure function is equal to

$$W_V(x_1^0, x_2^0) = PD_V(x_1^0, x_2^0) \tag{2.3}$$

The τ and x_F behaviour of the hadronic structure function does not depend on the interaction scale Q^2.

To determine the second order differential DY correction term $\Delta(t_1,t_2,Q^2)$ in the limit $t_1, t_2 \to 1$, we only need the soft gluon part of the first order corrections. Therefore, we will concentrate our attention on the first order results for the $q\bar{q}$ subprocess, in the limit $t_1, t_2 \to 1$. The exact expressions for the $q\bar{q}$ subprocess and the contribution due to the qg subprocess can be found in ref. [1].

The calculation of the contributions to the differential DY correction term can be done in quite a similar way as for the total cross-section (see section II.5). The $q\bar{q}$ subprocess is again divided into two parts. From the virtual gluon process

$$q(p_1) + \bar{q}(p_2) \to V(q) \quad \text{with one loop corrections} \tag{2.4}$$

we have

$$\hat{W}^{(1),V}(t_1,t_2,Q^2,\varepsilon) = 2\ \delta(1-t_1)\ \delta(1-t_2)\ \mathrm{Re}F^{(1)}(q^2) \tag{2.5}$$

where $F^{(1)}(q^2)$ is the first order quark formfactor (see eq. (II.5.3)). Of course, also the bremsstrahlung process

$$q(p_1) + \bar{q}(p_2) \to V(q) + g(k) \tag{2.6}$$

has to be taken into account. The contribution from this process, in the limit $t_1, t_2 \to 1$, is equal to

$$\hat{W}^{(1),S+H}(t_1,t_2,Q^2,\varepsilon) \overset{t_1,t_2\to 1}{=} g_b^2 s_n C_F (1-t_1)^{-1+\frac{1}{2}\varepsilon}\ (1-t_2)^{-1+\frac{1}{2}\varepsilon}(Q^2)^{\frac{1}{2}\varepsilon}\ \frac{4}{\Gamma(1+\frac{1}{2}\varepsilon)} \tag{2.7}$$

The limit $t_1, t_2 \to 1$ corresponds to the soft limit $x \to 1$ for the total cross-section, because one has

$$x = \frac{Q^2}{s} = \frac{\tau}{x_1 x_2} = \frac{x_1^0}{x_1}\frac{x_2^0}{x_2} = t_1 t_2 \tag{2.8}$$

Therefore, the terms $(1-t_i)^{-1+\varepsilon/2}$ have to be treated in the same manner as the $(1-x)^{-1+\varepsilon}$ in eq. (II.5.8), viz.

$$(1-t_i)^{-1+\frac{1}{2}\varepsilon} = \frac{2}{\varepsilon}\,\delta(1-t_i)\,\delta_i^{\frac{1}{2}\varepsilon} + \theta(1-t_i-\delta_i)\,(1-t_i)^{-1+\frac{1}{2}\varepsilon} \tag{2.9}$$

We have introduced two infrared cutoffs δ_1 and δ_2, which have the same purpose as the δ in case of the total cross-section. In this way $\hat{W}^{(1),S+H}$ is divided into three parts

$$\hat{W}^{(1),S+H} \equiv \hat{W}^{(1),SS} + \hat{W}^{(1),SH} + \hat{W}^{(1),HH} \tag{2.10}$$

with

$$\hat{W}^{(1),SS} = \delta(1-t_1)\,\delta(1-t_2)\,g_b^2 s_n C_F\,(Q^2)^{\frac{1}{2}\varepsilon}\,\delta_1^{\frac{1}{2}\varepsilon}\,\delta_2^{\frac{1}{2}\varepsilon}\,\frac{16}{\varepsilon^2}\frac{1}{\Gamma(1+\frac{1}{2}\varepsilon)} \tag{2.11}$$

$$\hat{W}^{(1),SH} = \delta(1-t_1)\,\theta(1-\delta_2-t_2)\,g_b^2 s_n C_F \times(1-t_2)^{-1+\frac{1}{2}\varepsilon}\,(Q^2)^{\frac{1}{2}\varepsilon}\,\delta_1^{\frac{1}{2}\varepsilon}\,\frac{8}{\varepsilon}\frac{1}{\Gamma(1+\frac{1}{2}\varepsilon)} + \left(t_1 \longleftrightarrow t_2\right) \tag{2.12}$$

$$\hat{W}^{(1),HH} = \theta(1-\delta_1-t_1)\,\theta(1-\delta_2-t_2)\,\hat{W}^{(1),S+H} \tag{2.13}$$

Combining these results we find

$$\hat{W}^{(1)}(t_1,t_2,Q^2,\varepsilon) = \hat{W}^{(1),V} + \hat{W}^{(1),SS} + \hat{W}^{(1),SH} + \hat{W}^{(1),HH} = \tag{2.14}$$
$$= \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{Q^2}{\mu^2}\right)^{\frac{1}{2}\varepsilon}\left\{\frac{1}{\varepsilon}\,\delta(1-t_1)\,P_0^{qq}(t_2) + \frac{1}{\varepsilon}\,P_0^{qq}(t_1)\,\delta(1-t_2) + w_0(t_1,t_2)\right\}$$

with

$$w_0(t_1,t_2) \overset{t_1,t_2\to 1}{=}$$
$$C_F\Big[\delta(1-t_1)\,\delta(1-t_2)\Big\{12\zeta(2)-16+\varepsilon\Big(-4\zeta(3)-\frac{21}{2}\zeta(2)+16\Big)\Big\}+$$
$$+\,\delta(1-t_1)\Big\{4\mathcal{D}_1(t_2)+\varepsilon\Big(-\zeta(2)\mathcal{D}_0(t_2)+\mathcal{D}_2(t_2)\Big)\Big\}+\Big(t_1\longleftrightarrow t_2\Big)+$$
$$+4\mathcal{D}_0(t_1)\mathcal{D}_0(t_2)+\varepsilon\Big(2\mathcal{D}_0(t_1)\mathcal{D}_1(t_2)+2\mathcal{D}_1(t_1)\mathcal{D}_0(t_2)\Big)\Big] \tag{2.15}$$

We have also given the order ε terms of $w_0(t_1,t_2)$, because we will need them in the next section. The first order splitting function $P_0(t_i)$ has been defined in eq. (II.5.13). Comparing the above equation with $\hat{\mathcal{F}}_2^{(1),q}$ in eq. (II.5.12), we see that they both have the same pole structure, as expected from the mass factorization theorem.

Using the mass factorization formula in eq. (1.7) we find for the first order part of differential DY correction term

$$\begin{aligned}
\Delta_0(t_1,t_2,Q^2) &= \\
&= \hat{W}^{(1)}(t_1,t_2,Q^2,\varepsilon) - \delta(1-t_1)\,\hat{\mathcal{F}}_2^{(1),q}(t_2,Q^2,\varepsilon) - \hat{\mathcal{F}}_2^{(1),q}(t_1,Q^2,\varepsilon)\,\delta(1-t_2) \\
&= w_0(t_1,t_2) - \delta(1-t_1)\,f_0(t_2) - f_0(t_1)\,\delta(1-t_2) \\
&= \left(\frac{\alpha_s}{4\pi}\right) C_F \Big\{\delta(1-t_1)\,\delta(1-t_2)\Big[2+20\zeta(2)\Big] + \\
&\qquad + 3\delta(1-t_1)\,\mathcal{D}_0(t_2) + 3\mathcal{D}_0(t_1)\,\delta(1-t_2) + 4\mathcal{D}_0(t_1)\,\mathcal{D}_0(t_2)\Big\} \qquad (2.16)
\end{aligned}$$

Notice that the $D_i(t_{1,2})$'s in the above expression are completely fixed by the pole structure of the $\hat{W}^{(1),V}$ and $\hat{W}^{(1),SS}$. This fact will be used to determine the second order soft gluon contributions in the RG method.

Because our second method to determine the $O(\alpha_s^2)$ corrections to the differential cross-section uses the Mellin transform of the DY correction term, we will now give the double Mellin transform of $\Delta_0(t_1,t_2,Q^2)$. It is defined by

$$\Delta_0^{(n_1,n_2)}(Q^2) = \int_0^1 dt_1 \int_0^1 dt_2\; t_1^{n_1-1}\; t_2^{n_2-1}\; \Delta_0(t_1,t_2,Q^2) \overset{n_1,n_2\to\infty}{=} \qquad (2.17)$$
$$\left(\frac{\alpha_s}{4\pi}\right)\left\{\frac{1}{2}\gamma_0^{(K)}\ln n_1 \ln n_2 + \frac{1}{2}\tilde{\gamma}_0\Big(\ln n_1 + \ln n_2\Big) + C_F\Big(2+20\zeta(2)\Big)\right\}$$

where $\gamma_0^{(K)}$ and $\tilde{\gamma}_0$ can be found in Appendix J. Comparing $\Delta_0^{(n_1,n_2)}(Q^2)$ with the Mellin transform of the total DY correction term, $\Delta^{(n)}(Q^2)$, we find

$$\Delta_{0,dif}^{(n_1,n_2)}(Q^2) = \Delta_{0,tot}^{(n)}(Q^2) \qquad \text{for } n_1 = n_2 = n \to \infty \qquad (2.18)$$

This identity will be exploited in the Mellin transform method. In the above equation we have included the subscripts 'dif' and 'tot' for clarity.

3 The second order differential Drell-Yan correction term

In the introduction we mentioned two methods to determine the second order soft gluon contributions to the differential DY correction term $\Delta(t_1,t_2,Q^2)$. We will now work them out in more detail.

We will first discuss the renormalization group (RG) method [2]. The drawback of this method is that it only fixes the coefficients of the $D_i(t_{1,2})$ terms. The $\delta(1-t_1)\ \delta(1-t_2)$ part, which is important for the normalization of the cross-section, cannot be determined in this way. Nevertheless, this method is interesting, because it can also be applied to the total cross-section. We have used it to check the calculations of the parton structure functions $\hat{W}(x,Q^2,\varepsilon)$ and $\hat{\mathcal{F}}_2(x,Q^2,\varepsilon)$.

The second way to find the soft gluon contributions is the Mellin transform method [2,3]. Combining it with the RG method also the $\delta(1-t_1)\ \delta(1-t_2)$ can be computed.

3.1 The renormalization group method

In section 2 we observed that the first order soft gluon contributions were completely fixed by the poles of $\hat{W}^{(1),V}$ and $\hat{W}^{(1),SS}$. Because this will also be the case at higher orders in α_s, the first step will be to find the pole structure of $\hat{W}(t_1, t_2, Q^2, \varepsilon)$ at order α_s^2.

For this purpose it is convenient to work with the Mellin transform of the parton structure function

$$\hat{W}^{(n_1,n_2)}(Q^2,\varepsilon) = \int_0^1 dt_1 \int_0^1 dt_2\ t_1^{n_1-1}\ t_2^{n_2-1}\ \hat{W}(t_1,t_2,Q^2,\varepsilon) \tag{3.1}$$

According to the mass factorization theorem, one can isolate the initial state collinear divergences in the following way

$$\begin{aligned}&\hat{W}^{(n_1,n_2)}(\alpha_s(\mu^2), Q^2/\mu^2, \varepsilon) = \\ &\quad C_{dij}^{(n_1,n_2)}(\alpha_s(\mu^2), Q^2/\mu^2)\ A^{n_1}(\alpha_s(\mu^2),1,\varepsilon)\ A^{n_2}(\alpha_s(\mu^2),1,\varepsilon)\end{aligned} \tag{3.2}$$

The above expression should be compared with eq. (II.4.7). Notice that we now have double Mellin transforms instead of single ones and that the mass factorization scale M has been chosen to be equal to the renormalization scale μ.

The Wilson coefficient $C_{dij}^{(n_1,n_2)}$ satisfies the renormalization group equation (see eq. (II.4.14))

$$\left\{\mu\frac{\partial}{\partial\mu} + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s} - \gamma_{qq}^{n_1} - \gamma_{qq}^{n_2}\right\} C_{dij}^{(n_1,n_2)}(\alpha_s(\mu^2), Q^2/\mu^2) = 0 \tag{3.3}$$

where γ_{qq}^n is the anomalous dimension, which we found in eq. (II.5.17). Expanding it in α_s, we have

$$\gamma_{qq}^n(\alpha_s) = \left(\frac{\alpha_s}{4\pi}\right)\gamma_0^n + \left(\frac{\alpha_s}{4\pi}\right)^2\gamma_1^n + \cdots \tag{3.4}$$

Furthermore, one can show that the operator matrixelement A^n is given by

$$\begin{aligned}&A^n(\alpha_s(\mu^2),1,\varepsilon) = \\ &\quad 1 - \left(\frac{\alpha_s}{4\pi}\right)\frac{1}{\varepsilon}\gamma_0^n + \left(\frac{\alpha_s}{4\pi}\right)^2\left\{\left(\frac{1}{2}(\gamma_0^n)^2 - \beta_0\gamma_0^n\right)\frac{1}{\varepsilon^2} - \frac{1}{2}\gamma_1^n\frac{1}{\varepsilon}\right\}\end{aligned} \tag{3.5}$$

here β_0 is the lowest order coefficient of the β-function (see eq. (III.4.17)).

Combining eqs. (3.2), (3.3) and (3.5) we find for the *unrenormalized** parton structure function

$$\begin{aligned}&\hat{W}_u^{(n_1,n_2)}(Q^2,\varepsilon) = \\ &\quad 1 + g_n^2(Q^2)^{\frac{1}{2}\varepsilon}\left\{-(\gamma_0^{n_1}+\gamma_0^{n_2})\frac{1}{\varepsilon} + w_0^{(n_1,n_2)}\right\} + \\ &\quad + g_n^4(Q^2)^{\varepsilon}\left\{\left[\frac{1}{2}(\gamma_0^{n_1}+\gamma_0^{n_2})^2 + \beta_0(\gamma_0^{n_1}+\gamma_0^{n_2})\right]\frac{1}{\varepsilon^2} + \right. \\ &\quad \left. -\left[\frac{1}{2}(\gamma_1^{n_1}+\gamma_1^{n_2}) + (\gamma_0^{n_1}+\gamma_0^{n_2})\,w_0^{(n_1,n_2)} + 2\beta_0 w_0^{(n_1,n_2)}\right]\frac{1}{\varepsilon} + w_1^{(n_1,n_2)}\right\}\end{aligned} \tag{3.6}$$

---

*To obtain the unrenormalized parton structure function, one has to invert the renormalization prescription given in eq. (III.4.11).

The expressions for γ_0^n, γ_1^n and β_0 can be found in Appendix J. Furthermore, $w_0^{(n_1,n_2)}$ is the Mellin transform of $w_0(t_1,t_2)$ in eq. (2.15) Therefore, the only unknown quantity in the above equation is $w_1^{(n_1,n_2)}$. All the information about the soft gluon contributions is contained in this function. In the subsequent part of this subsection we will explain how $w_1^{(n_1,n_2)}$ can be obtained in the limit $n_1, n_2 \to \infty$, or equivalently $w_1(t_1,t_2)$ in the limit $t_1, t_2 \to 1$.

From the calculation of the first order parton structure function $\hat{W}^{(1)}$ it is clear, that it is sufficient to know the $\delta(1-t_1)\delta(1-t_2)$ terms, $\hat{W}^{(1),V}$ and $\hat{W}^{(1),SS}$, to determine $w_0(t_1,t_2)$. This will also be true at higher orders of α_s. We conjecture that the $\delta(1-t_1)\delta(1-t_2)$ part of the unrenormalized parton structure function $\hat{W}_u(t_1,t_2,Q^2,\varepsilon)$ can be written as

$$\begin{aligned}
&\hat{W}_u^{SS+V}(t_1,t_2,\delta_1,\delta_2,Q^2,\varepsilon) \equiv \\
&\equiv \delta(1-t_1)\,\delta(1-t_2) + \sum_{i=1}^{\infty}\left\{\hat{W}^{(i),V} + \hat{W}^{(i),SS}\right\} = \\
&= \delta(1-t_1)\,\delta(1-t_2)\,|F(Q^2,\varepsilon)|^2\, B_{dif}(\delta_1,\delta_2,Q^2,\varepsilon) \qquad (3.7)
\end{aligned}$$

where $F(Q^2,\varepsilon)$ is the quark formfactor, which determines the virtual corrections to the parton structure function. The function B_{dif} fixes the soft behaviour of $\hat{W}^{SS+V}$. This conjecture has been verified for the total cross-section by our explicit calculation in the previous chapter (see eq. (III.4.3)). Furthermore, we assume that B_{dif} is of the form

$$B_{dif}(\delta_1,\delta_2,Q^2,\varepsilon) = 1 + g_n^2(Q^2)^{\frac{1}{2}\varepsilon}\,\delta_1^{\frac{1}{2}\varepsilon}\,\delta_2^{\frac{1}{2}\varepsilon}\,B_{dif}^{(1)}(\varepsilon) + g_n^4(Q^2)^{\varepsilon}\,\delta_1^{\varepsilon}\,\delta_2^{\varepsilon}\,B_{dif}^{(2)}(\varepsilon) \qquad (3.8)$$

This assumption follows from the behaviour of the phase space integrals. The first order part of B_{dif} is given by (see eq. (2.11))

$$g_n^2(Q^2)^{\frac{1}{2}\varepsilon}\,\delta_1^{\frac{1}{2}\varepsilon}\,\delta_2^{\frac{1}{2}\varepsilon}\,B_{dif}^{(1)}(\varepsilon) = \hat{W}^{(1),SS} \qquad (3.9)$$

The second order term, $B_{dif}^{(2)}(\varepsilon)$, has yet to be calculated. This is done by combining eqs. (3.6) and (3.7). Taking the inverse Mellin transform of $\hat{W}_u^{(n_1,n_2)}(Q^2,\varepsilon)$, the pole structure of the $\delta(1-t_1)\delta(1-t_2)$ term in $\hat{W}_u(t_1,t_2,Q^2,\varepsilon)$ can be computed up to $O(\alpha_s^2)$. By definition this is the pole structure of the function $\hat{W}_u^{SS+V}$. Because of this identity we find for $B_{dif}^{(2)}(\varepsilon)$

$$\begin{aligned}
&B_{dif}^{(2)}(\varepsilon) = \\
&C_F^2\left\{\frac{128}{\varepsilon^4} - 96\zeta(2)\frac{1}{\varepsilon^2} + \frac{224}{3}\zeta(3)\frac{1}{\varepsilon} + C^{C_F}\right\} + \\
&+C_AC_F\left\{-\frac{88}{3}\frac{1}{\varepsilon^3} + \frac{1}{\varepsilon^2}\left(\frac{268}{9} - 8\zeta(2)\right) + \frac{1}{\varepsilon}\left(28\zeta(3) + 22\zeta(2) - \frac{808}{27}\right) + C^{C_A}\right\} + \\
&+n_fC_F\left\{\frac{16}{3}\frac{1}{\varepsilon^3} - \frac{40}{9}\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{112}{27} - 4\zeta(2)\right) + C^{n_f}\right\} \qquad (3.10)
\end{aligned}$$

We now have the pole terms of $B_{dif}^{(2)}(\varepsilon)$. These fix the coefficients of the distributions $D_i(t_{1,2})$ in $w_1(t_1,t_2)$. However, the constants C^{C_F}, C^{C_A} and C^{n_f}, which determine the $\delta(1-t_1)\delta(1-t_2)$ part of $w_1(t_1,t_2)$, cannot be computed with the RG method.

