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Dr. M. DE HAAS. *Measurements concerning the coëfficient of viscosity of methyl chloride in absolute measure between the boiling point and the critical state.* [1])

Although the change of the coëfficient of viscosity of methyl chloride with the temperature between the boiling point and the critical state has been examined by Dr. L. M. J. STOEL [2]), yet only relative numbers were found. My intention was to derive in two ways absolute values for the coëfficient of viscosity from the times of discharge then obtained. In the first place by determining the dimensions of the apparatus, and deriving from them the constants in the formula for μ (with which we design the coëfficient of viscosity), calculated for his experiments by Dr. STOEL [3]). Secondly by using the same apparatus for experiments with a liquid whose coëfficient is known from other experiments. The execution of this purpose however was rendered impossible by the breaking of the original apparatus. Nevertheless the knowledge of the absolute values was

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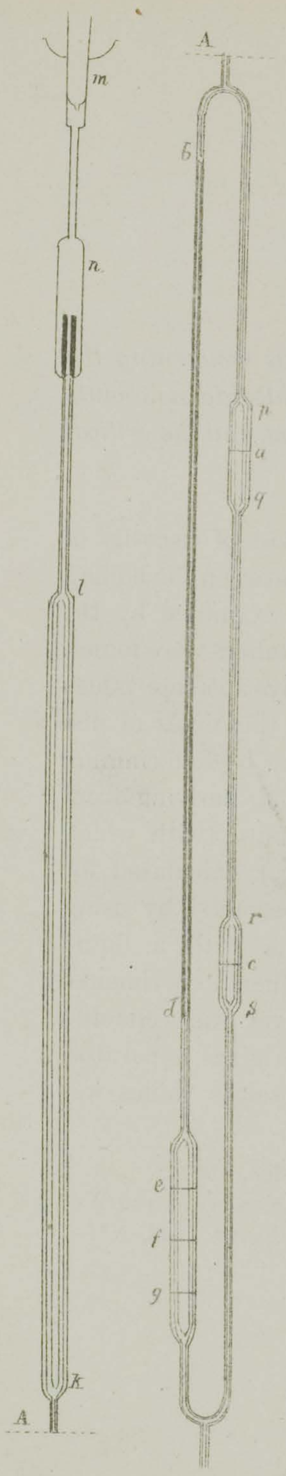
[1]) Published in extenso Dissertation, Leiden, 1894.

[2]) Dissertation, Leiden, 1891. Verslagen Kon. Acad. van Wet. Amsterdam, 28 Febr. 1891. Physikalische Revue 1892, N°. 5. Comm. Lab. of Physics Leiden, N°. 2.

[3]) Communications etc. N°. 2, p. 6.



particularly desirable for testing the law of corresponding states. This is the reason why I repeated Dr. Stoel's observations in such a way, that the absolute values of the coëfficient of viscosity might be established. In constructing another apparatus I avoided at the same time a difficulty in the method applied by Dr. Stoel [1]). As the tube in which the methyl chloride was pressed upward and the other branch of the O-shaped tube, had a different diameter, it was impossible to find the pressure, under which the methyl chloride was really pressed through the capillary tube, unless a most uncertain correction for the capillarity was applied, which appears having been rather considerable in Dr. Stoel's experiments. So I constructed the tube in such a manner that the parts pq and rs whereon the marks a and c are engraved, obtained the same diameter as eg. With this remodelled apparatus, measurings were done with water as discharging liquid, which were extended beyond 100°. The results

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[1]) Communications etc. N°. 2. p. 7.

were compared with those obtained by Slotte [1]) for the temperatures between 20° and 90°, and with those of Mützel [2]) and others for the ordinary temperature. Sufficient agreement being found it appeared that the apparatus was fit for absolute determinations. The following results we obtained from the experiments with water beyond 100° C.:

Temp.	μ
124°.0	0.002232
142°.2	0.001925
153°.0	0.001805

The experiments on methyl chloride were now repeated, and some modifications were made in the manner of cleaning and filling the tube, as well as in the circulation of the bath. Dr. Stoel already mentioned that his results had still to be corrected for the deviation of the observed pressures from the law of Poiseuille. In repeating his experiments I paid particular attention to the influence of the deviations and in calculating the results I took it into account.

First of all it is found by the application of the formula of Reynolds [3]) that the velocity of the methyl chloride in the capillary tube does not reach the »critical" value (at which the motion in the whole tube ceases to be in accordance with the suppositions, which are made in deducing Poiseuille's law) in

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[1]) Wied. Ann. 20 p. 257, 1883; Ofversigt of Finska Vetensk. Soc. Forhandl. 32, p. 116, 1890, vide also Beibl. 16, p. 182, 1892.

[2]) Wied. Ann. 43, p. 15, 1891.

