Ned. T. Natuurk. 11, July—August 1944.

Fundamental difficulties of a theory of particles*)

by H. A. Kramers

§ 1. *Difficulties in the classical electron theory.*

We may make a distinction; at least for practical purposes, between 'real' (A) and 'formal' (B) difficulties of a theory:

A. A number of facts are not explained, although they are related to explained facts, or they may even be in contradiction with the theory.

B. Lack of coherence, which may become apparent either in theoretical incompleteness, or in logical inconsistency.

Both real and formal incompleteness (which often go together, but not always) are a difficulty, but not an objection. A relatively 'open' theory more easily avoids the danger of dogmatizing than a relatively 'closed' theory. No small part of the resistance which the theory of relativity had to overcome was due to the beautiful closed form of classical mechanics.

As an introduction we first think of the difficulties of the classical electron theory, some 40 or 50 years ago. Dispersion (γ), Fresnel's aether drag, Zeeman-effect (α), cathode rays (β) and secondary electrons were interpreted by Lorentz on the basis of the field equations and the equations of notion:

$$m_{exp}\ddot{x} = K_x + e\Big(E_x + \frac{1}{c}(\dot{y}H_z - \dot{z}H_y)\Big) + \frac{2e^2}{3c^3}\dddot{x}. \qquad (1)$$

The effects α (1896), β (1897) and γ (in 1898) led in the course of this development to the establishment of the existence of the *negative electron*. In eq. (1) m is the experimental mass, K the binding force (Lorentz usually puts $K_x = -\alpha x$); E and H are the 'incident' external electromagnetic field. The last term is the radiation reaction (computed with retardation).

Concerning A: The spectral laws of Balmer and Rydberg were not explained. Are they perhaps even in contradiction with the theory? Nor was the anomalous Zeeman effect explained; the attempts of Lorentz and Voigt were somewhat at variance with the 'spirit' of the theory. Concerning B: the theory was certainly incomplete, in view of the unanswerable questions with

*) Address given at the Symposion on Elementary Particles at Utrecht, 14 April 1944. Translated by the editor.

respect to the structure of the electron. Is the theory perhaps even inconsistent? One should bear in mind, that (1) arose from Lorentz' model in the following way (we omit the H term):

$$m_0 \ddot{x} = K_x + eE_x - f\frac{e^2}{ac^2}\ddot{x} + \frac{2e^2}{3c^3}\dddot{x} + g\frac{ae^2}{c^4}\ddddot{x} + \ldots \quad (2)$$

Here m_0 is the inertial mass, a the electron radius, while f and g are numerical factors depending on the structure. To obtain eq. (1), which is the basis of practically all interpretable phenomena, one had to put

1) $$m_{exp} = m_0 + f\frac{e^2}{ac^2}. \quad (3)$$

If the second term on the right, the electromagnetic mass, is of the same order as m_{exp}, then a becomes of the order 10^{-13} cm (conventional electron radius). That this is small compared to atomic dimensions gave rise to a certain amount of satisfaction.

2) E (and H) are practically constant inside the electron. Even to-day the wave length of the hardest of the known γ rays ($h\nu \sim 15$ MeV; $\lambda \sim 10^{-11}$ cm) is large compared to 10^{-13} cm.

3) The $\ddddot{x}$ term and the following terms are negligible when applied to interpretable phenomena.

The hope of obtaining information about m_0 itself (from the change of mass with velocity) vanished, when relativity theory, and with it Lorentz' contractible electron, were adopted. This suggested that the inscrutability of the structure of the electron should be established as a principle. For instance, it inspired the various modifications of the original theory, in which the electron was 'really' a point. One should recall on the one hand the attempts and considerations of Mie, Hilbert, Born-Infeld, *et al.*, on the other hand the idea, resumed in 1938 by Dirac, to let a tend to zero and m_0 to minus infinity, in such a way that m_{exp} remains finite and positive. Apart from the fact that so far all these theories have yielded no concrete physical result, it seems to me, that they do not do justice to the 'spirit' of the original theory (i.e., the part of 'truth' contained in the theory).

Theories like that of Poincaré, with its stress field inside the electron, which keeps the charge together, still make use of a structure but try to give a relativistic justification for it. I do not wish to point out the difficulties encountered in this direction. They are just as 'academic' as the questions relating to the point electron. For there was already an asymptotic, approximate theory (starting from (1)), which is structure independent: in this theory the electron was characterized by the charge e and the mass m_{exp} and it was possible to interpret the facts to a certain extent.

2. *Difficulties in the modern theory of particles.*

Many new particles have been added since the discovery of the negaton. First of all appeared the positive nuclei in 1911, of which, however, only the proton could claim the title of elementary particle. The whole edifice of quantum theory arose. The fact that the conventional electron radius is small compared to atomic dimensions, turned out to be a consequence of the smallness of $e^2/\hbar c \cong 1/137$. It is true that wave functions ψ are used, but the fact that these are functions of 'the coordinates of the particles', indicates that in quantum theory, just as previously in eq. (1), one operates with point particles: one has to do – at least that is what one thinks – with a consistent quantization of the above mentioned asymptotic theory. A huge quantity of experimental material is interpreted by the modern theory. The particles are characterized not only by their charge and mass, but also by their spin; the latter shows up in the appearance of a set of wave functions ψ_1, ψ_2, ... instead of a single one, more or less in analogy with the classical description of the electromagnetic field by six space-time functions. We ask again about the real and formal difficulties *A* and *B*.