Substituting the expression for $B^{(2)}_{dif}(\varepsilon)$ in eq. (3.7) we find for $w_1(t_1,t_2)$

$$
\begin{aligned}
w_1(t_1,t_2) &\overset{t_1,t_2\to 1}{=} \\
&\delta(1-t_1)\,\delta(1-t_2)\Big\{\quad \text{unknown}\quad\Big\}+ \\
&+\delta(1-t_1)\Big\{C_F^2\Big[8\mathcal{D}_3(t_2)+\Big(16\zeta(2)-64\Big)\mathcal{D}_1(t_2)+32\zeta(3)\mathcal{D}_0(t_2)\Big]+ \\
&\qquad +C_AC_F\Big[-\frac{22}{3}\mathcal{D}_2(t_2)+\Big(\frac{268}{9}-8\zeta(2)\Big)\mathcal{D}_1(t_2)+ \\
&\qquad\qquad +\Big(28\zeta(3)+\frac{44}{3}\zeta(2)-\frac{808}{27}\Big)\mathcal{D}_0(t_2)\Big]+ \\
&\qquad +n_fC_F\Big[\frac{4}{3}\mathcal{D}_2(t_2)-\frac{40}{9}\mathcal{D}_1(t_2)+\Big(\frac{112}{27}-\frac{8}{3}\zeta(2)\Big)\mathcal{D}_0(t_2)\Big]\Big\}+ \\
&+\Big(t_1\longleftrightarrow t_2\Big)+ \\
&+C_F^2\Big[48\mathcal{D}_1(t_1)\mathcal{D}_1(t_2)+24\Big(\mathcal{D}_0(t_1)\mathcal{D}_2(t_2)+\mathcal{D}_2(t_1)\mathcal{D}_0(t_2)\Big)+ \\
&\qquad +\Big(16\zeta(2)-64\Big)\mathcal{D}_0(t_1)\mathcal{D}_0(t_2)\Big]+ \\
&+C_AC_F\Big[-\frac{44}{3}\Big(\mathcal{D}_0(t_1)\mathcal{D}_1(t_2)+\mathcal{D}_1(t_1)\mathcal{D}_0(t_2)\Big)+ \\
&\qquad +\Big(\frac{268}{9}-8\zeta(2)\Big)\mathcal{D}_0(t_1)\mathcal{D}_0(t_2)\Big]+ \\
&+n_fC_F\Big[\frac{8}{3}\Big(\mathcal{D}_0(t_1)\mathcal{D}_1(t_2)+\mathcal{D}_1(t_1)\mathcal{D}_0(t_2)\Big)-\frac{40}{9}\mathcal{D}_0(t_1)\mathcal{D}_0(t_2)\Big] \qquad (3.11)
\end{aligned}
$$

We have used the relation (see eq. (II.5.15))

$$
\ln^{i+1}\delta \longleftrightarrow (i+1)\,D_i(t) \qquad (3.12)
$$

to determine the coefficients of the distributions $D_i(t)$.

We now still have to perform mass factorization. Using eqs. (III.4.14), (1.7) and (3.6) one can show that the differential Drell-Yan correction term can be written as (cf. eq. (III.4.21))

$$
\begin{aligned}
&\Delta(t_1,t_2,Q^2)= \\
&\qquad \delta(1-t_1)\,\delta(1-t_2)+\left(\frac{\alpha_s(Q^2)}{4\pi}\right)\Delta_0(t_1,t_2)+\left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2\Delta_1(t_1,t_2) \qquad (3.13)
\end{aligned}
$$

with

$$
\begin{aligned}
\Delta_0(t_1,t_2) &= w_0(t_1,t_2)-\delta(1-t_1)\,f_0(t_2)-f_0(t_1)\,\delta(1-t_2) \\
\Delta_1(t_1,t_2) &= w_1(t_1,t_2)-\delta(1-t_1)\,f_1(t_2)-f_1(t_1)\,\delta(1-t_2)+ \\
&\quad -f_0(t_1)\,f_0(t_2)-f_0(t_1)\otimes\Delta_0(t_1,t_2)-\Delta_0(t_1,t_2)\otimes f_0(t_2) \qquad (3.14)
\end{aligned}
$$

An explicit expression for the differential DY correction term can be found in the next subsection.

3.2 The Mellin transform method

The Mellin transform method is based on the following relation between the total and differential cross-sections

$$\frac{d\sigma}{dQ^2} = \int_{\tau-1}^{1-\tau} dx_F \frac{d^2\sigma}{dQ^2 dx_F} \tag{3.15}$$

Performing the Mellin transform with respect to τ on both sides of the above equation, one obtains

$$\Delta_{tot}^{(n)}(Q^2) = \Delta_{dif}^{(n_1,n_2)}(Q^2) \qquad \text{for } n_1 = n_2 = n \tag{3.16}$$

where the Mellin transforms of the total and differential Drell-Yan correction terms are defined by

$$\Delta_{tot}^{(n)}(Q^2) = \int_0^1 dx\, x^{n-1}\, \Delta(x, Q^2) \tag{3.17}$$

and

$$\Delta_{dif}^{(n_1,n_2)}(Q^2) = \int_0^1 dt_1 \int_0^1 dt_2\, t_1^{n_1-1}\, t_2^{n_2-1}\, \Delta(t_1, t_2, Q^2) \tag{3.18}$$

An example of this relation at order α_s can be found in eq. (2.18).

In section III.5.2 we have presented a resummation formula (eq. (III.5.6)) for $\Delta_{tot}^{(n)}(Q^2)$, which is exact up to order α_s^2. Because of the above identity the differential DY correction term $\Delta_{dif}^{(n_1,n_2)}(Q^2)$ obeys the same resummation formula for $n_1 = n_2 = n$. We now have to restore the n_1 and n_2 to find the correct exponentiation prescription for the differential cross-section. There are two guidelines to search for the right equation. Firstly, one has to require that for $n_1 = n_2 = n$ the formula for the total cross-section is reproduced. Secondly, $\Delta_{dif}^{(n_1,n_2)}(Q^2)$ is symmetric in n_1 and n_2. Thus, we find

$$\left(\frac{\alpha_s(Q^2/n)}{4\pi}\right) \longrightarrow \left(\frac{\alpha_s(Q^2/\sqrt{n_1 n_2})}{4\pi}\right) \tag{3.19}$$

and

$$\ln n \longrightarrow \frac{1}{2}\left(\ln n_1 + \ln n_2\right) \tag{3.20}$$

However, it is not quite clear how we should replace $\ln^2 n$, because

$$\ln^2 n \longrightarrow a \ln^2 n_1 + b \ln n_1 \ln n_2 + a \ln^2 n_2 \qquad \text{with } 2a + b = 1 \tag{3.21}$$

fulfils both requirements. In eq. (2.18) we have calculated the Mellin transform of the first order differential DY correction term. From this equation we see that we should take $a = 0$ and $b = 1$, or

$$\ln^2 n \longrightarrow \ln n_1 \ln n_2 \tag{3.22}$$

To check whether this is also correct for the $O(\alpha_s^2)$ part of the resummation formula, we have used the results from the RG method. There we have been able to compute all the distributions $D_i(t_{1,2})$, or equivalently all the $\ln^i n_{1,2}$ in the Mellin language. We have verified that the above replacement is indeed correct.

Making the right substitutions, the Mellin transform of differential DY correction term is found to be equal to

$$\Delta_{dif}^{(n_1,n_2)}(Q^2) \overset{n_1,n_2\to\infty}{=}$$
$$\exp\left\{\left(\frac{\alpha_s^{(n_1n_2)}}{4\pi}\right)\left[\frac{1}{2}\gamma_0^{(K)}\ln n_1\ln n_2+\frac{1}{2}\tilde{\gamma}_0\left(\ln n_1+\ln n_2\right)+C_F\left(2+20\zeta(2)\right)\right]\right\}$$
$$\times\left\{1+\left(\frac{\alpha_s^{(n_1n_2)}}{4\pi}\right)^2\left[\frac{1}{2}\left(\gamma_1^{(K)}-\beta_0\tilde{\gamma}_0\right)\ln n_1\ln n_2+\right.\right.$$
$$+\frac{1}{2}\left(\tilde{\gamma}_1+\beta_0C_F(-8\zeta(2)-16)\right)\left(\ln n_1+\ln n_2\right)+$$
$$+C_F^2\left(-2-17\zeta(2)+72\zeta(3)-\frac{76}{5}\zeta(2)^2\right)+$$
$$+C_AC_F\left(\frac{215}{9}+\frac{2267}{9}\zeta(2)-36\zeta(3)-\frac{194}{5}\zeta(2)^2\right)+$$
$$\left.\left.+n_fC_F\left(-\frac{38}{9}-\frac{362}{9}\zeta(2)\right)\right]\right\} \tag{3.23}$$

with

$$\left(\frac{\alpha_s^{(n_1n_2)}}{4\pi}\right)=\left(\frac{\alpha_s(Q^2/\sqrt{n_1n_2})}{4\pi}\right) \tag{3.24}$$

Because the above exponentiation is exact up to $O(\alpha_s^2)$, it also fixes the second order coefficient of the $\delta(1-t_1)\delta(1-t_2)$ term in the differential DY correction term. Therefore, combining the renormalization group and the Mellin transform methods the complete soft gluon contribution can be calculated up to order α_s^2.

Finally, the differential Drell-Yan correction term can be obtained by expanding the exponent in the above expression and taking the inverse Mellin transform. In the limit $t_1, t_1 \to 1$ it is equal to

$$\Delta(t_1,t_2,Q^2) \overset{t_1,t_2\to 1}{=}$$
$$\delta(1-t_1)\delta(1-t_2)\left\{1+\left(\frac{\alpha_s(Q^2)}{4\pi}\right)C_F\left[2+20\zeta(2)\right]+\right.$$
$$+\left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2\left[C_F^2\left(\frac{964}{5}\zeta(2)^2+72\zeta(3)+14\zeta(2)\right)+\right.$$
$$+C_AC_F\left(-\frac{194}{5}\zeta(2)^2-36\zeta(3)+\frac{2366}{9}\zeta(2)+\frac{215}{9}\right)+$$
$$\left.\left.+n_fC_F\left(-\frac{380}{9}\zeta(2)-\frac{38}{9}\right)\right]\right\}+$$
$$+\delta(1-t_1)\left\{\left(\frac{\alpha_s(Q^2)}{4\pi}\right)C_F\ 3\ \mathcal{D}_0(t_2)+\right.$$

$$
\begin{aligned}
&+\left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2\left[C_F^2\left\{(9-16\zeta(2))\mathcal{D}_1(t_2)+\right.\right.\\
&+\left.\left(24\zeta(3)+36\zeta(2)+\frac{15}{2}\right)\mathcal{D}_0(t_2)\right\}+\\
&+C_AC_F\left\{-11\mathcal{D}_1(t_2)+\left(\frac{57}{2}-12\zeta(3)\right)\mathcal{D}_0(t_2)\right\}+\\
&\left.\left.+n_fC_F\left\{2\mathcal{D}_1(t_2)-5\mathcal{D}_0(t_2)\right\}\right]\right\}+\\
&+\left(t_1\longleftrightarrow t_2\right)+\\
&+\left(\frac{\alpha_s(Q^2)}{4\pi}\right)C_F\,4\,\mathcal{D}_0(t_1)\mathcal{D}_0(t_2)+\\
&+\left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2\left[C_F^2\left\{32\mathcal{D}_1(t_1)\mathcal{D}_1(t_2)+24\left(\mathcal{D}_1(t_1)\mathcal{D}_0(t_2)+\mathcal{D}_0(t_1)\mathcal{D}_1(t_2)\right)+\right.\right.\\
&+\left.(80\zeta(2)+17)\mathcal{D}_0(t_1)\mathcal{D}_0(t_2)\right\}+\\
&+C_AC_F\left\{-\frac{44}{3}\left(\mathcal{D}_1(t_1)\mathcal{D}_0(t_2)+\mathcal{D}_0(t_1)\mathcal{D}_1(t_2)\right)+\right.\\
&+\left.\left(\frac{268}{9}-8\zeta(2)\right)\mathcal{D}_0(t_1)\mathcal{D}_0(t_2)\right\}+\\
&\left.+n_fC_F\left\{\frac{8}{3}\left(\mathcal{D}_1(t_1)\mathcal{D}_0(t_2)+\mathcal{D}_0(t_1)\mathcal{D}_1(t_2)\right)-\frac{40}{9}\mathcal{D}_0(t_1)\mathcal{D}_0(t_2)\right\}\right]
\end{aligned}
\tag{3.25}
$$

The exact first order Drell-Yan correction term can be found in ref. [1].

4 The NA10 experiment

The main reason, we were interested in calculating the differential cross-section up to order α_s^2, was the anomalous scaling observed by the NA10 collaboration. They claimed that the x_F behaviour of their experimental data could not be described by the first order calculations done in QCD. However, recently we were told [4] that the NA10 data are being reassessed and that it is not sure whether the anomalous scaling will survive the re-evaluation of the data. Therefore, we feel that we should wait for the new data, before making detailed studies of the NA10 experiment.

Nevertheless, it is interesting to look at some numerical results for the differential cross-section. In the NA10 experiment the process $\pi^- + W \rightarrow \mu^+\mu^- + X$ is studied at a C.M. energy of 19.1 GeV. The differential cross-section $d^2\sigma/(d\sqrt{\tau}dx_{\mathrm{F}})$, which is measured by the NA10 collaboration, can be calculated using

$$
\frac{d^2\sigma}{d\sqrt{\tau}dx_{\mathrm{F}}} = \frac{2S\tau\sqrt{\tau}}{x_1^0 + x_2^0}\,\sigma_\gamma(Q^2)W_\gamma(x_1^0, x_2^0, Q^2) \tag{4.1}
$$

where the hadronic structure function $W_\gamma(x_1^0, x_1^0, Q^2)$ is given by eq. (1.6).

