Dr. P. ZEEMAN. *Measurements concerning* KERR's *phenomenon with normal polar reflection from iron and cobalt.*

§ 1. The observations till now known on the phase in KERR's phenomenon were in very good accordance with Prof. LORENTZ's theory, if to the calcutaled phase was added a nearly constant amount: SISSINGH's phase.

That SISSINGH's phase S proved to be nearly constant gave rise to series of measurements which all gave for this quantity, with the same substance, a nearly constant value within wide limits of the angle of incidence. These limits were f. i. with polar reflection: for cobalt 43° and 73°, for nickel 25° and 75°. But it appears from the graphical representation and the adjoined table, communicated in my name to the Academy in April 1894, that in the case of nickel S seems to diminish with the angle of incidence.

It is true the differences between the theoretical (for which S = Const.) and the observed phases were just within the limits of the probable errors, but at 75° the direction of the deviation was contrary to the direction at 25° and on the whole a certain bias in the discrepancies was undeniable. Nothing but a continued investigation could decide whether we had to deal here with a casual error or with a real deviation.

In the meantime, the supposition that SISSINGH's phase would be constant also beyond the limits of the experimental investigation, had been made the foundation of theoretical speculations by GOLDHAMMER [1]) and afterwards by WIND in still close connection with Prof. LORENTZ's theory [2]). Thus it became a matter of the highest importance to investigate whether SISSINGH's phase has really the same value even at an angle of incidence in the immediate vicinity of 0° — which investigation was already prepared at the time of the publication of my academical thesis [3]). If we succeeded in determining the phase at such an angle of incidence, we should be able to form a wide better judgment on the value of the mentioned theories.

The limits between which S is known would then still be extended in the case of polar reflection with 50° for iron, with 43° for cobalt and with 25° for nickel. The methods of null- and minimum-rotations which were of so high a value in the KERR-investigation, became wholly impracticable at smaller angles of incidence and still more so at 0°. Consequently it was necessary to try for this kind of investigations a new method which accordingly was found by using a $\frac{\lambda}{4}$ plate of DE SENARMONT.

[1]) GOLDHAMMER. Wied. Ann. Bd. 46, p. 71. 1892.

[2]) WIND. Verslag Akad. Afd. Natuurkunde, September 1894.

[3]) ZEEMAN. Dissertatie p. 53 en Verslag Akad. Afd. Natuurkunde, April 1894, p. 179. (Communications No. 10.)

§ 2. The use of a plate, for which the difference of phase of vibrations, parallel to two principal directions, is just a quarter of a wave-length, for analyzing elliptically polarized light, is sufficiently known.

From the position which we must give to the plate and to the analyzer in order to extinguish the light, we can easily deduce the ratio of the amplitudes in two arbitrary directions and the difference of their phases.

In this form, however, the method requires for a definite colour plates of strictly determined thickness and if f. i. we wish to make measurements for D-light with quartz-plates (for accurate measurements to be preferred to mica!) the grinding of such a plate requires extraordinary skill on the part of the optician; unless we leave it to chance for what colour the thickness of the plate will just [1]) correspond to $\frac{\lambda}{4}$ and then make the observations with light of that special colour. But we can avoid this difficulty by eliminating the thickness of the plate from the measurements themselves. Exactly in the same way as with a $\frac{\lambda}{4}$ plate, we have also with a plate of any thickness and with homogeneous light two (or if you like four) different positions (which are

[1]) The thickness can be determined with white light and a spectroscope by putting the plate between 2 crossed or parallel nicols, the inclination of the principal direction to the planes of polarization being 45°. From the position of the dark and bright bands we immediately read the colours for which the plate is $\frac{\lambda}{4}$. This position can be determined with greater accuracy than one would suspect from the form of the curve, representing the intensity of the band-spectrum.

not at 90° from each other) in which the plate can be so used that the light can be extinguished by the analyzer; from these the required quantities can be found in the way, explained in § 4.

§ 3. *The apparatus.* The light of an electric lamp is analyzed by a Christie-Hilger [1]) spectroscope and made parallel by a collimator. Then it is polarized, reflected elliptically, polarized at 1° 23′ by the magnetized mirror and finally examined with the plate and analyzer (fig. 1).