[3]) Phil. Trans. 1886, p. 167.

the observations below 130°, while according to **Wilberforce** [1]) this should not be the case in any of my experiments. The greatest velocity attained in the experiments was 80 c.M. a second.

The correction, which should be applied here, is caused by the loss of energy, in consequence of irregular motion at the ends of the capillary tube. The complete discussion of this correction is impossible in the absence of a solution of the equations of motion of a viscous fluid for this case. **Hagenbach** [2]), **Reynolds** [3]), **Houba** [4]), **Couette** [5]), **Wilberforce** [6]) and others [7]) have given different expressions for the deviation of the pressure from the law of **Poisseuille**: all of them agree in this, that, in order to find the pressure, that should exist if the law of **Poiseuille** was fulfilled, they diminish the pressure really measured with an amount proportional to the square of the mean strength of current and to the density of the liquid. In accordance with the principle of the mechanical similarity the hypothesis was made, that for my experiments the deviation would in the same way depend on the strength of current and the density. As with this apparatus the pressure

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[1]) Phil. Mag. 31, 5ᵉ Ser. 407, 1891.

[2]) Pogg. Ann. 109, p. 385, 1860.

[3]) Phil. Trans. 1883, p. 981.

[4]) Dissertation, Leiden 1883, p. 96.

[5]) Ann. de Chim. et de Phys. 6ᵉ Série, T. 21, p. 500, 1890.

[6]) l. c. pag. 408.

[7]) Vide for instance the theoretical investigations of **Boussinesq**, C. R. 113, p. 9 and 49.

and consequently the strength of current changes every moment, the correction of the pressure must be inserted into the equation of motion; after integration we obtain the following result, that in order to find the true coëfficient the uncorrected coëfficient of viscosity has to be diminished by a term inversely proportional to the time of discharge and proportional to the density and a constant C′ [1]). By making two observations at each temperature with different pressure, we get the same number of values for C′. The fact, that C′ was found almost constant at different temperatures, shows us that we are entitled to the hypothesis mentioned above.

In this way we found:

Temp.	C′
50°	0.00367
60°	0.00349
70°	0.00374
80°	0.00336
90°	0.00340
100°	0.00325

As the greatest value, the correction attains in these experiments, amounts to about 3 percent, this result is sufficient to calculate the correction. By means of graphic interpolation the following values [2]) of μ were derived from 120 times of discharge observed,

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[1]) This quantity is not really constant but may still be a function of **Reynold**'s number $\left(\frac{D\rho v}{\mu}\right)$ as is supposed by **Wilberforce**.

[2]) The values differ slightly from those given in the Reports of the Academy of Amsterdam according to later results.

while STOEL'S numbers for the density of methyl chloride were made use of:

Temp.	*Uncorr.* $\mu \times 10^6$.	*Corr.* $\times 10^7$.	*Corr.* $\mu \times 10^6$.
10°	2032	86	2023
20°	1833	93	1834
30°	1671	100	1661
40°	1532	107	1521
50°	1411	113	1400
60°	1301	120	1289
70°	1196	126	1183
80°	1097	133	1084
90°	1001	140	987.4
100°	911.1	148	896.3
110°	822.8	156	807.2
120°	736.4	163	720.1
130°	651.6	172	634.4
136°.8	590.6	168	573.8

The formula $\mu = C e^{-\frac{T}{B}}$ (in which T represents the absolute temperature, and C and B are constants) given by Dr. STOEL as an approximate formula, not differing too much from his uncorrected observations [1], represents with the same differences the corrected values I found; only beyond 120° greater deviations are found.

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[1] Communications etc. No. 2 p. 7.

Dr. H. KAMERLINGH ONNES. *The coëfficient of viscosity for fluids in corresponding states.*

The foregoing communication gives the coëfficient of viscosity of methyl chloride in absolute value. These results enable us to test near the critical temperature the relation between the coëfficiënts of viscosity of two fluids at corresponding temperatures, which I have deduced [1]) from the theorem, that corresponding states are to be considered as mechanically similar forms of motion. [2])

This relation is:

$$\mu_1 : \mu_2 = \frac{\sqrt{M_1}}{\sqrt[3]{m_1^2}}\sqrt{T_{k_1}} \quad : \quad \frac{\sqrt{M_2}}{\sqrt[3]{m_2^2}}\sqrt{T_{k_2}},$$

where μ represents the coëfficient of viscosity, M the molecular weight, m the volume of the molecules and T_k the critical temperature, while the indices 1

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[1]) Theorie der vloeistoffen, Verh. der Kon. Acad. v. Wetensch. Amsterdam, 1881, tweede stuk, pag. 8. Beibl. 5, p. 718, 1881.

[2]) Compare for further applications of this theorem: VAN DER WAALS, Thermodynamische Theorie der Kapillarität unter Voraussetzung stetiger Dichteänderung. Ztschr. f. phys. Chemie XIII. 4. Zusatz 2, 3, 4.