As in the case of the total cross-section the $\delta(1-t_1)\delta(1-t_2)$ term can be exponentiated after convoluting the parton distribution functions with the Drell-

Yan correction term $\Delta(t_1, t_2, Q^2)$. We then have

$$W(x_1^0, x_2^0, Q^2) = R^{(1)}_{exact}(x_1^0, x_2^0, Q^2) + R^{(2)}_{app}(x_1^0, x_2^0, Q^2) +$$
$$+W^{(0)}(x_1^0, x_2^0, Q^2) \exp\left\{\left(\frac{\bar{\alpha}_s}{4\pi}\right) C_F \left[4L_1L_2 + 3L_1 + 3L_2 + 2 + 20\zeta(2)\right]\right\} \times$$
$$\times \left[1 + \left(\frac{\bar{\alpha}_s}{4\pi}\right)^2 \left\{C_F^2 \left[-8\zeta(2)\left(L_1^2 + L_2^2\right) + \left(24\zeta(3) - 24\zeta(2) + \frac{3}{2}\right)(L_1 + L_2) + \right.\right.\right.$$
$$\left. -\frac{36}{5}\zeta(2)^2 + 72\zeta(3) - 26\zeta(2) - 2\right] +$$
$$+C_A C_F \left[\left(\frac{367}{9} - 8\zeta(2)\right) L_1 L_2 + \left(\frac{193}{6} + \frac{110}{3}\zeta(2) - 12\zeta(3)\right)(L_1 + L_2) + \right.$$
$$\left. -\frac{194}{5}\zeta(2)^2 - 36\zeta(3) + \frac{2366}{9}\zeta(2) + \frac{215}{9}\right] +$$
$$\left.\left. +n_f C_F \left[-\frac{58}{9} L_1 L_2 - \left(\frac{17}{3} + \frac{20}{3}\zeta(2)\right)(L_1 + L_2) - \frac{380}{9}\zeta(2) - \frac{38}{9}\right]\right\}\right] \tag{4.2}$$

with $L_1 = \ln(1 - x_1^0)$ and $L_2 = \ln(1 - x_2^0)$. The modified coupling constant $\tilde{\alpha_s}$ is defined by

$$\tilde{\alpha_s} = \alpha_s\left(Q^2\sqrt{(1 - x_1^0)(1 - x_2^0)}\right) \tag{4.3}$$

To compare with the experimental data we calculate the following quantity

$$d\sigma^{(n)} = \int dx_{\mathrm{F}} \int d\sqrt{\tau}\, \frac{d^2\sigma^{(n)}}{d\sqrt{\tau}dx_{\mathrm{F}}} \tag{4.4}$$

where $d^2\sigma^{(n)}/(d\sqrt{\tau}dx_{\mathrm{F}})$ is the order α_s^n contribution to the differential cross section. The integrals in eq. (4.4) are taken over the $\sqrt{\tau} - x_{\mathrm{F}}$ cells given in table 2 of ref. [5]. We used the parton distribution functions given in ref. [6]. In the running coupling constant (eq. (III.4.24)) we took n_f=4 and $\Lambda_{\overline{\mathrm{MS}}}$=0.3GeV. The experimental cross section, denoted by $d\sigma_{exp}$, can be found in ref. [5].

We will study the lowest $\sqrt{\tau}$ bin of the NA10 experiment to get an impression of the O(α_s^2) correction. In fig. 1 we present the ratio $d\sigma_{exp}/d\sigma_{th}$. The theoretical cross-section $d\sigma_{th}$ is calculated in five approximations, viz.

A: $d\sigma_{th}^{\mathrm{A}} = d\sigma^{(0)}$: using the leading log approximation.

B: $d\sigma_{th}^{\mathrm{B}} = \left(d\sigma^{(0)} + d\sigma^{(1)}\right)_{app}$: the subscript 'app' denotes that only the soft contributions are taken into account.

C: $d\sigma_{th}^{\mathrm{C}} = \left(d\sigma^{(0)} + d\sigma^{(1)}\right)_{exact}$: in this case the exact first order result for the $q\bar{q}$ process [1] is used.

D: $d\sigma_{th}^{\mathrm{D}} = \left(d\sigma^{(0)} + d\sigma^{(1)} + d\sigma^{(2)}\right)$: the first order is calculated as in C.

E: $d\sigma_{th}^{\mathrm{E}} = \left(d\sigma^{(0)} + d\sigma^{(1)} + d\sigma^{(2)}\right)_{PI}$: the cross-section is calculated with the resummation formula.

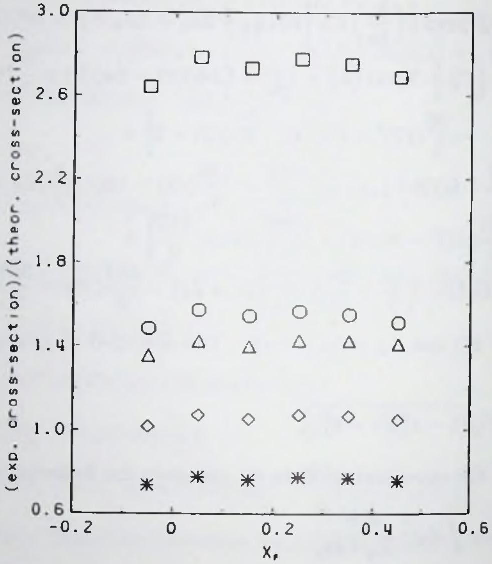

Fig. 1. The ratio $d\sigma_{exp}/d\sigma_{th}$ for the NA10 experiment
2 : $d\sigma_{th} = d\sigma^{A}_{th}$ ○ : $d\sigma_{th} = d\sigma^{B}_{th}$ △ : $d\sigma_{th} = d\sigma^{C}_{th}$
: $d\sigma_{th} = d\sigma^{D}_{th}$: $d\sigma_{th} = d\sigma^{E}_{th}$

Let us start with investigating the validity of the soft gluon approximation. Comparing $d\sigma^{B}_{th}$ and $d\sigma^{C}_{th}$, we see that it does not work that well for this $\sqrt{\tau}$ bin. However, for higher $\sqrt{\tau}$ bins the approximation improves.

Further, one observes that the second order result $d\sigma^{D}_{th}$ is in very good agreement with the experiment. Even so, one should be careful not to attach too much importance to this fact for several reasons. Firstly, the soft gluon approximation used in the second order calculation is not very reliable in this $\sqrt{\tau}$ bin (cf. $d\sigma^{B}_{th}$ vs $d\sigma^{C}_{th}$). Secondly, the experimental data may change due to the new analysis. Lastly, we find that the second order corrections ($\sim$ 95% of Born) are larger than the first order ones ($\sim$ 75% of Born). Therefore, one should start worrying about the convergence of the perturbation series and the reliability of perturbative QCD for this kind of fixed target experiments. The main problem of this type of experiments is the rather low energy, which causes the coupling constant to be large.

Finally, the perturbation improved result, $d\sigma^{E}_{th}$, gives rise to an overshoot.

5 Summary

We have presented two methods to determine the second order soft gluon contributions to the differential cross-section. Combining the two methods the differential DY correction term $\Delta(t_1, t_2, Q^2)$ could be computed completely up to order α_s^2 in the limit $t_1, t_2 \to 1$. Furthermore, quite analogously to the total cross-section, we have constructed a perturbation improved formula for the hadronic structure function $W_V(x_1^0, x_2^0, Q^2)$.

Unfortunately, we were not able to make detailed comparisons with the NA10 experiment. However, it is clear that the second order corrections turns out to be alarmingly large for fixed target experiments.

References

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Appendices

Appendix A: Some useful formulae for the Drell-Yan process

We will start with defining the parton structure functions $\hat{W}^{ab}$, with $(a,b) = (\mathrm{q},\bar{\mathrm{q}})$, (q,g), (q,q) or (g,g). For the definition of $\hat{W}^{ab}$ one has to consider the process

$$\begin{array}{lll} \text{parton } a + \text{parton } b & \rightarrow & \mathrm{V} + m \text{ partons} \\ & & \quad | \\ & & \quad \rightarrow \text{lepton pair} \end{array} \tag{A.1}$$

where V is a vector boson (γ, Z or W).

In fig. 1 we show the kinematics of the above process. Denoting the amplitude

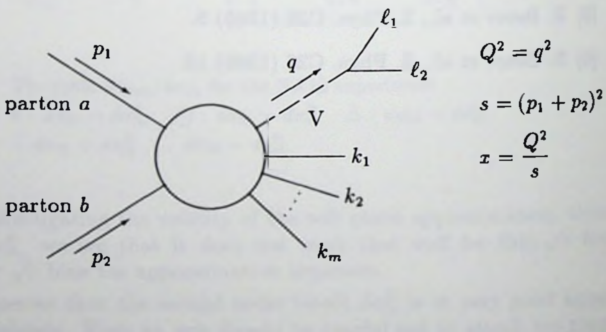


Fig. 1. The kinematics of the DY process at the parton level

of this process by A_{V}^{ab} the differential cross-section is given by

$$\begin{aligned} d\sigma^{\mathrm{V},ab} &= (2\pi)^4\, \delta\left(p_1 + p_2 - \ell_1 - \ell_2 - \sum_{i=1}^{m} k_i\right) \frac{1}{2s}\, |A_{\mathrm{V}}^{ab}|^2 \\ & \frac{d^4\ell_1}{(2\pi)^3}\, \delta^+(\ell_1^2)\, \frac{d^4\ell_2}{(2\pi)^3}\, \delta^+(\ell_2^2) \left\{\prod_{i=1}^{m} \frac{d^4k_i}{(2\pi)^3}\, \delta^+(k_i^2)\right\} \end{aligned} \tag{A.2}$$

where $|A_V^{ab}|^2$ is averaged over the initial state quantum numbers and summed over the final state ones.

The squared amplitude $|A_V^{ab}|^2$ can be decomposed into a product of two tensors

$$|A_V^{ab}|^2 = T_{\mu\nu}^{V,ab}\ L_V^{\mu\nu}\ \frac{1}{(Q^2 - M_V^2)^2 + M_V^2\Gamma_V^2} \tag{A.3}$$

where $T_{\mu\nu}^{V,ab}$ belongs to the process

$$\text{parton } a + \text{parton } b \rightarrow V + m \text{ partons} \tag{A.4}$$

and $L_V^{\mu\nu}$ is the lepton tensor, describing the decay of the vector boson into a lepton pair. Further, M_V is the mass of the vector boson and Γ_V its width.

To obtain the cross-section $d\sigma/dQ^2$ we rewrite the δ-function representing the momentum conservation, in the following way

$$\delta\left(p_1 + p_2 - \ell_1 - \ell_2 - \sum_{i=1}^{m} k_i\right) = \int d^4q\ dQ^2\ \theta(q^0)\ \delta(q^2 - Q^2)\ \delta(q - \ell_1 - \ell_2)\ \delta\left(p_1 + p_2 - q - \sum_{i=1}^{m} k_i\right) \tag{A.5}$$

Combining the above three equations the Drell-Yan cross-section is found to be

$$\begin{aligned} \frac{d\sigma^{V,ab}}{dQ^2} = & \frac{1}{16\pi^3}\frac{1}{s}\frac{1}{(Q^2 - M_V^2)^2 + M_V^2\Gamma_V^2} \\ & \int d^4\ell_1 \int d^4\ell_2\ L_V^{\mu\nu}\ \delta^+(\ell_1^2)\ \delta^+(\ell_2^2)\ \delta(q - \ell_1 - \ell_2) \\ & \int \frac{d^4q}{(2\pi)^3}\ \delta^+(q^2 - Q^2)\ \left\{\prod_{i=1}^{m} \int \frac{d^4k_i}{(2\pi)^3}\ \delta^+(k_i^2)\right\} \\ & (2\pi)^4\ \delta\left(p_1 + p_2 - q - \sum_{i=1}^{m} k_i\right) T_{\mu\nu}^{V,ab} \end{aligned} \tag{A.6}$$

This equation can be simplified, because due to gauge invariance we have

$$\int d^4\ell_1 \int d^4\ell_2\ L_V^{\mu\nu}\ \delta^+(\ell_1^2)\ \delta^+(\ell_2^2)\ \delta(q - \ell_1 - \ell_2) = \left(q^\mu q^\nu - q^2\ g^{\mu\nu}\right) L^V(q^2) \tag{A.7}$$

Using this relation the Drell-Yan cross-section becomes

$$\begin{aligned} \frac{d\sigma^{V,ab}}{dQ^2} = & -\frac{1}{16\pi^3}\frac{1}{(Q^2 - M_V^2)^2 + M_V^2\Gamma_V^2}\ \frac{Q^2}{s}\ L^V(Q^2) \\ & \int \frac{d^4q}{(2\pi)^3}\ \delta^+(q^2 - Q^2)\ \left\{\prod_{i=1}^{m} \int \frac{d^4k_i}{(2\pi)^3}\ \delta^+(k_i^2)\right\} \\ & (2\pi)^4\ \delta\left(p_1 + p_2 - q - \sum_{i=1}^{m} k_i\right)\ g^{\mu\nu}\ T_{\mu\nu}^{V,ab} \end{aligned} \tag{A.8}$$

If the coupling of the vector boson to the quarks is given by $i(A + B\gamma^5)\gamma^\mu$ one can write $g^{\mu\nu}\ T_{\mu\nu}^{V,ab}$ as

$$g^{\mu\nu}\,\mathrm{T}^{\mathrm{V},ab}_{\mu\nu} = (A^2+B^2)\,g^{\mu\nu}\,\langle M^{ab}_{\mu} M^{ab\dagger}_{\nu}\rangle_{av} \equiv C^{\mathrm{V},ab}\,g^{\mu\nu}\,\langle M^{ab}_{\mu} M^{ab\dagger}_{\nu}\rangle_{av} \tag{A.9}$$

Notice that the coefficients A and B will depend on the type of vector boson and the flavour of the quarks. Further, M^{ab}_{μ} is the amplitude of the process

$$\text{parton } a + \text{parton } b \rightarrow \mathrm{V} + m \text{ partons} \tag{A.10}$$

where the vertex $i(A+B\gamma^5)\gamma^{\mu}$ has been replaced by $i\gamma^{\mu}$. The brackets $\langle\cdots\rangle_{av}$ indicate averaging over the initial state quantum numbers and summation over the final state ones. We are now ready to define the parton structure function $\hat{W}^{ab}(x,Q^2,\varepsilon)$, viz.

$$\frac{d\sigma^{\mathrm{V},ab}}{dQ^2} = \frac{\tau}{x_1x_2}\frac{1}{8\pi^2}\frac{1}{N}\frac{1}{(Q^2-\mathrm{M}_{\mathrm{V}}^2)^2+\mathrm{M}_{\mathrm{V}}^2\Gamma_{\mathrm{V}}^2}\mathrm{L}^{\mathrm{V}}(Q^2)\;C^{\mathrm{V},ab}\hat{W}^{ab}(x,Q^2,\varepsilon) \tag{A.11}$$

with

$$\hat{W}^{ab}(x,Q^2,\varepsilon) = -\frac{N}{2\pi}\frac{1}{(1+\frac{1}{2}\varepsilon)}\int\frac{d^nq}{(2\pi)^{n-1}}\,\delta^+(q^2-Q^2)\left\{\prod_{i=1}^{m}\int\frac{d^nk_i}{(2\pi)^{n-1}}\,\delta^+(k_i^2)\right\}$$
$$(2\pi)^n\,\delta\left(p_1+p_2-q-\sum_{i=1}^{m}k_i\right)\;g^{\mu\nu}\,\langle M^{ab}_{\mu}M^{ab\dagger}_{\nu}\rangle_{av} \tag{A.12}$$

here N is the number of colours.

Notice that we have extended the number of space time dimensions from 4 to n, to regularize the divergences in the phase space integrals. Furthermore, the normalization of the parton structure functions is chosen in such a way that at the lowest order we find

$$\hat{W}^{q\bar{q}}(x,Q^2,\varepsilon) = \delta(1-x) \tag{A.13}$$

We will now give the formulae needed to determine the contributions to the Drell-Yan cross-section from the $q\bar{q}$ and qg subprocesses. Substituting eq. (A.11) in the parton model formula (eq. (II.2.1)) we find for the $q\bar{q}$ process

$$\frac{d\sigma^{\mathrm{V},q\bar{q}}}{dQ^2} = \tau\;\sigma_{\mathrm{V}}(Q^2,\mathrm{M}_{\mathrm{V}}^2)\times$$
$$\int_0^1 dx_1\int_0^1 dx_2\int_0^1 dx\;\delta(\tau-x_1x_2x)\;PD_{\mathrm{V}}^{q\bar{q}}(x_1,x_2)\;\hat{W}^{q\bar{q}}(x,Q^2,\varepsilon) \tag{A.14}$$

A similar formula can be derived for the qg subprocess (just replace $q\bar{q}$ in the above equation by qg). The explicit expressions for σ_{V} and PD_{V} are:

$$\mathrm{V}=\gamma\;:\quad \sigma_{\gamma}(Q^2) = \frac{4\pi\alpha^2}{3Q^4}\frac{1}{N} \tag{A.15}$$

$$PD_{\gamma}^{q\bar{q}}(x_1,x_2) = \sum_{q} e_q^2\left\{q(x_1)\bar{q}(x_2)+\bar{q}(x_1)q(x_2)\right\} \tag{A.16}$$

$$PD_{\gamma}^{qg}(x_1,x_2) = \sum_{q} e_q^2\left\{q(x_1)+\bar{q}(x_2)\right\}g(x_2)+\left[1\leftrightarrow 2\right] \tag{A.17}$$

$$V = Z \quad : \quad \sigma_Z(Q^2, M_Z{}^2) = \frac{\pi\alpha^2}{192\sin^4\theta_W\cos^4\theta_W}\,\frac{1}{N}\,\frac{1+\left[1-4\sin^2\theta_W\right]^2}{(Q^2-M_Z{}^2)^2+M_Z{}^2\Gamma_Z^2} \tag{A.18}$$

$$PD_Z^{q\bar{q}}(x_1,x_2) = \sum_q C_q\left\{q(x_1)\bar{q}(x_2)+\bar{q}(x_1)q(x_2)\right\} \tag{A.19}$$

$$PD_Z^{qg}(x_1,x_2) = \sum_q C_q\left\{q(x_1)+\bar{q}(x_1)\right\}g(x_2)+\left[1\leftrightarrow 2\right] \tag{A.20}$$

$$\text{with } C_q = 1+\left\{1-4|e_q|\sin^2\theta_W\right\}^2 \tag{A.21}$$

$$V = W^- \quad : \quad \sigma_W(Q^2, M_W{}^2) = \frac{\pi\alpha^2}{12\sin^4\theta_W}\,\frac{1}{N}\,\frac{1}{(Q^2-M_W{}^2)^2+M_W{}^2\Gamma_W^2} \tag{A.22}$$

$$PD_W^{q\bar{q}}(x_1,x_2) = \cos^2\theta_C\left\{\bar{u}(x_1)d(x_2)+\bar{c}(x_1)s(x_2)\right\}+ $$
$$+\sin^2\theta_C\left\{\bar{u}(x_1)s(x_2)+\bar{c}(x_1)d(x_2)\right\}+\left[1\leftrightarrow 2\right] \tag{A.23}$$

$$PD_W^{qg}(x_1,x_2) = \left\{\bar{u}(x_1)+d(x_1)+\bar{c}(x_1)+s(x_1)\right\}g(x_2)+\left[1\leftrightarrow 2\right] \tag{A.24}$$

In the above equations α is the fine structure constant and θ_W and θ_C are the weak and Cabibo mixing angles, respectively.