A. Are there facts with which the theory cannot cope, which may even be in contradiction with it, and a closer investigation of which will perhaps throw light on the nature of the particles? The last question implies for example the question whether the value of $e^2/\hbar c$ can be 'understood', and whether it will be possible to say more concerning the spin multiplicities and the masses than is known at present. The somewhat surprising answer to this question is: *probably*. The situation appears to be as follows.

α) Facts that do not involve explicitly the structure of the nucleus. The results from double electron scattering (see L. Rosenfeld, Ned. Tijdschrift Natuurk. **10**, 53, 1943) seem to be in contradiction with the theory. Repetition of these experiments is desirable. Furthermore: it is well known that so far great mathematical difficulties made a precise calculation of the wave lengths and intensities of spectral lines for atom or molecule wellnigh impossible, even if the corresponding Schrödinger equation were exactly known. For light atoms the latter is known well enough. Computations like those of Hylleraas make a contradiction between facts and theory improbable in this case. For heavy atoms, however, one is faced with a many-body problem, in which relativistic effects yield an important contribution. Now the theory is not yet able to state the Hamiltonian more exactly than to $(v/c)^2$ terms. Only where the reduction to a one-electron problem is justifiable, can one probably rely on Dirac's theory. This means that v^4/c^4 effects in the wave lengths of the spectral lines (and occasionally even v^2/c^2 effects in their intensities) must exist, against which the theory is still powerless. Unfortunately, because of the above mentioned mathematical difficulties, it has so far been impossible to use this experimental source to obtain more data about the relativistic interaction of electrons.

β) There are many facts which have a bearing on the nuclear structure and which the theory cannot deal with. From Rosenfeld's lecture we know, however, that the theory is here – even in non-relativistic approximation – still far from unambiguous. Was it not possible recently for Pais to improve the theory of the photo-effect of the deuteron? Hence it may be presumed, but it is not certain, that the known facts are already a source from which one may draw essentially new knowledge about the theory of particles; it may be possible to include them all within the prevailing scheme of wave functions and Hamiltonian. It must be granted that for problems like the one of the magnetic moment of the proton such an optimistic, or rather pessimistic, expectation is likely to be wrong.

B. In its theoretical aspect the present-day quantum theory of particles is not only open, and unfinished, nay, there is even a lack of logical consistency. This lies in the notorious *divergencies* of sums or integrals, which occur in the calculation of certain effects and clearly demonstrate that the theory is wrong. But still not wholly useless – which is the remarkable thing in the present situation. Guided by physical or other considerations, one has been able to draft so-called *subtraction prescriptions*, which make the mentioned sums or integrals convergent. Example: in a situation in which there is only one light quantum present in space the expression for E^2 in a space point diverges. One subtracts the expression for E^2 at that same point for the case where no light quantum is present. The procedure to accomplish this subtraction comes readily to mind; the result is a convergent expression, which we use confidently, not least of all because it exhibits automatically the desired correspondence with the analogous classical situation. In many other cases, though not in this case, the situation in quantum theory resembles very much the divergence of the electromagnetic mass of an electron if one lets its radius tend to zero. Concerning a famous divergence noticed by Dirac (viz., in the expression for $E_1 - E_2 - h\nu$ in a Bohr radiation jump) it was shown by Serpe, that it is exactly parallel to the divergence of the electromagnetic mass; but for other divergences, in particular those which occur in the further elaboration of hole theory, such a correspondence is no longer present. Here the electromagnetic mass, for instance, does not tend to infinity as $1/a$, but as $\log 1/a$. One assertion can be made: the hope that the difficulties of a classical relativistic electron theory (dualism: particle-field) would disappear of themselves through the mathematical mechanism of the quantization prescriptions has turned out to be in vain. One may say that it is the 'fault' of the relativity theory, which only permits 'contact forces' (and no direct forces at distance) between particles, for this is the quantum theoretical translation of the idea of the relativistic field theory of classical physics (where the interaction between two fields is described by their values and the values of their derivatives in one and the same space-time point). The unfortunate thing is that this relativistic character of the theory is often jeopardized, or perhaps destroyed, by the subtraction prescriptions now in vogue.

3. *Attempts to solve the difficulties.*

At present three currents can be distinguished among the attempts to attack the difficulties methodically. Among the methodical attempts I do not include the well-known program: the x-y-z-t description of the relativity theory is to be dropped and replaced with something more profound involving some 'smallest length' ($\sim 10^{-13}$ cm) and a 'smallest time'. That is not to say that it should not be valuable to keep this program in mind, and that the eventual solution might not be regarded as a realisation of it, but I have (in spite of certain authors) the feeling that it does not exhibit the necessary promise as a starting point for a methodical investigation; it is for me, so to say, too mathematical, has too few contacts with experiment.