The mirrors. In the same way as on former occasions [2]) I used for mirrors disks of polished metal of 5 mM. diameter; they were fixed with shellac to the top of a truncated cone, being the end of a soft-iron cylinder of 23 m.M. thickness and about 6 cM. length. This cylinder is lengthened by a thinner piece on which a screw is cut, by which it could form the continuation of the remaining part of the magnetic cone. By this arrangement the mirror could easily be taken from the apparatus and replaced in the same position. In this way alterations of the surface could be detected without difficulty by examining the optical constants [3]).

The spectrometer was so mounted that in this complementary examination the collimator and polarizer were used in unaltered position.

The electro-magnet. After a few modifications required by my investigation, I could avail myself of an electromagnet, constructed formerly by Dr. Siertsema

[1]) Zeeman. Archives Néerl. I. 27, p. 259. 1893.
[2]) Zeeman. l. c. p. 281.
[3]) Zeeman. l. c. pp. 270, 283.

in order to produce a strong field for another purpose.

In fig. 2 and fig. 1 we have a vertical and horizontal sketch of the apparatus. It consists principally of three soft-iron parallelopipeda. The lowest piece measures $50 \times 10 \times 10$ cM., the two erect pieces measure $10 \times 10 \times 14$ and $10 \times 10 \times 17$ cM. The left carries just a little above the middle the cylinder *C* into which the piece with the mirror is screwed. The right one *B*, I had made in such a way that it serves as a submagnet [1]) and at the same time does not disturb the passage of the rays. The produced axis of the cylinder is also that of the truncated cone *D* of about 4 cM. length.

The whole piece *P* is pierced by a partly cylindrical and for the rest conical canal, the width of which is 6 mM. near the mirror and 22 mM. at the other end. The coil, as a rule used with 20 Amp., was wound with insolated copper-wire of 4 mM. thickness; during the experiments the elevation of temperature was insignificant. The screw-bolts *T* press the parallelopipeda strongly together by which the magnetic resistance is diminished and displacement of the parts is made impossible.

The distance from the mirror to the front of the sub-magnet was about 3 mM.

In this position of the sub-magnet the mirror was normally magnetized. I could convince myself of this fact by bringing very small pieces of a watch-spring near the mirror; they stood normal to the plane of the mirror even close to the edge.

[1]) Compare du Bois. Phil. Mag. (5) Vol. 29, 1890, p. 293.

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As it was of course of great importance to give to the mirror its exact position, the electro-magnet (weighing with its accessories 90 K.G.) was placed upon a large, carefully worked, turning-lathe-support. Rotations round a vertical axis and horizontal displacements in two directions perpendicular to each other, could in this way be effected without difficulty. Also the angle of rotation could be rudely read on a divided circle; small rotations of the mirror round a horizontal axis were effected by putting thin copperplates under the electro-magnet. The whole apparatus was supported by four columns, resting on one of the pillars of the building.

The plate. The quartz-plate used was about 0,31 mM. thick and had a surface of 1×1 cM^2. By choosing quartz, which can be so nicely ground that the image of a wire-cross can be seen in the front, I was enabled to effect the delicate adjustment of the plate. In fig. 3 (scale $^1/_2$ of the original size) the manner is indicated in which the plate is placed before the analyzer, while fig. 4 gives a perspective drawing of the whole measuring-arrangement.

I fixed the plate P with collodion on three spots to a perforated piece of cork, placed in a tube, all the required adjustments of which could be effected by means of 3 screws pushing it forward and 3 others drawing it backward. C is a copper disk, to which the lever is attached which is used for the delicate adjustments.

The rude adjustment of the plate was made with the circle D which I took from a JAMIN spectrometer

and also the reductional observations necessary for reducing to minutes the rotations read in scale divisions. Rotations of the whole tube round $A\,A$ were regulated by a fine screw in the tail-piece of the trunnion-plate by which $A\,A$ is supported, rotations round a vertical axis having also been taken into consideration in the construction. The tube B was kept down by a counterpoise G. By the sketched arrangement I could effect that the plane of the plate P was normal to the incident beam of light which at the same time coincided with the axis of the tube B. It became clear to me that, if in one position of the plate these conditions were satisfied, only small fluctuations of the image of the cross-wires occurred when it was turned and that consequently the construction of the tube was quite satisfactory.

The analyzer and polarizer were the same that were formerly used with the null- and minimum-rotations [1]).