For completeness we will also give the formulae for the Z and W production rates

$$\sigma_Z = \frac{\pi^2\alpha}{12\sin^2\theta_W\cos^2\theta_W}\,\frac{1}{S}\,W_Z(\tau = M_Z^2/S, Q^2 = M_Z{}^2) \tag{A.25}$$

and

$$\sigma_W = \frac{\pi^2\alpha}{3\sin^2\theta_W}\,\frac{1}{S}\,W_W(\tau = M_W^2/S, Q^2 = M_W{}^2) \tag{A.26}$$

where S is the C.M. energy of the collider and W_V is the hadronic DY structure function.

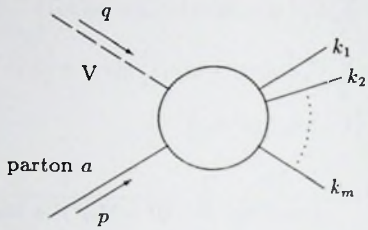
Appendix B: The parton tensor $\hat{H}^a_{\mu\nu}$

In this Appendix we will give the definitions of the parton tensor $\hat{H}^a_{\mu\nu}$ and the parton structure function $\hat{\mathcal{F}}_2^a(x, Q^2, \varepsilon)$ in the case of dimensional regularization. In order to do this we look at the process ($a = q$ or g)

$$\text{parton } a + \gamma \to m \text{ partons} \tag{B.1}$$

In fig. 2 we have given the kinematics of this process. Denoting the amplitude of this process by M^a_μ, the parton tensor $\hat{H}^a_{\mu\nu}$ is given by

$$\hat{H}^a_{\mu\nu}(x, Q^2, \varepsilon) =$$
$$\left\{\prod_{i=1}^m \int \frac{d^n k_i}{(2\pi)^{n-1}}\,\delta^+(k_i^2)\right\}(2\pi)^n\,\delta\left(q+p-\sum_{i=1}^m k_i\right)\,(M^a_\mu M^{a\dagger}_\nu)_{av} \tag{B.2}$$



$$Q^2 = -q^2 \qquad x = \frac{-q^2}{2p\cdot q}$$

Fig. 2. The kinematics of the DIS process at the parton level

where the bracktes $\langle \cdots \rangle_{av}$ indicate averaging over the quantum numbers of parton a and summation over those of the final state partons.

The parton structure function $\hat{\mathcal{F}}_2^a(x, Q^2, \varepsilon)$ can be obtained by applying the following projection operator to the parton tensor $\hat{\mathrm{H}}^a_{\mu\nu}$

$$\hat{\mathcal{F}}_2^a(x, Q^2, \varepsilon) = -\frac{1}{4\pi}\frac{1}{(1+\frac{1}{2}\varepsilon)}\left\{g^{\mu\nu} + 12\frac{x^2}{q^2}\left(1+\frac{1}{3}\varepsilon\right)p^\mu p^\nu\right\}\hat{\mathrm{H}}^a_{\mu\nu} \tag{B.3}$$

Comparing the above expression with eq. (II.2.14) we see that extra factors ε's appear due to the n-dimensional regularization scheme.

Appendix C: The renormalization group equations

An important tool for the study of QCD is the so called renormalization group equation (RGE) [1]. It can be used to predict the asymptotic behaviour of Green's functions.

Let us look at an amputated Green's function with k external gluons an ℓ external quarks, which we denote by

$$\mathrm{G}_{\mathrm{u}}^{k,\ell}(p, \alpha_s^{\mathrm{u}}, \varepsilon) \tag{C.1}$$

The subscript 'u' indicates that we are dealing with unrenormalized quantities. The p represents all the external momenta and $\varepsilon = \mathrm{n} - 4$, with n the number of space time dimensions (dimensional regularization is assumed to have been used).

The Green's function $\mathrm{G}_{\mathrm{u}}^{k,\ell}$ contains UV divergences, which appear as poles in ε. It can be made finite by applying the renormalization procedure

$$A^a_{\mu,\mathrm{u}} \longrightarrow Z_3^{\frac{1}{2}}(\mu, \alpha_s^{\mathrm{R}}, \varepsilon)\, A^a_{\mu,\mathrm{R}} \tag{C.2}$$

$$\psi_{\mathrm{u}} \longrightarrow Z_2^{\frac{1}{2}}(\mu, \alpha_s^{\mathrm{R}}, \varepsilon)\, \psi_{\mathrm{R}} \tag{C.3}$$

$$\alpha_s^{\mathrm{u}} \longrightarrow Z_g^2(\mu, \alpha_s^{\mathrm{R}}, \varepsilon)\, \alpha_s^{\mathrm{R}} \tag{C.4}$$

where A^a_μ, ψ and α_s are the gluon field, the quark field and the strong coupling constant, respectively. The Z's are the renormalization constants. The renormalized Green's function $G_R^{k,\ell}$ is given by

$$G_R^{k,\ell}(p,\alpha_s^R,\mu) = \lim_{\varepsilon\to 0} Z_3^{\frac{1}{2}k}(\mu,\alpha_s^R,\varepsilon)\, Z_2^{\frac{1}{2}\ell}(\mu,\alpha_s^R,\varepsilon)\, G_u^{k,\ell}\left(p, Z_g^2(\mu,\alpha_s^R,\varepsilon)\alpha_s^R,\varepsilon\right) \tag{C.5}$$

The parameter μ, appearing in the above equations, is the so called renormalization scale. It is introduced in the $\overline{\text{MS}}$ scheme to ensure that the renormalized fields and the renormalized coupling constant keep their correct physical mass dimensions. The choice of this parameter μ is completely arbitrary. Of course, the unrenormalized quantities do not depend on the renormalization scale μ. This can be used to derive the RGE for the renormalized Green's function $G_R^{k,\ell}$. We have

$$\mu\frac{d}{d\mu}G_u^{k,\ell}(p,\alpha_s^u,\varepsilon)\Big|_{\alpha_s^u \text{ fixed}} = 0 \tag{C.6}$$

Using eq. (C.5) the above equation can be rewritten into

$$\left\{\mu\frac{\partial}{\partial\mu} + \beta(\alpha_s^R)\frac{\partial}{\partial\alpha_s^R} - \frac{k}{2}\gamma_G(\alpha_s^R) - \frac{\ell}{2}\gamma_F(\alpha_s^R)\right\} G_R^{k,\ell}(p,\alpha_s^R,\mu) = 0 \tag{C.7}$$

where we have introduced

$$\beta(\alpha_s^R) = \lim_{\varepsilon\to 0}\mu\frac{d}{d\mu}\alpha_s^R(\mu,\alpha_s^u,\varepsilon)\Big|_{\alpha_s^u \text{ fixed}} \tag{C.8}$$

$$\gamma_G(\alpha_s^R) = \lim_{\varepsilon\to 0}\mu\frac{d}{d\mu}\ln\left(Z_3(\mu,\alpha_s^u,\varepsilon)\right)\Big|_{\alpha_s^u \text{ fixed}} \tag{C.9}$$

$$\gamma_F(\alpha_s^R) = \lim_{\varepsilon\to 0}\mu\frac{d}{d\mu}\ln\left(Z_2(\mu,\alpha_s^u,\varepsilon)\right)\Big|_{\alpha_s^u \text{ fixed}} \tag{C.10}$$

This formula is referred to as the renormalization group equation. The functions γ_G and γ_F are called the anomalous dimensions of the gluon and quark fields, respectively.

The aim is to use the RGE to determine the asymptotic behaviour of the Green's function $G_R^{k,\ell}$. For simplicity let us consider the equation

$$\left\{\mu\frac{\partial}{\partial\mu} + \beta(\alpha_s^R)\frac{\partial}{\partial\alpha_s^R} - \gamma(\alpha_s^R)\right\} G_R(p,\alpha_s^R,\mu) = 0 \tag{C.11}$$

We want to know what happens when $p \to \lambda p$.

Of course the above equation still holds when p is replaced by λp. Further, one has on dimensional grounds

$$G_R(\lambda p,\alpha_s^R,\mu) = \mu^D\Phi\left(\frac{\lambda p}{\mu},\alpha_s^R\right) \tag{C.12}$$

where Φ is a dimensionless function and D is the canonical mass dimension of G_R. From this equation one can deduce

$$\left\{\mu\frac{\partial}{\partial\mu}+\lambda\frac{\partial}{\partial\lambda}-D\right\}\ \mathrm{G_R}(\lambda p,\alpha_s^{\mathrm{R}},\mu)=0 \tag{C.13}$$

Combining eqs. (C.11) and (C.13) and introducing $t=-\ln\lambda$, we find

$$\left\{\frac{\partial}{\partial t}+\beta(\alpha_s^{\mathrm{R}})\frac{\partial}{\partial\alpha_s^{\mathrm{R}}}+D-\gamma(\alpha_s^{\mathrm{R}})\right\}\ \mathrm{G_R}(\mathrm{e}^{-t}p,\alpha_s^{\mathrm{R}},\mu)=0 \tag{C.14}$$

Defining the running coupling constant $\alpha_s(t)$ by

$$\left\{\begin{array}{lcl}\frac{d}{dt}\alpha_s(t) & = & \beta(\alpha_s(t))\\ & & \\ \alpha_s(0) & = & \alpha_s^{\mathrm{R}}\end{array}\right. \tag{C.15}$$

we finally have

$$\left\{\frac{d}{dt}+D-\gamma(\alpha_s^{\mathrm{R}})\right\}\ \mathrm{G_R}(\mathrm{e}^{-t}p,\alpha_s(t),\mu)=0 \tag{C.16}$$

Integrating the above equation we get

$$\mathrm{G_R}(\mathrm{e}^{t}p,\alpha_s^{\mathrm{R}},\mu)=\mathrm{G_R}(p,\alpha_s(t),\mu)\exp\left\{Dt-\int_0^t\gamma(\alpha_s(t'))\,dt'\right\} \tag{C.17}$$

Assuming $\alpha_s(t)$ goes to a fixed value α_s^{fix} for large t, one finds

$$\mathrm{G_R}(\lambda p,\alpha_s^{\mathrm{R}},\mu)\sim\lambda^{D-\gamma(\alpha_s^{fix})} \tag{C.18}$$

Contrary to the naively expected behaviour (λ^D), we find an extra term $-\gamma(\alpha_s^{fix})$ in the exponent. This explains the name anomalous dimension for the function γ.

Appendix D: The phase space integrals

The m-particle phase space integral for the DY process

$$\mathrm{q}(p_1)+\bar{\mathrm{q}}(p_2)\rightarrow\mathrm{V}(q)+\mathrm{g}(k_1)+\cdots+\mathrm{g}(k_{m-1}) \tag{D.1}$$

is defined by

$$\begin{aligned}\int dPS_m^{\mathrm{DY}} &= \int\frac{d^nq}{(2\pi)^{n-1}}\,\delta^+(q^2-Q^2)\prod_{j=1}^{m-1}\left(\int\frac{d^nk_j}{(2\pi)^{n-1}}\,\delta^+(k_j^2)\right)\times\\ &\qquad (2\pi)^n\delta^n\left(p_1+p_2-q-\sum_{j=1}^{m-1}k_j\right)\end{aligned} \tag{D.2}$$

For the DI process

$$\mathrm{V}(q)+\mathrm{q}(p_1)\rightarrow\mathrm{q}(p_2)+\mathrm{g}(k_1)+\cdots+\mathrm{g}(k_{m-1}) \tag{D.3}$$

we have

$$\int dPS_m^{\rm DI} = \int \frac{d^n p_2}{(2\pi)^{n-1}}\, \delta^+(p_2^2) \prod_{j=1}^{m-1} \left(\int \frac{d^n k_j}{(2\pi)^{n-1}}\, \delta^+(k_j^2) \right) \times$$
$$(2\pi)^n \delta^n \left(p_1 + q - p_2 - \sum_{j=1}^{m-1} k_j \right) \tag{D.4}$$

In the subsequent part of this Appendix we will work out the 2 and 3-particle phase space integrals. The 3-particle phase space integrals will be calculated in two different frames.

The 2-particle phase space integrals

For the DY process

$$\mathrm{q}(p_1) + \bar{\mathrm{q}}(p_2) \to \mathrm{V}(q) + \mathrm{g}(k) \tag{D.5}$$

the 2-particle phase space is given by

$$\int dPS_2^{\rm DY} = \frac{1}{(2\pi)^{n-2}} \int d^n q \int d^n k\; \delta^+(q^2 - Q^2)\; \delta^+(k^2)\; \delta^n(p_1 + p_2 - q - k) \tag{D.6}$$

In the C.M. frame of the incoming particles the momenta can be parametrized as

$$\begin{aligned} p_1 &= \frac{1}{2}\sqrt{s}\,(1,0,\cdots,0,0,1) \\ p_2 &= \frac{1}{2}\sqrt{s}\,(1,0,\cdots,0,0,-1) \\ k &= \frac{(s-Q^2)}{2\sqrt{s}}\,(1,0,\cdots,0,0,\cos\theta) \end{aligned} \tag{D.7}$$

where $s = (p_1 + p_2)^2$. In this frame eq. (D.6) is equal to

$$\int dPS_2^{\rm DY} = 2^{4-2n} \frac{\pi^{1-\frac{1}{2}n}}{\Gamma(\frac{1}{2}n - 1)} \left(\frac{s - Q^2}{s} \right)^{n-3} s^{\frac{1}{2}n-2} \int_0^\pi d\theta (\sin\theta)^{n-3} \tag{D.8}$$

Introducing the variables

$$x = \frac{Q^2}{s} \qquad \text{and} \qquad y = \frac{1}{2}(1 + \cos\theta) \tag{D.9}$$

we finally have

$$\int dPS_2^{\rm DY} = \frac{2\pi}{(4\pi)^{\frac{1}{2}n}} \frac{(Q^2)^{\frac{1}{2}n-2}}{\Gamma(\frac{1}{2}n - 1)}\, x^{2-\frac{1}{2}n}\, (1-x)^{n-3} \int_0^1 dy\, \{y(1-y)\}^{\frac{1}{2}n-2} \tag{D.10}$$

Quite analogously we find for the DIS process

$$\mathrm{V}(q) + \mathrm{q}(p_1) \to \mathrm{q}(p_2) + \mathrm{g}(k) \tag{D.11}$$

the following results. By definition we have

$$\int dPS_2^{\rm DI} = \frac{1}{(2\pi)^{n-2}} \int d^n p_2 \int d^n k \; \delta^+(p_2^2) \; \delta^+(k^2) \; \delta^n(p_1 + q - p_2 - k) \tag{D.12}$$

Again we parametrize the momenta in the C.M. system of the incoming particles, we then find ($s = (p_1 + q)^2$)

$$\begin{aligned} p_1 &= \frac{(s-q^2)}{2\sqrt{s}}(1,0,\cdots,0,0,1) \\ q &= \left(\frac{s+q^2}{2\sqrt{s}},0,\cdots,0,0,-\frac{(s-q^2)}{2\sqrt{s}}\right) \\ k &= \frac{1}{2}\sqrt{s}(1,0,\cdots,0,0,\cos\theta) \end{aligned} \tag{D.13}$$

and

$$\int dPS_2^{\rm DI} = 2^{4-2n} \frac{\pi^{1-\frac{1}{2}n}}{\Gamma(\frac{1}{2}n-1)} s^{\frac{1}{2}n-2} \int_0^\pi d\theta (\sin\theta)^{n-3} \tag{D.14}$$

For practical calculations it is convenient to use

$$x = \frac{-q^2}{2p_1 \cdot q} = \frac{-q^2}{(s-q^2)} \qquad \text{and} \qquad y = \frac{1}{2}(1+\cos\theta) \tag{D.15}$$

In terms of these variables the DI 2-particle phase space integral becomes

$$\int dPS_2^{\rm DI} = \frac{2\pi}{(4\pi)^{\frac{1}{2}n}} \frac{(-q^2)^{\frac{1}{2}n-2}}{\Gamma(\frac{1}{2}n-1)} \left(\frac{1-x}{x}\right)^{\frac{1}{2}n-2} \int_0^1 dy \, \{y(1-y)\}^{\frac{1}{2}n-2} \tag{D.16}$$

The 3-particle phase space integrations

We will present the 3-particle phase space integrals in two different frames. We will start with the parametrization of the phase space integrals in the C.M. frame of the incoming particles (see refs. [2,3]). Then we will compute the integrals in the C.M. system of the two outgoing gluons. The latter frame was first discussed in ref. [4].