These three currents are thus as follows:

I. The introduction of more new fields (with their associated particles), with the purpose that the nasty diverging expressions will cancel each other without additional subtraction prescriptions, so that the result is finite. Bopp and Stückelberg worked in this direction, by considering the electron as source not merely of an electromagnetic field, but besides of a new field, of the Yukawa type, with short range. On the one hand these theories are somewhat fictitious, and, not to mention other difficulties, not all divergencies are cut out. On the other hand, this kind of investigation yields various suggestions, which might turn out of great value. For instance, they may throw some light on the question of the difference between the proton and the neutron masses.

II. The contemplation of the classical particle theory, in order to derive from the correspondence postulate starting points for an improved quantum theory. To this belong in the first place the above mentioned theories of Born-Infeld and Dirac, which modify or re-interpret the classical theory in such a way that they permit to work with a real point particle. These classical theories, however, turn out to be extremely difficult, if not impossible, to quantize and I suspect that their lack of success is because they violate the spirit of the original classical theory. In the second place I here wish to mention the investigations of Opechowski and myself. These investigations brought to light the fact that the quantized interaction between an electrically charged particle and the electromagnetic field, which is in fact the prototype after which all later descriptions of interactions have been modelled, does not exhibit the full correspondence with Lorentz' classical electron theory, which after all one ought to have required. When comparing eq. (1) with eq. (2), one may say that the factor m_{exp} in (1) arose from a classical subtraction prescription relative to the electromagnetic momentum, which is embodied in eq. (3). This feature of the classical theory, which is closely related to the fact that E and H in (1) represent the external and not the total electromagnetic field (which diverges at the point of the electron), has been neglected, so to say, in the litterature, with the consequence that the usually advocated quantum theory of electron and radiation is not directly related to the asymptotic theory of Lorentz, whose results one would have liked to carry over into a quantized form. The fact that in spite of this numerous results could be derived in a satisfactory way (Dirac's derivation of Einstein's A's and B's and of the scattering formulae) is due to details into which I cannot now enter. Typical for the conventional quantum theory is that it is incapable of formulating a stationary state in which purely monochromatic light is being scattered by a bound or free electron, whereas in the classical electron theory this is a very simple problem. It also fails in other respects (example: Dirac's calculation about the shift of spectral lines, i.e., the already mentioned calculation of $E_1 - E_2 - h\nu$). We have been able to give, in a modest (among other things non-relativistic) approximation, a formulation of the interaction of electron and radiation which in our opinion is more correct, and which satisfies the correspondence postulate. Further investigation must decide whether, by continuing in this direction, one can develop a theory which yields a satisfactory, relativistically invariant formulation of the radiation-electron problem.

III. Heisenberg's recent investigations concerning the possibility of a relativistic description of the interaction that is not based on the use of a Hamiltonian with interaction terms in a Schrödinger equation. Heisenberg considers only free particles and introduces a formalism ('scattering matrix') by means of which the result of a short interaction (scattering) between these particles can be described. Formerly the scattering matrix could be derived from the Hamiltonian, but now we are to consider the scattering matrix as fundamental. We do not care whether a Schrödinger equation for particles in interaction exists; we do care which correspondence requirements exist and how the scattering matrix can obey them. It is interesting that the scattering matrix is also able in principle to answer the question, in which stationary states the particles considered can be bound together.

These are related to the existence and the position of zeros and poles of the eigenvalues of the scattering matrix, considered as a complex function of its arguments. Heisenberg could already give a (very simple) model of a two-particle system, in which a perfectly sharply relativistically determined stationary state occurs, while there are no divergence difficulties whatsoever.

However promising, this is still only a beginning, and in particular with regard to a correct description of the electromagnetic fields of photons I expect difficulties, which the investigations in this direction will have to overcome. Fortunately, Heisenberg's program is still open in several respects, and one may perhaps expect a great deal from a fortunate combination with further ideas.

REFERENCES.

1) H. A. Lorentz, The Theory of Electrons, p. 251 ff. (Teubner, Leipzig 1909).
2) V. F. Weisskopf, Phys. Rev. 56, 72, 1939.
3) F. Bopp, Ann. d. Physik 38, 345, 1940 and 42, 573, 1942.
E. C. G. Stueckelberg, Helv. Phys. Acta 14, 51, 1941.
4) M. Born and L. Infeld, Proc. Roy. Soc. (A) 143, 410, 1934; 144, 425, 1934; 147, 522, 1934.
E. Feenberg, Phys. Rev. 47, 148, 1935.
5) P. A. M. Dirac, Proc. Roy. Soc. (A) 167, 148, 1938.
M. H. L. Pryce, Proc. Roy. Soc. (A) 168, 389, 1938.
L. Infeld and P. R. Wallace, Phys. Rev. 57, 797, 1940.
6) J. Serpe, Physica 7, 133, 1940 and 8, 226, 1941.
W. Opechowski, Physica 8, 161, 1941.
7) W. Heisenberg, Zs. f. Physik 120, 513 and 673, 1942.