very weak primary current in order to choose the resistance in the by-current. If we thereafter make the primary current stronger, then another choice of the resistance would be necessary. We however did not observe any difference.

Dr. A. LEBRET. *Dissymmetry of the Hall-effect in bismuth for the opposite directions of the magnetic field.*

In all the plates of bismuth I used, the Hall-current, proved not to be of the same strength for the two opposite directions of magnetization.

Always using the described method of observation, I was urged to introduce for one direction of magnetization (A) an amount of resistance, wholly different from that for the opposite direction (B).

(The amount of the resistance to be chosen for the opposite directions of the primary current differed but very little, and was independent of its strength) [1].

If we trace the cause, which may produce this dissymmetry, we meet under B 7 b in the list of errors in the preceding communication thermo-electric currents, produced by the PELTIER-effect.

Since these only arise after the primary current ha passed a while, and as our method is a momentary one, it is not to be accepted that the right explanation should be found in them, chiefly on account of the great amount of the dissymmetry.

[1]) The primary current could vary from 1.5 till 7 ampères. The strength of the magnetic field was about 3000 (c. g. s.).

Moreover we discussed the error, which might arise from not attending to the remanent magnetism, but which has been avoided according to *B* 12.

Further the influence of the magnetic force of the primary current also proved not to be the cause.

Now we still find under *A* 2 the longitudinal thermomagnetic effect, the influence of which however is avoided by closing in the first place the magnetizing current and then the primary current.

Finally under *B* 8 we find a difference of potential, which exists already without a magnetic field. In that place however, it has been shown how this difference of potential may be neutralized with the aid of the by-current. But can it be now, that the by-current produces the euqipotentiality for a zero magnetic field, but does not warrant it, if the resistance of the branch which passes through the plate of bismuth increases in consequence of the magnetic field? We examined this for the first plate which showed us the dissymmetry and it proved that an increase of the resistance would produce a dissymmetry of a direction opposite to the really observed.

The plate being very thick and consequently of small resistance, it was a priori improbable that this should be the right explanation. In order to exclude all mistakes however, the whole by-current was omitted, the error of the incomplete equipotentiality being eliminated by performing the experiment as was communicated in *B a* 8 and 8 *b*. The dissymmetry arose exactly to the same amount as before, so that the cause is neither to be found in the by-current, nor in the variation of resistance of

the plate of bismuth according to the magnetic field.

We further made measurements, employing the usual method of deviation, which needs the correction for the different sensiblenesses according to the opponent directions of the magnetic field, but we found the dissymmetry as well as before.

Moreover we remark that by our method of compensation the difference of the sensibilities of the galvanometer for the two directions of the magnetization cannot have any influence.

The magnetic force was determined with the aid of a Rowland's proof-plate, and proved to be of the same amount for the opposite directions.

From all this it is plain that we have to do with a phenomenon, which cannot be explained with the aid of the phenomena already known.

Experiments with respect to the dissymmetry under various circumstances.

1. Turning the plate in such a manner that front and back change place.

We take the plate out of the vessel, turn it 180° about a vertical axis and put it back.

The dissymmetry proves now to be *reversed*, the Hall-effect, which before appeared to be stronger for the direction of magnetization *B*, now appearing to be stronger for the direction of magnetization *A* [1]). The

[1]) This was also found by van Aubel, as he communicates in the Archives des sciences physiques et naturelles de Genève 3. 33. 25 March 1895.

amount of the dissymmetry is the same. Hence it follows that there must be in the plate some difference, of what kind soever, the sign of which is determined by the normal, drawn to the outside from a previously fixed side of the plate.

2. Changing the primary and the secondary electrodes in such a manner that the primary current passes from C to D, A and B becoming the secondary electrodes, connected with the galvanometer.

The dissymmetry proves again to be *reversed*, the Hall-effect appearing stronger for the direction of magnetization A, when it was before stronger for the direction of magnetization B.

Again the amount of the dissymmetry was the same [1]).

Determination of an axis of symmetrie

Changing the primary and the secondary electrodes is theoretically the same as turning the plate in its plane round an angle of 90°.

The dissymmetry changing sign in turning 90° about the determining normal, must have passed the value of zero.

Therefore a round plate of bismuth was manufactured, to which the four electrodes could be fastened with

[1]) This experiment has properly been taken with regard to the possibility that the magnetic force, arising by the primary current itself, had rendered the determination of the equipotentiality in a zero magnetic field faulty. At that time I had not yet proved that this influence is too small to be measured. And we avoid the action of the magnetic force for the greater part by using the electrode divided into two parts. Instead of annulling the dissymmetry however, we found reversing.

the aid of small screws. These can easily be loosened and again be fastened after having somewhat turned the plate.

The dissymmetry appears again in this plate, changing sign after turning it in its own plane round an angle of 90°. And between these two positions we really found one, for which the dissymmetry had disappeared.

This fundamental position of the plate according to the primary electrodes may be indicated on it by a directrix. Turning from this position into both directions the appearing dissymmetry proved to be of different sign. The dissymmetry rose to the highest point after having turned round 45° (always in the plane of the plate), reckoning from that fundamental position. After having turned round an angle of 90° the dissymmetry again has disappeared, so that we might as well have chosen the latter position for the fundamental one. The only difference between these two positions is, that the dissymmetry appearing after turning in the same direction from each of these positions, acquires the other sign.

If the electrodes have not been fastened in the fundamental position, we obtain a dissymmetry changing sign also after turning about a vertical axis, i. e. after changing front and back. For, imagining the directrix indicated on the plate, we observe it acquiring a position exactly opposite to the position before the change. So the result of experiment I agrees entirely with the last result.