Frame I: The C.M. of the incoming particles [2,3]

We will start with the DY process

$$\mathrm{q}(p_1) + \bar{\mathrm{q}}(p_2) \rightarrow \mathrm{V}(q) + \mathrm{g}(k_1) + \mathrm{g}(k_2) \tag{D.17}$$

In this case we have (see eq. (D.2))

$$\int dPS_3^{\rm DY} = \frac{1}{(2\pi)^{2n-3}} \int d^n q \int d^n k_1 \int d^n k_2 \; \delta^+(q^2 - Q^2) \; \delta^+(k_1^2) \; \delta^+(k_2^2) \; \delta^n(p_1 + p_2 - q - k_1 - k_2) \tag{D.18}$$

In this frame the momenta can be parametrized in the following way

$$
\begin{aligned}
p_1 &= \frac{1}{2}\sqrt{s}(1,0,\cdots,0,0,1) \\
p_2 &= \frac{1}{2}\sqrt{s}(1,0,\cdots,0,0,-1) \\
k_1 &= \frac{s-s_2}{2\sqrt{s}}(1,0,\cdots,0,\sin\theta,\cos\theta) \\
k_2 &= \frac{s-s_1}{2\sqrt{s}}(1,0,\cdots,\sin\chi\sin\phi,\cos\chi\sin\theta+\sin\chi\cos\phi\cos\theta, \\
&\qquad \cos\chi\cos\theta-\sin\chi\cos\phi\sin\theta)
\end{aligned} \tag{D.19}
$$

with $s=(p_1+p_2)^2$, $s_1=(k_1+q)^2$ and $s_2=(k_2+q)^2$. In the above expressions χ is the angle between the momenta k_1 and k_2. It satisfies the relation

$$
\sin^2(\frac{1}{2}\chi) = \frac{s(s+Q^2-s_1-s_2)}{(s-s_1)(s-s_2)} \tag{D.20}
$$

Further, the DY 3-particle phase space integral is given by

$$
\begin{aligned}
\int dPS_3^{\mathrm{DY}} &= \frac{1}{(4\pi)^n}\frac{s^{1-\frac{1}{2}n}}{\Gamma(n-3)}\int_0^\pi d\theta\int_0^\pi d\phi\,(\sin\theta)^{n-3}(\sin\phi)^{n-4}\times \\
&\int_{Q^2}^{s} ds_1\int_{sQ^2/s_1}^{s+Q^2-s_1} ds_2\left\{(s_1s_2-sQ^2)(s+Q^2-s_1-s_2)\right\}^{\frac{1}{2}n-2}
\end{aligned} \tag{D.21}
$$

It is useful to rewrite the above equation into

$$
\begin{aligned}
\int dPS_3^{\mathrm{DY}} &= \frac{1}{(4\pi)^n}\frac{(Q^2)^{n-3}}{\Gamma(n-3)}x^{3-n}(1-x)^{2n-5}\int_0^\pi d\theta\int_0^\pi d\phi\,(\sin\theta)^{n-3}(\sin\phi)^{n-4} \\
&\int_0^1 dy\int_0^1 dz\,\{z(1-z)\}^{\frac{1}{2}n-2}\{y(1-y)\}^{n-3}\{1-(1-x)y\}^{1-\frac{1}{2}n}
\end{aligned} \tag{D.22}
$$

where the variables x, y and z are defined by

$$
\begin{aligned}
x &= \frac{Q^2}{s} \\
s_1 &= s\{1-(1-x)y\} \\
s_2 &= s\frac{\{x+(1-x)^2y(1-y)(1-z)\}}{1-(1-x)y}
\end{aligned} \tag{D.23}
$$

For the DIS process

$$
\mathrm{V}(q)+\mathrm{q}(p_1)\to\mathrm{q}(p_2)+\mathrm{g}(k_1)+\mathrm{g}(k_2) \tag{D.24}
$$

the 3-particle phase space integral is defined as

$$
\begin{aligned}
\int dPS_3^{\mathrm{DI}} &= \frac{1}{(2\pi)^{2n-3}}\int d^np_2\int d^nk_1\int d^nk_2\;\delta^+(p_2^2)\;\delta^+(k_1^2)\;\delta^+(k_2^2) \\
&\qquad \delta^n(p_1+q-p_2-k_1-k_2)
\end{aligned} \tag{D.25}
$$

Quite similar to the DY process the momenta can be written as

$$\begin{aligned}
p_1 &= \frac{(s-q^2)}{2\sqrt{s}}(1,0,\cdots,0,0,1) \\
q &= \left(\frac{s+q^2}{2\sqrt{s}},0,\cdots,0,0,-\frac{(s-q^2)}{2\sqrt{s}}\right) \\
k_1 &= \frac{s-s_2}{2\sqrt{s}}(1,0,\cdots,0,\sin\theta,\cos\theta) \qquad \text{(D.26)}\\
k_2 &= \frac{s-s_1}{2\sqrt{s}}(1,0,\cdots,\sin\chi\sin\phi,\cos\chi\sin\theta+\sin\chi\cos\phi\cos\theta, \\
&\qquad \cos\chi\cos\theta-\sin\chi\cos\phi\sin\theta) \qquad \text{(D.27)}
\end{aligned}$$

where $s=(p_1+q)^2$, $s_1=(k_1+p_2)^2$ and $s_2=(k_2+p_2)^2$. Again χ is the angle between k_1 and k_2, which now satisfies

$$\sin^2(\frac{1}{2}\chi)=\frac{s(s-s_1-s_2)}{(s-s_1)(s-s_2)} \qquad \text{(D.28)}$$

Further, we have

$$\begin{aligned}
\int dPS_3^{\rm DI} &= \frac{1}{(4\pi)^n}\frac{s^{1-\frac{1}{2}n}}{\Gamma(n-3)}\int_0^\pi d\theta\int_0^\pi d\phi\,(\sin\theta)^{n-3}(\sin\phi)^{n-4}\times \\
&\quad \int_0^s ds_1\int_0^{s-s_1}\{s_1s_2(s-s_1-s_2)\}^{\frac{1}{2}n-2} \qquad \text{(D.29)}
\end{aligned}$$

Introducing the variables x, y and z

$$\begin{aligned}
x &= \frac{-q^2}{2p_1\cdot q}=\frac{-q^2}{(s-q^2)} \\
s_1 &= sy \\
s_2 &= (1-y)zs \qquad \text{(D.30)}
\end{aligned}$$

we find

$$\begin{aligned}
\int dPS_3^{\rm DI} &= \frac{1}{(4\pi)^n}\frac{(-q^2)^{n-3}}{\Gamma(n-3)}\left(\frac{1-x}{x}\right)^{n-3}\int_0^\pi d\theta\int_0^\pi d\phi\,(\sin\theta)^{n-3}(\sin\phi)^{n-4} \\
&\quad \int_0^1 dy\int_0^1 dz\; y^{\frac{1}{2}n-2}(1-y)^{n-3}\{z(1-z)\}^{\frac{1}{2}n-2} \qquad \text{(D.31)}
\end{aligned}$$

Frame II: The C.M. of the two outgoing gluons [4]

We will start with the Drell-Yan process given in eq. (D.17). Introducing $K=k_1+k_2$ and $s_{12}=K^2$ we can rewrite eq. D.18 into

$$\begin{aligned}
&\int dPS_3^{\rm DY}=\frac{1}{(2\pi)^{2n-3}} \\
&\quad\int d^nq\int d^nK\int ds_{12}\,\delta^+(q^2-Q^2)\,\delta^+(K^2-s_{12})\,\delta^n(p_1+p_2-q-K)\times \\
&\quad\int d^nk_1\int d^nk_2\,\delta^+(k_1^2)\,\delta^+(k_2^2)\,\delta^n(K-k_1-k_2) \qquad \text{(D.32)}
\end{aligned}$$

The 3-particle phase space is now divided into two Lorentz invariant parts. The first part is most easily computed in the C.M. frame of the incoming particles. The second one is calculated in the C.M. frame of k_1 and k_2. In this frame the momenta can be parametrized as follows

$$\begin{aligned}
k_1 &= \frac{1}{2}\sqrt{s_{12}}\,(1,0,\cdots,\sin\phi\sin\theta,\cos\phi\sin\theta,\cos\theta)\\
k_2 &= \frac{1}{2}\sqrt{s_{12}}\,(1,0,\cdots,-\sin\phi\sin\theta,-\cos\phi\sin\theta,-\cos\theta)\\
p_1 &= \frac{(s-t)}{2\sqrt{s_{12}}}\,(1,0,\ldots,0,0,1)\\
q &= \left(\frac{s-Q^2-s_{12}}{2\sqrt{s_{12}}},0,\cdots,0,|\vec{q}|\sin\psi,|\vec{q}|\cos\psi\right)\\
&\quad |\vec{q}| = \frac{\sqrt{\lambda(s,Q^2,s_{12})}}{2\sqrt{s_{12}}}\\
&\quad \cos\psi = \frac{(s-Q^2)(u-Q^2)-s_{12}(t+Q^2)}{(s-t)\sqrt{\lambda(s,Q^2,s_{12})}}
\end{aligned} \tag{D.33}$$

where $t = 2p_1\cdot q$, $u = 2p_2\cdot q$, $s = (p_1+p_2)^2$, $s_{12} = s-t-u+Q^2$ and $\lambda(a,b,c)$ is the Källen function. Using this parametrization we find

$$\begin{aligned}
\int dPS_3^{\mathrm{DY}} &= \frac{1}{(4\pi)^n}\frac{s^{1-\frac{1}{2}n}}{\Gamma(n-3)}\int_0^\pi d\theta\int_0^\pi d\phi\,(\sin\theta)^{n-3}(\sin\phi)^{n-4}\times\\
&\quad\int_{Q^2}^{s} du\int_{sQ^2/u}^{s+Q^2-u} dt\,\{ut-sQ^2\}^{\frac{1}{2}n-2}(s_{12})^{\frac{1}{2}n-2}
\end{aligned} \tag{D.34}$$

Introducing the variables x, y and z

$$\begin{aligned}
x &= \frac{Q^2}{s}\\
u &= s(1-(1-x)y)\\
t &= s\frac{\{x+(1-x)^2y(1-y)(1-z)\}}{1-(1-x)y}
\end{aligned} \tag{D.35}$$

we get

$$\begin{aligned}
\int dPS_3^{\mathrm{DY}} &= \frac{1}{(4\pi)^n}\frac{(Q^2)^{n-3}}{\Gamma(n-3)}x^{3-n}(1-x)^{2n-5}\int_0^\pi d\theta\int_0^\pi d\phi\,(\sin\theta)^{n-3}(\sin\phi)^{n-4}\\
&\quad\int_0^1 dy\int_0^1 dz\,\{y(1-y)\}^{n-3}\,\{z(1-z)\}^{\frac{1}{2}n-2}\,\{1-y(1-z)\}^{1-\frac{1}{2}n}
\end{aligned} \tag{D.36}$$

The phase space integral for the DIS process, given in eq. (D.25) can be handled in a similar way. In the C.M. frame of k_1 and k_2 we can parametrize the momenta as

$$k_1 = \frac{1}{2}\sqrt{s_{12}}\,(1,0,\cdots,\sin\phi\sin\theta,\cos\phi\sin\theta,\cos\theta)$$

$$
\begin{aligned}
k_2 &= \frac{1}{2}\sqrt{s_{12}}\,(1,0,\cdots,-\sin\phi\sin\theta,-\cos\phi\sin\theta,-\cos\theta) \\
p_1 &= \frac{(s-t-q^2)}{2\sqrt{s_{12}}}\,(1,0,\cdots,0,0,1) \\
p_2 &= \frac{(s-s_{12})}{2\sqrt{s_{12}}}\,(1,0,\cdots,0,\sin\psi,\cos\psi) \\
& \cos\psi = 1-\frac{2s_{12}t}{(s-t-q^2)(s-s_{12})}
\end{aligned} \tag{D.37}
$$

with $t = 2p_1 \cdot p_2$, $u = 2p_2 \cdot q$, $s = (p_1+q)^2$ and $s_{12} = s-t-u$. Applying this parametrization to eq. D.25 we obtain

$$
\begin{aligned}
\int dPS_3^{\mathrm{DI}} &= \frac{1}{(4\pi)^n}\frac{(s-q^2)^{3-n}}{\Gamma(n-3)}\int_0^\pi d\theta\int_0^\pi d\phi\;(\sin\theta)^{n-3}(\sin\phi)^{n-4} \\
&\int_0^{s-q^2} dt\int_{tq^2/(s-q^2)}^{s-t} du\;(s_{12})^{\frac{1}{2}n-2}(t)^{\frac{1}{2}n-2}\left\{(s-q^2)u-q^2t\right\}^{\frac{1}{2}n-2}
\end{aligned} \tag{D.38}
$$

This result can be rewritten into

$$
\begin{aligned}
\int dPS_3^{\mathrm{DI}} &= \frac{1}{(4\pi)^n}\frac{(-q^2)^{n-3}}{\Gamma(n-3)}\left(\frac{1-x}{x}\right)^{n-3}\int_0^\pi d\theta\int_0^\pi d\phi\;(\sin\theta)^{n-3}(\sin\phi)^{n-4} \\
&\int_0^1 dy\int_0^1 dz\;y^{\frac{1}{2}n-2}(1-y)^{n-3}\{z(1-z)\}^{\frac{1}{2}n-2}
\end{aligned} \tag{D.39}
$$

where the variables x, y and z are defined by

$$
\begin{aligned}
x &= \frac{-q^2}{2p_1\cdot q} = \frac{-q^2}{(s-q^2)} \\
u &= \{1-x-y-(1-x)(1-y)z\}\,(s-q^2) \\
t &= y(s-q^2)
\end{aligned} \tag{D.40}
$$

Appendix E: The second order quark form factor

In this Appendix we present the results for the second order quark form factor. The techniques used in our computations can be found in ref. [2]. The results of the diagrams in fig. III.3 (see page 33) are

$$
S = C_1\left\{-\frac{8}{\varepsilon^3}+\frac{14}{\varepsilon^2}+\frac{1}{\varepsilon}\left(2\zeta(2)-\frac{53}{2}\right)+\frac{355}{8}-\frac{7}{2}\zeta(2)-\frac{32}{3}\zeta(3)\right\} \tag{E.1}
$$

$$
\begin{aligned}
QL &= C_2\left\{-\frac{8}{3}\frac{1}{\varepsilon^3}+\frac{56}{9}\frac{1}{\varepsilon^2}-\frac{1}{\varepsilon}\left(\frac{2}{3}\zeta(2)+\frac{353}{27}\right)+\right. \\
&\left.+\frac{7541}{324}+\frac{14}{9}\zeta(2)-\frac{26}{9}\zeta(3)\right\}
\end{aligned} \tag{E.2}
$$

$$
GL = C_3\left\{\frac{20}{3}\frac{1}{\varepsilon^3}-\frac{152}{9}\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}\left(\frac{5}{3}\zeta(2)+\frac{1969}{54}\right)+\right.
$$

$$+\frac{65}{9}\zeta(3) - \frac{38}{9}\zeta(2) - \frac{43165}{648}\Big\} \tag{E.3}$$

$$QV = C_4\left\{\frac{8}{\varepsilon^3} + \frac{1}{\varepsilon^2}(8\zeta(2) - 22) + \frac{1}{\varepsilon}\left(\frac{109}{2} - 20\zeta(2) - 4\zeta(3)\right) + \right.$$

$$\left. +\frac{8}{5}\zeta(2)^2 + \frac{59}{3}\zeta(3) + \frac{91}{2}\zeta(2) - \frac{911}{8}\right\} \tag{E.4}$$

$$GV = C_3\left\{\frac{4}{\varepsilon^4} - \frac{1}{\varepsilon^2}(\zeta(2) + 5) + \frac{1}{\varepsilon}\left(\frac{16}{3}\zeta(3) - \zeta(2) + \frac{73}{4}\right) + \right.$$

$$\left. +\frac{1}{2}\zeta(3) - \frac{57}{40}\zeta(2)^2 + \frac{15}{4}\zeta(2) - \frac{663}{16}\right\} \tag{E.5}$$

$$C = C_4\left\{\frac{16}{\varepsilon^4} - \frac{32}{\varepsilon^3} + \frac{1}{\varepsilon^2}(64 - 28\zeta(2)) + \frac{1}{\varepsilon}\left(\frac{244}{3}\zeta(3) + 32\zeta(2) - 116\right) + \right.$$

$$\left. +204 - 58\zeta(2) - \frac{53}{2}\zeta(2)^2 - \frac{380}{3}\zeta(3)\right\} \tag{E.6}$$

$$L = C_1\left\{\frac{16}{\varepsilon^4} - \frac{16}{\varepsilon^3} + \frac{1}{\varepsilon^2}(4\zeta(2) + 34) + \frac{1}{\varepsilon}\left(4\zeta(2) - \frac{92}{3}\zeta(3) - \frac{101}{2}\right) + \right.$$

$$\left. +\frac{152}{3}\zeta(3) + \frac{103}{10}\zeta(2)^2 - \frac{35}{2}\zeta(2) + \frac{631}{8}\right\} \tag{E.7}$$

with

$$C_1 = g_n^4\, C_F^2\, (-q^2)^\varepsilon$$
$$C_2 = g_n^4\, n_f C_F\, (-q^2)^\varepsilon$$
$$C_3 = g_n^4\, C_A C_F\, (-q^2)^\varepsilon$$
$$C_4 = g_n^4\left(C_F - \frac{1}{2}C_A\right) C_F(-q^2)^\varepsilon$$

The second order form factor is given by

$$F^{(2)}(q^2) = 2S + QL + GL + 2QV + 2GV + C + L \tag{E.8}$$

Appendix F: One loop scalar integrals

Using the Passarino-Veltman reduction scheme [5] the calculation of the virtual corrections to the one gluon bremsstrahlung process comes down to computing the following four scalar integrals.