§ 4. *Calculation of the experiments.* The current was taken strong enough to neglect the disturbing influence of the permanent magnetism residual from former magnetizations on the final magnetic state [2]). From the principle of images it follows that with a reversal of the magnetization the amplitude of the magnetic component of the light changes its sign only [3]), but that the phase remains the same. So there is no

[1]) SISSINGH. Archives Néerl. T. 27. p. 189.
[2]) DU BOIS. Phil. Mag. (5) Vol. 29. p. 296.
[3]) ZEEMAN. l. c. p. 283.

objection against using the difference of the rotations of the analyzer and also of those of the plate at + and — magnetization instead of using rotations from the principal positions. The demagnetization which takes so much time and the difficult determination of the principal position at an angle of incidence of 0° are in this manner avoided. Besides, an occasional residual magnetism can have no influence on the calculation of the phase by the smallness of the magnetic component of the light.

With respect to the signs used I have still to mention the following. It is known that one sees two dark spots near the bright centre of the interference-phenomenon, exhibited by a quartz-plate placed between two crossed nicols in convergent monochromatic light [1]). On the edge of the tube containing the plate, I have marked by two scratches the direction which is perpendicular to these and call it: direction 2; direction 1 coinciding with the scratches. I call rotations positive if they are (for an observer placed before the object-glass of the collimator) in a direction opposite to that of the hands of a watch. It has already been remarked (§ 2) that there are two positions (the difference of which is not 180°) of the plate and the analyzer in which no light is transmitted. The rotations of the plate will for convenience sake be denoted by 2φ, those of the analyzer with respect to the plate by ψ'. The rotation φ_2 be 0 if with crossed nicols direction 2 coincides with the plane of polarization of

[1]) KOHLRAUSCH. Prakt. Physik. p. 193. 1892.

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the incident light; φ_1, on the contrary, if direction 1 is perpendicular to it, ψ_2' and ψ_1' are in these cases also 0° and 90°.

According to E. WIEDEMANN [1]) the following equations (derived in accordance with KICHHOFF's indications) hold if we use the above-mentioned notation and if B denotes the phase of the magnetic component of the light:

$$\left.\begin{aligned} tg\,\frac{B-C}{2} &= cotg\,\frac{A}{2}\,\frac{Sin\,(\psi'-\varphi)}{Sin\,(\psi'+\varphi)} \\ tg\,\frac{B+C}{2} &= cotg\,\frac{A}{2}\,\frac{Cos\,(\psi'-\varphi)}{Cos\,(\psi'+\varphi)} \end{aligned}\right| \;\ldots\ldots (1)$$

In these equations A is an auxiliary angle dependent on the thickness of the plate and given by

$$Cos.\ A = \frac{tg\,(\psi_1'-\psi_2')}{tg\,(\varphi_1-\varphi_2)} \;\ldots\ldots (2)$$

C is an auxiliary angle of no consequence for our purpose.

In (1) both systems for ψ' and φ must be used. Thus 2 results for B are obtained the accordance of which is a measure for the obtained accuracy.

§ 5. *Measurements on iron.* To one position of the polarizer belong eight positions of the quartz-plate, viz. 4 for each position of the analyzer. In each position 4 adjustments are made at + and 4 at — magnetization. Like in all similar observations we may take for granted that the result of those 8 positions will be free from

[1]) E. WIEDEMANN. Berichte der K. S. Gesellschaft der Wissenschaften. 1872, p. 279.

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all systematic errors, resulting from imperfections of the different parts of the apparatus. The observation in the three other positions of the polarizer should then give the same result within the limits of the probable errors. This proved to be the case.

In a more extensive publication the arrangement of the measurements will be described in detail

For one position of the polarizer I give the double values of the rotations, with which those got from the diametrical positions have already been combined.

Iron-mirror D-light.

	Position of the plate.	Rotation of the plate.	Rotation of the analyzer.
Position of the polar. 218°.7			
» » anal. 4°	2	35′.8	18′5.
	1	24′.5	43′.2
» » anal. 184°	2	36′.5	16′.5
	1	24′.3	44′5.