Modification of the Theory of the Hall-effect.

As has been shown by Goldhammer [1]), the Hall-effect

[1]) Wied. Ann. 31 p. 370.

may be described by supposing that the metal used becomes aeolotropic by the magnetic field, in consequence of which the following relations of MAXWELL concerning the current and the decrease of potential exist:

$$\frac{\partial p}{\partial x} = -K_{11}\ u - K_{12}\ v - K_{13}\ w$$

$$\frac{\partial p}{\partial y} = -K_{21}\ u - K_{22}\ v - K_{23}\ w$$

$$\frac{\partial p}{\partial z} = -K_{31}\ u - K_{32}\ v - K_{33}\ w.$$

Since two dimensions will suffice in order to explain the phenomenon, this becomes:

$$\frac{\partial p}{\partial x} = -K_{11}\ u - K_{12}\ v$$

$$\frac{\partial p}{\partial y} = -K_{21}\ u - K_{22}\ v.$$

Always however two perpendicular directions may be found, for which the equations become:

$$\frac{\partial p}{\partial x} = -K_{11}\ u - H v$$

$$\frac{\partial p}{\partial y} = H u - K_{22}\ v.$$

These directions we call the principal axes.

GOLDHAMMER supposes there will be no reason to infer a difference between K_{11} and K_{22}, and so he equalizes these values. So the equations have been obtained, which BOLTZMANN introduced for describing the Hall-

effect, which are in fact the same as those given by LORENTZ [1]).

The difference of potential at the secondary electrodes really is:

$$e = H \frac{I}{d} \quad ^{2})$$

so that we must consider H to be dependent on the used substance (coefficient of HALL) and of the magnetic force. Supposing that after reversing the magnetic force H only changes sign, retaining the same amount, the Hall-effect results symmetrical. And it is easily shown that the symmetry persists for all axes, as the equations remain the same after having turned the system of coordinates.

Supposing however that K_{11} and K_{22} are of a different value, we again can show that the resulting difference of potential is $e = H \frac{I}{d}$, if we fasten the primary electrodes to the plate in the direction of the principal axis, for which the equations have been given. It only changes sign, retaining the same amount, if we reverse the magnetic field. So for this principal axis the Hall-effect is symmetrical.

But now transforming the equations for another direction, the new axis making an angle x with the old ones, they become

$$\frac{\partial p}{\partial \xi} = u_1 \left(-K_{11} \cos^2 \alpha - K_{22} \sin^2 \alpha\right) + v_1 \left\{-H + \sin \alpha \cos \alpha \left(-K_{22} + K_{11}\right)\right\}$$

1) Arch. Neérl. XIX p. 123.

2) d is the thickness of the plate.

$$\frac{\partial p}{\partial \eta} = u_1 \left\{ + H + \sin\alpha \cos\alpha (-K_{22} + K_{11}) \right\} + v_1 (-K_{11} \sin^2\alpha - K_{22} \cos^2\alpha)$$

The difference of potential resulting at the secondary electrodes, the primary wires having been fastened in the direction of one of the new axes, is for one direction of magnetization:

$$e = \left\{ H + \tfrac{1}{2} \sin 2\alpha (-K_{22} + K_{11}) \right\} \frac{I}{d}$$

and for the opposite direction of magnetization

$$e = -\left\{ H - \tfrac{1}{2} \sin 2\alpha (-K_{22} + K_{11}) \right\} \frac{I}{d}$$

Hence it follows that the absolute values of the difference of potential are different, so that the dissymmetry is described by supposing the resistance of the plate of bismuth placed in the magnetic field, to be different in two perpendicular directions.

Variation of the dissymmetry with the strength of the magnetic field.

The dissymmetry now being ascribed to the difference of the resistance arising by the magnetic field, the question rises if the dissymmetry depends on the strength of the magnetic field just as the resistance itself.

The experiments, which will make this out, were performed in the position differing about 45° from the fundamental position [1]).

[1]) These measurements were performed without the use of the by-current. Also it was determined (in accordance with B 8 b of the former communication) with the aid of the compensative current, how strong the secondary current for a zero magnetic field was, the amount being deducted from the currents arising for the various magnetic fields.

The dissymmetry observed proved to increase with the strength of the magnetic field, the increase for small values of the magnetic field being somewhat slower than that of the square of the magnetic force, and for great values of the magnetic field somewhat more than the increase of the magnetic force itself.

As it appears from the experiments of DRUDE and of HENDERSON [1]), that also the increase of the resistance of bismuth with the magnetic force follows the same law, this may be considered to confirm our explanation of the dissymmetry.

The following values result from calculating

$\frac{M}{\text{about } 1500}$	$H\,C$	$2.65\ (K_{11}-K_{22})\ C$	$H\,M\,C^1$
1	1	1	1
1.44	1.23	1.82	1.77
1.83	1.36	2.59	2.49
2.32	1.42	3.58	3.29

M, H and $K_{11}-K_{22}$ indicating in absolute measure the magnetic force, the Hall-effect and the dissymmetry, C and C^1 being constants of less importance.

The variation of $(K_{11}-K_{22})$ agreeing in some degree with the variation of the product $H\ M$, we have added an account of the relative value of this product.

We intend to examine the dissymmetry in a plate, cut out of a great crystal or an agglomeration of crystals all grown together in the same direction.

[1]) WIED. Ann. 53, p. 912.