1. For the selfenergy we have (see fig. 3.a) :

$$\int \frac{d^n l}{(2\pi)^n} \frac{1}{l^2(l+p)^2} = -i(4\pi)^{-\frac{1}{2}n}(-p^2)^{\frac{1}{2}\varepsilon}\frac{2}{\varepsilon}\,\frac{\Gamma(1-\frac{1}{2}\varepsilon)\Gamma^2(1+\frac{1}{2}\varepsilon)}{\Gamma(2+\varepsilon)} \tag{F.1}$$

2. For the vertex correction we have two possibilities (see fig. 3.b).

   (a) $p_1^2 = p_2^2 = 0,\ p_3^2 \neq 0$

$$\int \frac{d^n l}{(2\pi)^n} \frac{1}{l^2(l+p_1)^2(l-p_2)^2} =$$

$$-i(4\pi)^{-\frac{1}{2}n}(-p_3^2)^{-1+\frac{1}{2}\varepsilon}\frac{4}{\varepsilon^2}\,\frac{\Gamma(1-\frac{1}{2}\varepsilon)\Gamma^2(1+\frac{1}{2}\varepsilon)}{\Gamma(1+\varepsilon)} \tag{F.2}$$

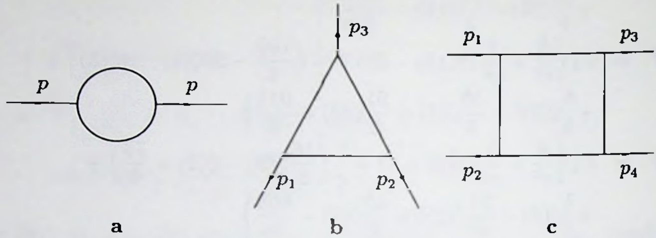


Fig. 3. Scalar diagrams

(b) $p_1^2 \neq 0$, $p_2^2 \neq 0$ and $p_3^2 = 0$

$$\int \frac{d^n l}{(2\pi)^n} \frac{1}{l^2(l+p_1)^2(l-p_2)^2} = \tag{F.3}$$

$$i(4\pi)^{-\frac{1}{2}n} \frac{4}{\varepsilon^2} \frac{\Gamma(1-\frac{1}{2}\varepsilon)\Gamma^2(1+\frac{1}{2}\varepsilon)}{\Gamma(1+\varepsilon)} \frac{1}{(p_1^2 - p_2^2)} \left\{(-p_1^2)^{\frac{1}{2}\varepsilon} - (-p_2^2)^{\frac{1}{2}\varepsilon}\right\}$$

3. In the case of $p_1^2 = p_2^2 = p_3^2 = 0$, $p_4^2 \neq 0$, the four point function is given by (see fig. 3.c) :

$$\int \frac{d^n l}{(2\pi)^n} \frac{1}{l^2(l+p_1)^2(l-p_2)^2(l-p_2-p_3)^2} =$$

$$-i(4\pi)^{-\frac{1}{2}n} \frac{8}{\varepsilon^2} \frac{\Gamma(1-\frac{1}{2}\varepsilon)\Gamma^2(1+\frac{1}{2}\varepsilon)}{\Gamma(1+\varepsilon)} \left\{ \frac{(-p_4^2)^{\frac{1}{2}\varepsilon}}{P_{12}P_{23}} F(1,\frac{\varepsilon}{2};1+\frac{\varepsilon}{2};-\frac{P_{13}\,p_4^2}{P_{12}P_{23}}) + \right.$$

$$\left. - \frac{(-P_{12})^{\frac{1}{2}\varepsilon}}{P_{12}P_{23}} F(1,\frac{\varepsilon}{2};1+\frac{\varepsilon}{2};-\frac{P_{13}}{P_{23}}) - \frac{(-P_{23})^{\frac{1}{2}\varepsilon}}{P_{12}P_{23}} F(1,\frac{\varepsilon}{2};1+\frac{\varepsilon}{2};-\frac{P_{13}}{P_{12}}) \right\} \tag{F.4}$$

with $P_{ij} = (p_i + p_j)^2$ and $F(a,b;c;x)$ is the hypergeometric function.

Appendix G: The 3-particle phase space soft integrals

For the calculation of the two gluon bremsstrahlung and the quark pair production processes we have to compute 3-particle phase space integrals. In section III.3 we are only interested in the $\delta(1-x)$ part of the second order contributions. This part can be isolated by introducing soft integrals, defined by

$$\text{"soft integral"} = \int_{1-\delta}^{1} dx \int dPS_3\, |A|^2 \tag{G.1}$$

where x is the Drell-Yan/Bjorken variable, δ the infrared cut-off introduced in eq. (II.5.8) and dPS_3 represents either the DY or DI phase space integration. As δ is assumed to be very small, we may put $x = 1$ in $|A|^2$, wherever it is possible. This reduces the number of integrals that we have to compute considerably.
The notation we use for our propagators is

$$P_{ij} = (l_i + l_j)^2 \tag{G.2}$$

where the momenta l_i are defined as

	DY	DI
l_1	p_1	p_1
l_2	p_2	$-p_2$
l_3	$-k_1$	$-k_1$
l_4	$-k_2$	$-k_2$
l_5	$-q$	q

When working in the C.M. frame of the outgoing gluons, we perform partial fractioning in the following way

$$\begin{aligned} \frac{1}{P_{13}P_{14}} &= \frac{1}{S_1}\left\{\frac{1}{P_{13}} + \frac{1}{P_{14}}\right\} \\ \frac{1}{P_{23}P_{24}} &= \frac{1}{S_2}\left\{\frac{1}{P_{23}} + \frac{1}{P_{24}}\right\} \end{aligned} \tag{G.3}$$

where $S_1 = P_{13} + P_{14}$ and $S_2 = P_{23} + P_{24}$. This ensures that the angular part of the 3-particle phase space integrals is of the from

$$\int_0^\pi d\theta \int_0^\pi d\phi \frac{(\sin\theta)^{n-3}(\sin\phi)^{n-4}}{(a + b\cos\theta)^i (A + B\cos\theta + C\cos\phi\sin\theta)^j} \tag{G.4}$$

We distinguish two cases

1. $a^2 = b^2$ and $A^2 = B^2 + C^2$
   This condition is fulfilled by nearly all integrals, we encountered. In this case we have

$$\int_0^\pi d\theta \int_0^\pi d\phi \frac{(\sin\theta)^{n-3}(\sin\phi)^{n-4}}{(1 - \cos\theta)^i (1 - \cos\chi\cos\theta - \sin\chi\cos\phi\sin\theta)^j} = \tag{G.5}$$
$$2^{1-i-j}\,\pi\, \frac{\Gamma(\frac{1}{2}n - 1 - j)\Gamma(\frac{1}{2}n - 1 - i)}{\Gamma(n - 2 - i - j)} \frac{\Gamma(n-3)}{\Gamma^2(\frac{1}{2}n - 1)} F(i, j; \frac{n}{2} - 1; \cos^2\frac{\chi}{2})$$

2. $a^2 \neq b^2$ and $A^2 = B^2 + C^2$
   This happens when we are dealing with a so called "massive" propagator. The only example we found is

$$\int dPS_3^{\mathrm{DI}} \frac{(-q^2)^3}{P_{24}P_{25}P_{34}P_{35}} \tag{G.6}$$

Here P_{35} is the "massive" propagator. The angular part of this integral is of the form

$$\int_0^\pi d\theta \int_0^\pi d\phi \frac{(\sin\theta)^{n-3}(\sin\phi)^{n-4}}{(D-\cos\theta)(1-\cos\chi\cos\theta-\sin\chi\cos\phi\sin\theta)} = \pi\frac{\varepsilon}{4}\frac{\Gamma(1+\varepsilon)}{\Gamma^2(1+\frac{1}{2}\varepsilon)}$$
$$\times \int_0^1 dv \int_0^1 dw \frac{(1-v)^{-1+\frac{1}{2}\varepsilon}w^{-1+\frac{1}{2}\varepsilon}(1-w)^{\frac{1}{2}\varepsilon}}{\frac{1}{2}(D-1)+\frac{1}{2}(1-\cos\chi)v(1-w)+(1-v)(1-w)} \quad \text{(G.7)}$$

This equality can be found using the method described in Appendix A of ref. [2]. The problem of this integral is that eq. (G.7) cannot be written in as elegant a form as eq. (G.6). Therefore we had to replace the angular part of the 3-particle phase space integral by eq. (G.7) and had to perform the integrations using brute force methods.

We will now list some of the 3-particle phase space integrals. To carry out the integrations we have made extensive use of ref. [6].

Drell-Yan soft integrals

The Drell-Yan soft integral is defined by

$$\int_{1-\delta}^1 dx \int dPS_3^{\mathrm{DY}} \text{"propagators"} \quad \text{(G.8)}$$

All the DY soft integrals have an overall factor

$$\frac{\pi}{(4\pi)^n}(Q^2)^\varepsilon \delta^{2\varepsilon}$$

In our list of DY soft integrals we have left out this factor. Moreover, we have used the shorthand notation

$$A = \frac{\Gamma^2(1+\frac{1}{2}\varepsilon)}{\Gamma(1+\varepsilon)}$$

<u>List of DY soft integrals</u>

$$\text{"propagators"} \longrightarrow \int_{1-\delta}^1 dx \int dPS_3^{\mathrm{DY}} \text{"propagators"}$$

$$\frac{(Q^2)^2}{P_{15}P_{25}P_{34}} = 4\frac{1}{(1+\varepsilon)}\frac{1}{\varepsilon^3}A$$

$$\frac{(Q^2)^2}{P_{13}P_{15}P_{34}} = \frac{8}{\varepsilon^4}A$$

$$\frac{(Q^2)^2}{P_{13}P_{24}P_{34}} = \frac{24}{\varepsilon^4}e^{(\varepsilon\gamma_E)}\left\{1-\frac{23}{12}\varepsilon^2\zeta(2)+3\varepsilon^3\zeta(3)-\frac{11}{480}\zeta(2)^2\right\}$$

$$\frac{(Q^2)^3}{S_1S_2P_{13}P_{24}} = \frac{8}{\varepsilon^4}A$$

$$\frac{(Q^2)^3}{S_1S_2P_{13}P_{23}} = \frac{8}{\varepsilon^4}A$$

$$\frac{(Q^2)^3}{P_{13}P_{15}P_{24}P_{25}} = \frac{8}{\varepsilon^4}A$$

$$\frac{(Q^2)^3}{P_{13}P_{15}P_{23}P_{25}} = \frac{8}{\varepsilon^4}A$$

$$\frac{Q^2(P_{13})^2}{P_{25}P_{25}P_{34}P_{34}} = -\frac{4}{3}\frac{(1+\frac{1}{4}\varepsilon)}{(1+\frac{1}{3}\varepsilon)(1+\varepsilon)(1-\frac{1}{2}\varepsilon)}\frac{1}{\varepsilon^2}A$$

$$\frac{Q^2P_{13}P_{23}}{P_{15}P_{25}P_{34}P_{34}} = -\frac{2}{3}\frac{(1+\frac{1}{2}\varepsilon)}{(1+\frac{1}{3}\varepsilon)(1-\frac{1}{2}\varepsilon)}\frac{1}{\varepsilon^3}A$$

Deep inelastic soft integrals

To determine the $\delta(1-x)$ part of the two gluon bremsstrahlung and quark pair production processes we had to calculate 52 DI soft integrals. We have selected a set of the more difficult ones and listed them here. The DI soft integral is defined as

$$\int_{1-\delta}^{1} dx \int dPS_3^{\text{DI}} \text{"propagators"} \tag{G.9}$$

All the DI soft integrals have an overall factor

$$\frac{\pi}{(4\pi)^n}(-q^2)^\varepsilon\delta^\varepsilon\frac{1}{\Gamma(1+\varepsilon)}$$

which we have omitted in the following list.

<u>List of DI soft integrals</u>

$$\text{"propagators"} \longrightarrow \int_{1-\delta}^{1} dx \int dPS_3^{\text{DI}} \text{"propagators"}$$

$$\frac{(-Q^2)^2}{P_{15}P_{23}P_{25}} = -\frac{4}{\varepsilon^2}\left\{\zeta(2)-\frac{7}{2}\varepsilon\zeta(3)+\frac{9}{4}\varepsilon^2\zeta(2)^2\right\}$$

$$\frac{(-Q^2)^2}{S_2P_{13}P_{23}} = -\frac{16}{\varepsilon^4}\left\{1-\varepsilon^2\zeta(2)+\frac{15}{8}\varepsilon^3\zeta(3)-\frac{123}{160}\varepsilon^4\zeta(2)^2\right\}$$

$$\frac{(-Q^2)^2}{S_2P_{13}P_{24}} = -\frac{8}{\varepsilon^4}\left\{1-\frac{1}{4}\varepsilon^2\zeta(2)-\frac{3}{4}\varepsilon^3\zeta(3)+\frac{147}{160}\varepsilon^4\zeta(2)^2\right\}$$

$$\frac{(-Q^2)^2}{P_{13}P_{24}P_{34}} = -\frac{24}{\varepsilon^4}\left\{1-\frac{3}{4}\varepsilon^2\zeta(2)+\varepsilon^3\zeta(3)-\frac{33}{160}\varepsilon^4\zeta(2)^2\right\}$$

$$\frac{(-Q^2)^2}{P_{23}P_{25}P_{34}} = -\frac{8}{\varepsilon^4}\left\{1-\frac{1}{4}\varepsilon^2\zeta(2)-\frac{3}{4}\varepsilon^3\zeta(3)+\frac{147}{160}\varepsilon^4\zeta(2)^2\right\}$$

$$\frac{(-Q^2)^3}{P_{13}P_{15}P_{24}P_{25}} = \frac{8}{\varepsilon^4}\left\{1-\frac{1}{4}\varepsilon^2\zeta(2)-\frac{3}{4}\varepsilon^3\zeta(3)+\frac{147}{160}\varepsilon^4\zeta(2)^2\right\}$$

$$\frac{(-Q^2)^3}{S_1S_2P_{13}P_{23}} = \frac{24}{\varepsilon^4}\left\{1-\frac{3}{4}\varepsilon^2\zeta(2)+\frac{2}{3}\varepsilon^3\zeta(3)+\frac{51}{160}\varepsilon^4\zeta(2)^2\right\}$$

$$\frac{(-Q^2)^3}{S_1S_2P_{13}P_{24}} = \frac{16}{\varepsilon^4}\left\{1-\frac{3}{4}\varepsilon^2\zeta(2)+\frac{5}{4}\varepsilon^3\zeta(3)-\frac{99}{160}\varepsilon^4\zeta(2)^2\right\}$$

$$\frac{(-Q^2)^3}{P_{24}P_{25}P_{34}P_{35}} = \frac{16}{\varepsilon^4}\left\{1-\frac{3}{4}\varepsilon^2\zeta(2)+\frac{5}{4}\varepsilon^3\zeta(3)-\frac{99}{160}\varepsilon^4\zeta(2)^2\right\}$$

All the integrals that we have listed can be carried out in the C.M. frame of the outgoing gluons. But for some DI soft integrals it is easier to work in the C.M. frame of the incoming particles.
For example

$$\frac{(-q^2)^2}{P_{13}P_{23}P_{24}} = \frac{(-q^2)^2}{S_2P_{13}P_{23}} + \frac{(-q^2)^2}{S_2P_{13}P_{24}} =$$
$$-24\frac{\pi}{(4\pi)^n}(-q^2)^\epsilon S^\epsilon \frac{1}{\varepsilon^4}\frac{\Gamma^3(1+\frac{1}{2}\varepsilon)}{\Gamma(1+\varepsilon)\Gamma(1+\frac{3}{2}\varepsilon)} \tag{G.10}$$

In the C.M. frame of the outgoing gluons we had to perform partial fractioning to handle the angular integrations. This results in two phase space integrals, which are listed above. However, in the C.M. frame of the incoming particles the phase space integral in eq. (G.10) could be calculated directly. It was even possible to express its result in terms of Γ-functions.

Appendix H: The $n_f C_F$ colour part of the quark pair production

In this Appendix we give the exact results for the $n_f C_F$ contributions from the quark pair production processes (see section III.3.4).