From this and 3 other series it now follows:

	φ_1	ψ'_1	φ_2	ψ'_2
Pol. 38.7	$-11.2^5 \pm 0.40$	$-9.2^5 \pm 0.55$	$-90^\circ - 18.5 \pm 0.32$	$90^\circ + 9.5^5 \pm 0.60$
128.7	-11.9 ± 0.55	-9.3 ± 0.68	$-90^\circ - 18.3 \pm 0.36$	$90^\circ + 9.7 \pm 0.55$
218.7	-12.2 ± 0.36	$-9.7^5 \pm 0.54$	$-90^\circ - 18.1 \pm 0.35$	$90^\circ + 9.6 \pm 0.47$
308.7	-11.8 ± 0.36	-8.8 ± 0.47	$-90^\circ - 19.7 \pm 0.48$	$90^\circ + 10.5^5 \pm 0.59$

The whole results to:

$$-11.79 \pm 020 \quad -9.23 \pm 0.27 \quad -90^\circ - 18.51 \pm 018 \quad 90^\bullet + 9.82 \pm 0.27$$

The adjoined numbers are the probable errors of the adjustments, expressed in minutes, like the rotations.

The optical constants of the mirrors were:

before the experiments	$I = 76^\circ 42'$	$H = 28^\circ 7'$	D-light.
after » »	$I = 77^\circ 12'$	$H = 27^\circ 18'$	
mean value	$I = 76^\circ 57'$	$H = 27^\circ 44'$	

The two values for the phase B (taken in the same way as in my former measurements) are:

From position 1.	From position 2.
$B = 30^\circ 26'$	$B = 31^\circ 11'$

mean value $B = 30^\circ 30' \pm 1^\circ 26'$

From this value and from the phase calculated with the measured optical constants, follows for SISSINGH's phase at 0°:

$$S_{Fe} = 69^\circ 19'.$$

With iron-equatorially SISSINGH found $S = 85^\circ$ and with iron-polarly I found $S = 80^\circ$ at an angle of incidence $i = 51^\circ 22'$. So my result is that in this way we find another value for S beyond the formerly given limits.

§ 6. *Measurements on Ccbalt.* Here I contented myself (considering what has been said at the beginning of the preceding §) with the 8 series, each of which I put together from 8 observations. The results are the following.

Cobalt-mirror. D-light.

φ_1	ψ_1'	φ_2	ψ_2'
$-15'.33 \pm 0.19$	$-3'.73 \pm 0.32$	$-90^\circ - 18'.26 \pm 0.28$	$-90^\circ + 3'.56 \pm 0.42$

The optical constants were:

before the observations	$I = 77^\circ 0'$	$H = 31^\circ 24'$
after » »	$I = 76^\circ 40'$	$H = 31^\circ 40'$
mean value	$I = 76^\bullet 50'$	$H - 31^\circ 34'$

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Hence it follows for B

from position 1. $B = 11^\circ 19'$ — from position 2. $B = 10^\circ 55'$

mean value $B = 11^\circ 10' \pm 3^\circ$.

Calculating the theoretical phase with the given optical constants and comparing B with it, we have:

$$S_{Co} = 42^\circ 4'$$

while formerly I found at angles of incidence greater than 43° on an average $S = 49^\circ 30'$.

Comparing the observations of this § and of § 5 with the former, it would consequently follow, that S diminishes at very small angles of incidence. For the nearer confirmation of this result however, a comparison of the method of the null- and minimum-rotations with the method now followed is necessary. Therefore I intend to make measurements by the two methods at the same angle and with the same mirror.

In order to ascertain myself now already of the reliability of the results of the method now followed (the figures formerly obtained were found accordant by two mutually controlling methods), I have made:

§ 7. *Measurements on a silver-mirror.* At an angle of incidence of 30° I measured the difference of phase of the rays polarized in and perpendicularly to the plane of incidence. For this purpose I gave to the polarizer deflections of 1° from the principal position to the left and to the right. The ellipse now obtained is completely analogous to the one found above, the difference of phase of the components amounting to

about 11° if we calculate it by CAUCHY's formulae from the measured optical constants.

The measurements with the plate gave results which agreed with the calculated values within the limits of the observational errors.

§ 8. In order to have a complete view of the rate of change of SISSINGH's phase it will be desirable to make further measurements between 0° and the limits of the former angles of incidence, even if the measurements mentioned at the end of § 6 confirm (and the probable errors entitle us to this expectation) the values hitherto found at greater angles of incidence. For the present we must doubt whether one of the theories framed is capable of a complete explanation of KERR's phenomenon. A complete theory will also have to account for the fact that the succession in magnitude of SISSINGH's phase for iron, cobalt, nickel (and probably magnetite) is the same as that of their maxima of magnetization.