The virtual corrections are due to the diagram QL in fig. III.3 (p. 33). We therefore have (see eq. (E.2))

$$\hat{W}^{n_f,V} = 2\ \delta(1-x)\ \mathrm{Re}QL \tag{H.1}$$

and

$$\hat{\mathcal{F}}_2^{n_f,V} = 2\ \delta(1-x)\ \mathrm{Re}QL \tag{H.2}$$

Further, one also has contributions from the processes

$$\mathrm{q} + \bar{\mathrm{q}} \to \mathrm{V} + \mathrm{q} + \bar{\mathrm{q}} \tag{H.3}$$

and

$$\mathrm{V} + \mathrm{q} \to \mathrm{q} + \mathrm{q} + \bar{\mathrm{q}} \tag{H.4}$$

If one is only interested in the $n_f C_F$ colour part of the above processes, not all the diagrams in figs. III.8 and III.9 (p. 35 and 36) have to be taken into account. For the Drell-Yan process one only has to consider the diagrams indicated by A. In case of the DIS process only the squared amplitudes of the diagrams denoted by A and B (not including the interference terms between A and B) contribute. Due to the simple topology of these diagrams the 3-particle phase space integrals can be reduced to effectively performing 2-particle phase space integrations. We then find

$$\begin{aligned}\hat{W}^{n_f,S+H} = {}& g_n^4 n_f C_F \frac{(1+\frac{1}{2}\varepsilon)}{(1+\frac{1}{3}\varepsilon)}\frac{\Gamma^2(1+\frac{1}{2}\varepsilon)}{\Gamma(2+\varepsilon)}(Q^2)^\varepsilon x^{-\varepsilon}(1-x)^{-1+2\varepsilon} \\ & \left\{\frac{16}{3}\frac{1}{\varepsilon^2}(1+x^2) + \frac{1}{\varepsilon}\left[-\frac{8}{3}(1+x^2)\ln x - \frac{16}{3}(1-x)^2\right] + \right. \\ & -8(1+x^2)\zeta(2) - \frac{8}{3}x^2\mathrm{Li}_2(1-x) + \frac{2}{3}(1-x^2)\ln^2 x + \\ & \left. -\frac{8}{3}x(1-x)\ln x + 12(1-x)^2\right\}\end{aligned} \tag{H.5}$$

and

$$\hat{\mathcal{F}}_2^{n_f,S+H} = g_n^4 n_f C_F \frac{(1+\frac{1}{2}\varepsilon)}{(1+\frac{1}{3}\varepsilon)}\frac{\Gamma^2(1+\frac{1}{2}\varepsilon)}{\Gamma(2+\varepsilon)}(-q^2)^{\varepsilon} x^{-\varepsilon}(1-x)^{-1+\varepsilon}$$
$$\Big\{\frac{8}{3}\frac{1}{\varepsilon^2}(1+x^2) + \frac{1}{\varepsilon}\Big[-\frac{4}{3}(1+x^2)\ln x + \frac{8}{3}(2+3x)(1-x) - 4\Big] +$$
$$-\frac{1}{3}(1+x^2)\Big[8\zeta(2) + 4\mathrm{Li}_2(1-x) + \ln^2 x\Big] +$$
$$+4x^2\ln x - \frac{22}{3}(1+2x)(1-x) + \frac{19}{3}\Big\} \qquad \text{(H.6)}$$

Of course, the $(1-x)^{-1+a\varepsilon}$ terms have to be treated properly as discussed in section II.5. The soft contributions are

$$\hat{W}^{n_f,S} = \sigma_D^{DY} \qquad \text{(H.7)}$$

and

$$\hat{\mathcal{F}}_2^{n_f,S} = n_f C_F \text{ part of } \sigma_D^{DI} \qquad \text{(H.8)}$$

The expressions for σ_D^{DY} and σ_D^{DI} can be found in eqs. (III.3.12) and (III.3.13), respectively. Further, we have for the hard parts

$$\hat{W}^{n_f,H} = \theta(1-x-\delta)\ \hat{W}^{n_f,S+H} \qquad \text{(H.9)}$$

and

$$\hat{\mathcal{F}}_2^{n_f,H} = \theta(1-x-\delta)\ \hat{\mathcal{F}}_2^{n_f,S+H} \qquad \text{(H.10)}$$

Putting everything together, the renormalized parton structure functions $\hat{W}^{n_f}$ and $\hat{\mathcal{F}}_2^{n_f}$ are given by

$$\hat{W}^{n_f}(x,Q^2,\varepsilon) = \hat{W}^{n_f,V} + \hat{W}^{n_f,S} + \hat{W}^{n_f,H} - \frac{4}{3}n_f\left(\frac{\alpha_s}{4\pi}\right)\frac{1}{\varepsilon}\hat{W}^{(1),q\bar{q}} \qquad \text{(H.11)}$$

$$\hat{\mathcal{F}}_2^{n_f}(x,Q^2,\varepsilon) = \hat{\mathcal{F}}_2^{n_f,V} + \hat{\mathcal{F}}_2^{n_f,S} + \hat{\mathcal{F}}_2^{n_f,H} - \frac{4}{3}n_f\left(\frac{\alpha_s}{4\pi}\right)\frac{1}{\varepsilon}\hat{\mathcal{F}}_2^{(1),q} \qquad \text{(H.12)}$$

where $\hat{W}^{(1),q\bar{q}}$ and $\hat{\mathcal{F}}_2^{(1),q}$ are the first order contributions*, which can be found in eqs. (II.5.34) and (II.5.12). These terms are needed in the above expressions for renormalization (see also eq. (III.4.11)). Furthermore, in eqs. (H.11) and (H.12) one has to make the replacement

$$g_n^2 \longrightarrow \left(\frac{\alpha_s}{4\pi}\right)(\mu^2)^{-\frac{1}{2}\varepsilon} \qquad \text{(H.13)}$$

*Notice that one has to extend the expressions given in section II.5 to include terms of order ε.

The Drell-Yan correction term due to the contributions given in this Appendix, is equal to

$$\Delta^{n_f}(x,Q^2) \;=\; \hat{W}^{n_f}(x,Q^2,\varepsilon) - 2\,\hat{\mathcal{F}}_2^{n_f}(x,Q^2,\varepsilon) = \tag{H.14}$$
$$= \Delta^{n_f}_{soft}(x,Q^2) + \Delta^{n_f}_{reg}(x,Q^2) + \frac{2}{3} n_f \left(\frac{\alpha_s}{4\pi}\right) \ln\left(\frac{Q^2}{\mu^2}\right) \Delta_0^{q\bar{q}}(x,Q^2)$$

with

$$\Delta^{n_f}_{soft}(x,Q^2) \;=\; n_f C_F \left(\frac{\alpha_s}{4\pi}\right)^2 \Big\{\delta(1-x)\,\left[\frac{16}{3}\zeta(3) - \frac{340}{9}\zeta(2) - \frac{38}{9}\right] + $$
$$+8\mathcal{D}_2(x) - \frac{44}{9}\mathcal{D}_1(x) - \left(10 + \frac{16}{3}\zeta(2)\right)\mathcal{D}_0(x)\Big\} \tag{H.15}$$

and

$$\Delta^{n_f}_{reg}(x,Q^2) \;=\; n_f C_F \left(\frac{\alpha_s}{4\pi}\right)^2 \Big\{-4(1+x)\ln^2(1-x) + \frac{4}{3}\frac{\ln^2 x}{(1-x)} + $$
$$-\frac{16}{3}\frac{(1+x^2)}{(1-x)}\ln x \ln(1-x) + \frac{8}{3}\frac{\mathrm{Li}_2(1-x)}{(1-x)} + $$
$$+\left(-\frac{80}{9} + \frac{40}{9}x\right)\ln(1-x) + \left(4 - \frac{16}{3}x - \frac{32}{3}x^2\right)\frac{\ln x}{(1-x)} + $$
$$+\left(\frac{188}{9} + \frac{8}{3}\zeta(2)\right)x + \frac{56}{3} + \frac{8}{3}\zeta(2)\Big\} \tag{H.16}$$

Further, $\Delta_0^{q\bar{q}}(x,Q^2)$ is the first order contribution to the Drell-Yan correction term (see eq. (II.5.42)). Notice that this term can be absorbed into the running coupling constant using the definition in eq. (III.4.24).

Appendix I: Mellin transforms

For some of our calculations we need to know the Mellin transforms of the distributions $\mathcal{D}_i(x)$. A very elegant way of determining these Mellin transforms in the limit $n \to \infty$ is given in Appendix B of ref. [8].

One uses the identities

$$\int_0^1 dx\; x^{n-1}(1-x)^{-1+\varepsilon} = \frac{\Gamma(n)\Gamma(\varepsilon)}{\Gamma(n+\varepsilon)} \overset{n\to\infty}{=} \frac{1}{\varepsilon} n^{-\varepsilon}\;\Gamma(1+\varepsilon) \tag{I.1}$$

and

$$(1-x)^{-1+\varepsilon} \;=\; \frac{1}{\varepsilon}\,\delta^{\varepsilon}\,\delta(1-x) + \theta(1-\delta-x)\,(1-x)^{-1+\varepsilon} =$$
$$= \frac{1}{\varepsilon}\,\delta(1-x) + \sum_{i=0}^{\infty}\frac{\varepsilon^i}{i!}\mathcal{D}_i(x) \tag{I.2}$$

Expanding the above equations in ε, one finds for the Mellin transforms

$$D_i^{(n)} = \int_0^1 dx\; x^{n-1}\mathcal{D}_i(x) \tag{I.3}$$

x-language	n-language
$\mathcal{D}_i(x)$	$D_i^{(n)}$ $(n \to \infty)$
$\mathcal{D}_0(x)$	$-\ln n'$
$\mathcal{D}_1(x)$	$\frac{1}{2}\ln^2 n' + \frac{1}{2}\zeta(2)$
$\mathcal{D}_2(x)$	$-\frac{1}{3}\ln^3 n' - \zeta(2)\ln n' - \frac{2}{3}\zeta(3)$
$\mathcal{D}_3(x)$	$\frac{1}{4}\ln^4 n' + \frac{3}{2}\zeta(2)\ln^2 n' + 2\zeta(3)\ln n' + \frac{27}{20}\zeta(2)^2$

where $\ln n' = \ln n + \gamma_E$.

Appendix J: Miscellaneous

In this Appendix we will give some useful formulae for the calculation of second order soft contributions. At the end of this Appendix the explicit expressions of the parton structure functions $\hat{W}(x, Q^2, \varepsilon)$ and $\hat{\mathcal{F}}_2(x, Q^2, \varepsilon)$ can be found.

The splitting function $P^{qq}(x)$

The splitting function $P^{qq}(x)$ (see eq. (II.5.17)) is known up to order α_s^2 [7]. Expanding it in α_s we have

$$P^{qq}(x) = \left(\frac{\alpha_s}{4\pi}\right) P_0(x) + \left(\frac{\alpha_s}{4\pi}\right)^2 P_1(x) + \cdots \tag{J.1}$$

In the limit $x \to 1$ the functions $P_0(x)$ and $P_1(x)$ are equal to

$$\begin{aligned}
P_0(x) &\overset{x\to 1}{=} C_F\left\{6\,\delta(1-x) + 8\,\mathcal{D}_0(x)\right\} \\
P_1(x) &\overset{x\to 1}{=} \delta(1-x)\Big\{C_F^2(3 - 24\zeta(2) + 48\zeta(3)) + \\
&\qquad + C_A C_F\left(\frac{17}{3} + \frac{88}{3}\zeta(2) - 24\zeta(3)\right) + \\
&\qquad + n_f C_F\left(-\frac{2}{3} - \frac{16}{3}\zeta(2)\right)\Big\} + \\
&\quad + \mathcal{D}_0(x)\left\{C_A C_F\left(\frac{536}{9} - 16\zeta(2)\right) - \frac{80}{9}\, n_f C_F\right\}
\end{aligned} \tag{J.2}$$

The anomalous dimension γ_{qq}^n

The anomalous dimension γ_{qq}^n [7] is directly related to the splitting function $P^{qq}(x)$ through the Mellin transform

$$\gamma_{qq}^n = -\int_0^1 dx\ x^{n-1}\ P^{qq}(x) \tag{J.3}$$

We also write the anomalous dimensions as a series in α_s

$$\gamma_{qq}^n = \left(\frac{\alpha_s}{4\pi}\right)\gamma_0^n + \left(\frac{\alpha_s}{4\pi}\right)^2\gamma_1^n + \cdots \tag{J.4}$$

In the limit $n \to \infty$ the $O(\alpha_s^{i+1})$ part of the anomalous dimension, γ_i^n, can be written as

$$\gamma_i^n \overset{n\to\infty}{=} \gamma_i^{(K)} \ln n + \bar{\gamma}_i \tag{J.5}$$

The constituents γ_i^K and $\bar{\gamma}_i$ are found to be

$$\begin{aligned}
\gamma_0^{(K)} &= 8\ C_F \\
\gamma_1^{(K)} &= C_A C_F\left(\frac{536}{9} - 16\zeta(2)\right) - \frac{80}{9}\ n_f C_F \\
\bar{\gamma}_0 &= -6\ C_F \\
\bar{\gamma}_1 &= C_F^2\Big(-3 + 24\zeta(2) - 48\zeta(3)\Big) + \\
&\quad + C_A C_F\left(-\frac{17}{3} - \frac{88}{3}\zeta(2) + 24\zeta(3)\right) + \\
&\quad + n_f C_F\left(\frac{2}{3} + \frac{16}{3}\zeta(2)\right)
\end{aligned} \tag{J.6}$$

The β-function

The lowest order coefficient of the β-function

$$\beta(\alpha_s) = 2\ \alpha_s\left\{-\beta_0\left(\frac{\alpha_s}{4\pi}\right) + \cdots\right\} \tag{J.7}$$

is equal to

$$\beta_0 = \frac{11}{3}C_A - \frac{2}{3}n_f \tag{J.8}$$

The functions $w_0(x)$ and $f_0(x)$ up to order ε

The non-pole contributions w_0 and f_0 appearing in eqs. (III.4.13) and (III.4.14) can be found in section II.5. Extending them to include terms of order ε, we find

$$\begin{aligned}
w_0(x) \overset{x\to 1}{=} C_F\Big\{&\delta(1-x)\left[-16 + 8\zeta(2) + \varepsilon\left(16 - \frac{21}{2}\zeta(2)\right)\right] + \\
&+ 16\mathcal{D}_1(x) + \varepsilon\Big[8\mathcal{D}_2(x) - 6\zeta(2)\mathcal{D}_0(x)\Big]\Big\}
\end{aligned} \tag{J.9}$$

and

$$f_0(x) \stackrel{x\to 1}{=} C_F\left\{\delta(1-x)\left[-9-4\zeta(2)+\varepsilon\left(9+\frac{3}{4}\zeta(2)\right)\right]+4\mathcal{D}_1(x)+\right.$$
$$\left.-3\mathcal{D}_0(x)+\varepsilon\left[\mathcal{D}_2(x)-\frac{3}{2}\mathcal{D}_1(x)+\left(\frac{7}{2}-3\zeta(2)\right)\mathcal{D}_0(x)\right]\right\} \quad \text{(J.10)}$$

Convolutions

The calculation of second order corrections involves convolutions between the distributions $\mathcal{D}_i(x)$. We list here some useful results.

$$(D_0\otimes D_0)(x) \stackrel{x\to 1}{=} -\zeta(2)\delta(1-x)+2\mathcal{D}_1(x) \quad \text{(J.11)}$$

$$(D_0\otimes D_1)(x) \stackrel{x\to 1}{=} \zeta(3)\delta(1-x)+\frac{3}{2}\mathcal{D}_2(x)-\zeta(2)\mathcal{D}_0(x) \quad \text{(J.12)}$$

$$(D_0\otimes D_2)(x) \stackrel{x\to 1}{=} -\frac{4}{5}\zeta(2)^2\delta(1-x)+\frac{4}{3}\mathcal{D}_3(x)-2\zeta(2)\mathcal{D}_1(x)+2\zeta(3)\mathcal{D}_0(x) \quad \text{(J.13)}$$

$$(D_1\otimes D_1)(x) \stackrel{x\to 1}{=} -\frac{1}{10}\zeta(2)^2\delta(1-x)+\mathcal{D}_3(x)-2\zeta(2)\mathcal{D}_1(x)+2\zeta(3)\mathcal{D}_0(x) \quad \text{(J.14)}$$

The parton distributions $\hat{W}(x,Q^2,\varepsilon)$ and $\hat{\mathcal{F}}_2(x,Q^2,\varepsilon)$

Substituting the above formulae into eqs. (III.4.13) and (III.4.14) we find for the parton structure functions $\hat{W}(x,Q^2,\varepsilon)$ and $\hat{\mathcal{F}}_2(x,Q^2,\varepsilon)$ [9] the following expressions.

$$\hat{\mathcal{F}}_2(x,Q^2,\varepsilon) \stackrel{x\to 1}{=}$$
$$\delta(1-x)\left[1+\left(\frac{\alpha_s}{4\pi}\right)C_F\left(\frac{Q^2}{\mu^2}\right)^{\frac{1}{2}\varepsilon}\left\{\frac{6}{\varepsilon}-9-4\zeta(2)+\varepsilon\left(9+\frac{3}{4}\zeta(2)\right)\right\}+\right.$$
$$+\left(\frac{\alpha_s}{4\pi}\right)^2\left\{C_F^2\left(\frac{Q^2}{\mu^2}\right)^{\varepsilon}\left[\frac{1}{\varepsilon^2}\left(18-32\zeta(2)\right)+\frac{1}{\varepsilon}\left(56\zeta(3)-12\zeta(2)-\frac{105}{2}\right)+\right.\right.$$
$$\left.+\frac{118}{5}\zeta(2)^2-90\zeta(3)+\frac{91}{2}\zeta(2)+\frac{763}{8}\right]+$$
$$+C_AC_F\left[\frac{22}{\varepsilon^2}+\frac{1}{\varepsilon}\left(\frac{17}{6}+\frac{44}{3}\zeta(2)-12\zeta(3)\right)+\right.$$
$$+\frac{71}{5}\zeta(2)^2+\frac{140}{3}\zeta(3)-\frac{251}{3}\zeta(2)-\frac{5465}{72}+$$
$$\left.+\left(\frac{215}{6}+\frac{88}{3}\zeta(2)-12\zeta(3)\right)\ln\left(\frac{Q^2}{\mu^2}\right)-\frac{11}{2}\ln^2\left(\frac{Q^2}{\mu^2}\right)\right]+$$
$$+n_fC_F\left[-\frac{4}{\varepsilon^2}-\frac{1}{\varepsilon}\left(\frac{1}{3}+\frac{8}{3}\zeta(2)\right)+\frac{4}{3}\zeta(3)+\frac{38}{3}\zeta(2)+\frac{457}{36}+\right.$$
$$\left.\left.\left.-\left(\frac{19}{3}+\frac{16}{3}\zeta(2)\right)\ln\left(\frac{Q^2}{\mu^2}\right)+\ln^2\left(\frac{Q^2}{\mu^2}\right)\right]\right\}\right]+$$
$$+\left(\frac{\alpha_s}{4\pi}\right)C_F\left(\frac{Q^2}{\mu^2}\right)^{\frac{1}{2}\varepsilon}\left[\left\{\frac{8}{\varepsilon}-3+\varepsilon\left(\frac{7}{2}-3\zeta(2)\right)\right\}\mathcal{D}_0(x)+\right.$$

$$+\left(4-\frac{3}{2}\varepsilon\right)\mathcal{D}_1(x)+\varepsilon\mathcal{D}_2(x)\Big]+$$

$$+\left(\frac{\alpha_s}{4\pi}\right)^2\Bigg\{C_F^2\left(\frac{Q^2}{\mu^2}\right)^{\varepsilon}\Bigg[\Bigg\{\frac{48}{\varepsilon^2}-\frac{1}{\varepsilon}\Big(90+64\zeta(2)\Big)+$$

$$+8\zeta(3)+36\zeta(2)+\frac{237}{2}\Bigg\}\mathcal{D}_0(x)+$$

$$+\left\{\frac{64}{\varepsilon^2}-\frac{24}{\varepsilon}+20-96\zeta(2)\right\}\mathcal{D}_1(x)+\left(\frac{48}{\varepsilon}-30\right)\mathcal{D}_2(x)+\frac{56}{3}\mathcal{D}_3(x)\Bigg]+$$

$$+C_AC_F\Bigg[\Bigg\{\frac{88}{3}\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}\left(\frac{268}{9}-8\zeta(2)\right)+40\zeta(3)+\frac{44}{3}\zeta(2)-\frac{3155}{54}+$$

$$+\left(\frac{367}{9}-8\zeta(2)\right)\ln\left(\frac{Q^2}{\mu^2}\right)-\frac{22}{3}\ln^2\left(\frac{Q^2}{\mu^2}\right)\Bigg\}\mathcal{D}_0(x)+$$

$$+\left\{\frac{367}{9}-8\zeta(2)-\frac{44}{3}\ln\left(\frac{Q^2}{\mu^2}\right)\right\}\mathcal{D}_1(x)-\frac{22}{3}\mathcal{D}_2(x)\Bigg]+$$

$$+n_fC_F\Bigg[\Bigg\{-\frac{16}{3}\frac{1}{\varepsilon^2}-\frac{40}{9}\frac{1}{\varepsilon}+\frac{247}{27}-\frac{8}{3}\zeta(2)-\frac{58}{9}\ln\left(\frac{Q^2}{\mu^2}\right)+$$

$$+\frac{4}{3}\ln^2\left(\frac{Q^2}{\mu^2}\right)\Bigg\}\mathcal{D}_0(x)+\left\{-\frac{58}{9}+\frac{8}{3}\ln\left(\frac{Q^2}{\mu^2}\right)\right\}\mathcal{D}_1(x)+\frac{4}{3}\mathcal{D}_2(x)\Bigg]\Bigg\} \tag{J.15}$$

and

$$\hat{W}(x,Q^2,\varepsilon)\overset{x\to1}{=}$$

$$\delta(1-x)\Bigg[1+\left(\frac{\alpha_s}{4\pi}\right)C_F\left(\frac{Q^2}{\mu^2}\right)^{\frac{1}{2}\varepsilon}\left\{\frac{12}{\varepsilon}+8\zeta(2)-16+\varepsilon\left(16-\frac{21}{2}\zeta(2)\right)\right\}+$$

$$+\left(\frac{\alpha_s}{4\pi}\right)^2\Bigg\{C_F^2\left(\frac{Q^2}{\mu^2}\right)^{\varepsilon}\Bigg[\frac{1}{\varepsilon^2}\Big(72-128\zeta(2)\Big)+\frac{1}{\varepsilon}\Big(-189+72\zeta(2)+304\zeta(3)\Big)+$$

$$-\frac{24}{5}\zeta(2)^2-60\zeta(3)-196\zeta(2)+\frac{1279}{4}\Bigg]+$$

$$+C_AC_F\Bigg[\frac{44}{\varepsilon^2}+\frac{1}{\varepsilon}\left(\frac{17}{3}+\frac{88}{3}\zeta(2)-24\zeta(3)\right)+$$

$$-\frac{12}{5}\zeta(2)^2+28\zeta(3)+\frac{592}{9}\zeta(2)-\frac{1535}{12}+$$

$$+\left(-24\zeta(3)+\frac{193}{3}\right)\ln\left(\frac{Q^2}{\mu^2}\right)-11\ln^2\left(\frac{Q^2}{\mu^2}\right)\Bigg]+$$

$$+n_fC_F\Bigg[-\frac{8}{\varepsilon^2}+\frac{1}{\varepsilon}\left(-\frac{2}{3}-\frac{16}{3}\zeta(2)\right)+8\zeta(3)-\frac{112}{9}\zeta(2)+\frac{127}{6}+$$

$$-\frac{34}{3}\ln\left(\frac{Q^2}{\mu^2}\right)+2\ln^2\left(\frac{Q^2}{\mu^2}\right)\Bigg]\Bigg\}\Bigg]+$$

$$+\left(\frac{\alpha_s}{4\pi}\right)C_F\left(\frac{Q^2}{\mu^2}\right)^{\frac{1}{2}\varepsilon}\left\{\left(\frac{16}{\varepsilon}-6\zeta(2)\varepsilon\right)\mathcal{D}_0(x)+16\mathcal{D}_1(x)+8\varepsilon\mathcal{D}_2(x)\right\}+$$

$$+\left(\frac{\alpha_s}{4\pi}\right)^2\Bigg\{C_F^2\left(\frac{Q^2}{\mu^2}\right)^{\varepsilon}\Bigg[\Bigg\{\frac{192}{\varepsilon^2}-\frac{1}{\varepsilon}\Big(256+128\zeta(2)\Big)+$$

$$+512\zeta(3)-240\zeta(2)+256\Big\}\mathcal{D}_0(x)+$$

$$+\left\{\frac{256}{\varepsilon^2}+\frac{192}{\varepsilon}-576\zeta(2)-256\right\}\mathcal{D}_1(x)+\left\{\frac{384}{\varepsilon}+96\right\}\mathcal{D}_2(x)+\frac{896}{3}\mathcal{D}_3(x)\Big]+$$

$$+C_AC_F\left[\left\{\frac{176}{3}\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}\left(\frac{536}{9}-16\zeta(2)\right)+56\zeta(3)+\frac{176}{3}\zeta(2)-\frac{1616}{27}+\right.\right.$$

$$\left.+\left(\frac{536}{9}-16\zeta(2)\right)\ln\left(\frac{Q^2}{\mu^2}\right)-\frac{44}{3}\ln^2\left(\frac{Q^2}{\mu^2}\right)\right\}\mathcal{D}_0(x)+$$

$$\left.+\left\{\frac{1072}{9}-32\zeta(2)-\frac{176}{3}\ln\left(\frac{Q^2}{\mu^2}\right)\right\}\mathcal{D}_1(x)-\frac{176}{3}\mathcal{D}_2(x)\right]+$$

$$+n_fC_F\left[\left\{-\frac{32}{3}\frac{1}{\varepsilon^2}-\frac{80}{9}\frac{1}{\varepsilon}+\frac{224}{27}-\frac{32}{3}\zeta(2)-\frac{80}{9}\ln\left(\frac{Q^2}{\mu^2}\right)+\right.\right.$$

$$\left.\left.\left.+\frac{8}{3}\ln^2\left(\frac{Q^2}{\mu^2}\right)\right\}\mathcal{D}_0(x)+\left\{-\frac{160}{9}+\frac{32}{3}\ln\left(\frac{Q^2}{\mu^2}\right)\right\}\mathcal{D}_1(x)+\frac{32}{3}\mathcal{D}_2(x)\right]\right\} \qquad (J.16)$$

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Samenvatting

Hogere orde correcties op het Drell-Yan proces

In 1970 is door S.D. Drell en T.M. Yan een theoretische beschrijving gegeven voor de productie van massieve lepton-paren in hadron-hadron processen. Hun beschrijving is gebaseerd op het parton model van Feynman, waarin men aanneemt dat hadronen opgebouwd zijn uit vrije puntdeeltjes, de zgn. partonen. Drell en Yan postuleerden dat het lepton-paar geproduceerd wordt door de annihilatie van een parton en een anti-parton uit de twee botsende hadronen. Vanwege het succes van hun model wordt de lepton-paar productie vaak aangeduid met de naam 'Drell-Yan proces'.

Sindsdien is zowel theoretisch als experimenteel de nodige vooruitgang geboekt. Door de toepassing van QCD, een theorie voor de sterke wisselwerking, heeft men verfijningen kunnen aanbrengen aan het model van Drell en Yan. In het bijzonder was men in staat de eerste orde correctie op de lepton-paar productie te berekenen. Een verontrustend gegeven hierbij is dat deze correctie zodanig groot is dat men reden heeft tot twijfel aan de convergentie van de storingsreeks. Men heeft geprobeerd dit probleem op te lossen door te zoeken naar een hersommatie van de dominante bijdragen, zodat de nieuwe storingsreeks een beter convergentie gedrag vertoont. Meestal leidt dit soort procedures tot de exponentiatie van de eerste orde term. Het moge duidelijk zijn, dat de beste methode om de convergentie van de storingsreeks en ook de hersommatie procedures te toetsen, de berekening van de 2e orde QCD correcties op het Drell-Yan proces is.

Vanuit het oogpunt van het experiment zijn er ook redenen tot interesse in 2e orde QCD correcties. De fixed target experimenten bereiken tegenwoordig een zodanig hoge statistiek, dat nauwkeurige toetsingen van QCD mogelijk zijn. Ook de Z en W productie in de CERN en FNAL $p\bar{p}$-colliders zullen met grotere precisie gemeten worden. Door de toenemende nauwkeurigheid van de experimentele gegevens, ontstaat natuurlijk de behoefte aan betere theoretische voorspellingen.

Uit het voorgaande blijkt dat het zowel theoretisch als experimenteel van belang is om hogere orde correcties op het Drell-Yan proces te kennen. Aangezien de berekening van de complete 2e orde correctie zeer tijdrovend is, beperken we ons in dit proefschrift tot de berekening van de dominante 2e orde bijdragen. Dit houdt met name in, dat we alleen het $q\bar{q}$ proces in beschouwing nemen.

In hoofdstuk II bespreken we het Drell-Yan formalisme. In het bijzonder besteden wij aandacht aan de collineaire divergenties in de begin toestand. Voor de behandeling van deze singulariteiten voeren we het begrip massa-factorisatie in. Onze keuze voor de massa-factorisatie procedure verplicht ons behalve het Drell-Yan proces ook de zeer inelastische lepton-hadron verstrooiing in beschouwing te nemen. Dit hoofdstuk wordt afgesloten met een voorbeeld op orde α_s.

In het derde hoofdstuk geven we de resultaten van de 2e orde berekeningen voor het Drell-Yan proces en de zeer inelastische lepton-hadron verstrooiing. Verder bepalen we voor het eerst genoemde proces m.b.v. de massa-factorisatie procedure

de werkzame doorsnede $d\sigma/dQ^2$ tot op orde α_s^2. Met dit 2e orde resultaat bestuderen we een hersommatie procedure en bepalen we de werkzame doorsneden voor de Z en W productie in de CERN ($\sqrt{S}$=630 GeV) en FNAL ($\sqrt{S}$=1.8 TeV) p$\bar{\text{p}}$-colliders. Het blijkt dat op dit moment de experimentele onzekerheden nog te groot zijn om verschil te kunnen maken tussen de eerste en tweede orde correcties, maar hopelijk zal dit in de toekomst veranderen.

In het laatste hoofdstuk worden twee methoden besproken om, zonder expliciete berekeningen, de 2e orde bijdrage aan de werkzame doorsnede $d^2\sigma/(dQ^2dx_F)$ te bepalen. De eerste methode berust op de renormalisatie groep vergelijkingen, waaraan de correcties op het Drell-Yan proces en de zeer inelastische verstrooiing voldoen. De tweede maakt gebruik van een relatie tussen de Mellin getransformeerden van de werkzame doorsneden $d\sigma/dQ^2$ en $d^2\sigma/(dQ^2dx_F)$. Vervolgens vergelijken we de theoretisch gevonden resultaten met het NA10 experiment ($\sqrt{S}$=19.1 GeV). We vinden dan dat de convergentie van de storingsreeks bij dit soort lage energieen zeer twijfelachtig wordt.

Curriculum Vitae

van

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Na mijn eindexamen gymnasium β aan de Scholengemeenschap Augustinianum te Eindhoven begon ik in 1981 aan de studie natuurkunde aan de Rijksuniversiteit te Leiden. Het kandidaatsexamen natuurkunde met bijvakken wiskunde en sterrenkunde werd afgelegd in februari 1984. In juni 1986 volgde het doctoraalexamen theoretische natuurkunde met bijvak wiskunde. Mijn experimentele stage heb ik gelopen in de groep Moleculaire Natuurkunde o.l.v. Prof. Dr. J.P. Woerdman. Het doctoraal onderzoek is verricht onder begeleiding van Dr. W.L. van Neerven in de groep Hoge Energie Fysica van Prof. Dr. F.A. Berends.

Sinds september 1986 verricht ik als A.I.O. (assistent in opleiding) onder leiding van Prof. Dr. F.A. Berends en Dr. W.L. van Neerven onderzoek aan hogere orde correcties in QCD.

STELLINGEN

1. Voor de bepaling van de werkzame doorsneden voor W en Z productie bij de toekomstige hadron-colliders zal naast de berekening van de volledige 2e orde correctie op het Drell-Yan proces, ook de bestudering van het gedrag van de parton-distributiefuncties voor kleine fracties van de hadronimpuls van belang zijn.

2. De isotopen van rubidium kunnen worden gescheiden met behulp van hun eigen fluorescentie licht.

   S.N. Atutov en A.M. Shalagin, Pis'ma Astron. Zh. 14 (1988) 664.

3. In het licht van de precisie waarmee men de vervalsbreedte van het Z boson bij LEP gaat bepalen, mag bij de theoretische berekening ervan de massa van de bottom quark niet worden verwaarloosd.

4. Voor twee niet negatieve gehele getallen m en n geldt de relatie

$$\delta_{mn} = \sum_{j=0}^{m} \sum_{k=0}^{n} (-1)^{j+k} \binom{m}{j}^2 \binom{n}{k}^2 \Big/ \binom{m+n}{j+k}$$

   Probleem uit The Mathematical Intelligencer.

5. De 3-deeltjes faseruimte integraal voor het $n_f C_F$ gedeelte van de quarkpaar-productie kan door gebruik te maken van de eenvoudige topologie van de bijdragende diagrammen getransformeerd worden tot een effectieve 2-deeltjes faseruimte integraal.

6. De door Bengtsson et al. in de lichtkegel-ijk afgeleide interactie tussen drie massaloze deeltjes met verschillende, heeltallige spin, is niet uniek bepaald. Men kan de interactie uniek vastleggen door te eisen dat dit resultaat ook verkregen moet kunnen worden uit een manifest Lorentz covariante formulering die voldoet aan localiteit.

   A.K.H. Bengtsson et al., Class. Qu. Grav. 4 (1987) 1333.

7. Wanneer men de aarde beschouwt als een homogene bol met een straal R en een dichtheid ρ, wordt de tijdsduur t_{BA} om wrijvingsloos via een brachystochrone tunnel van een plaats A naar een plaats B te vallen, die hemelsbreed een afstand ℓ van elkaar gescheiden zijn, gegeven door

$$t_{BA} = \sqrt{\frac{3(2\pi R - \ell)\ell}{4\pi\rho G\ R^2}}$$

waarbij G de gravitatie constante is. Onder deze ideale omstandigheden kan men in ongeveer 40 minuten van Nederland naar Japan reizen.

8. In de literatuur bestaat een discrepantie tussen de resultaten verkregen voor de QED stralingscorrecties op het proces $e + P \rightarrow \ell + 'X'$ (zeer inelastische eP verstrooiing). Deze kan gemakkelijk worden opgelost met behulp van renormalisatiegroep technieken.

'Proceedings of the HERA workshop', ed. R.D. Peccei, Hamburg, October 12-14, 1987, Vols 1,2.

9. In veel gevallen kan men de aanleiding tot wrijving tussen Japanners en Nederlanders terugbrengen tot een van de twee onderstaande situaties:

   (a) de Nederlander heeft 'ja' gezegd, terwijl een 'nee' van hem verwacht werd

   (b) de Japanner heeft 'ja' gezegd, terwijl hij eigenlijk 'nee' bedoelde.

10. Het aantal mensen, dat bij een promotiereceptie met een oneven aantal mensen de hand schudt, is even.

T. Matsuura

Leiden, 22 juni 1989