COMMUNICATIONS

FROM THE

PHYSICAL LABORATORY

AT THE

UNIVERSITY OF LEIDEN

BY

PROF. DR. H. KAMERLINGH ONNES.
Director of the Laboratory.

No. 25—36.

APRIL 1893—JANUARY 1897.

EDUARD IJDO — PRINTER — LEIDEN.

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No. 25.

(REPRINT).

H KAMERLINGH ONNES. A device for illuminating scales read by reflexion (With two plates).

(*Translated from: Verslagen van de Afdeeling Natuurkunde der Kon. Akad. van Wetenschappen te Amsterdam*, **18 April 1896**, p. 311—317).

EDUARD IJDO — PRINTER — LEIDEN.

H. KAMERLINGH ONNES. *A device for illuminating scales read by reflexion.*

1. In the physical laboratory at Leiden for many years a method of reading scales with mirror and telescope has been used, in which a small source of light is made to give an intense and uniform illumination to long glass scales. The researches of SISSINGH, ZEEMAN, SIERTSEMA and LEBRET amongst others have demonstrated the convenience of this method. Fig. 1 shows the arrangement, which has been in use for many years for the measurement of low temperatures by means of a thermoelectric couple. This is the first opportunity I have had of describing my method.

Behind the transparent glass scale (as manufactured by HARTMANN and BRAUN) reflectors are placed obtained by cutting a concave mirror into strips [1]) (see figs. 1 and 2). The source of light is placed above the scale and each of the strips throws an image of the source of light upon the reading mirror of the measuring apparatus. When the reading telescope is focussed on the image of the scale the divisions are seen intensely black

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[1]) Originally I cut a broken concave mirror into strips, since these strips are especially manufactured for me by P. J. KIPP & ZONEN, J. W. GILTAY, opvolger, at Delft.

on a very uniformly and brilliantly illuminated-background. The excellence of the illumination obtained in this way has also been noticed by W. H. JULIUS, pag. 53 of his remarkable thesis (1888) he describes how he illuminated his galvanometer-scale by a paraffine-lamp a concave mirror being placed immediately behind the scale and points out the extreme precision with which the readings could be taken.

The method described has many advantages. It is independent of the daylight; the readings can be taken even in a room where the light is very bright, with a telescope magnifying 60 times or more; and scales divided into quarter millimeters might be used and tenth parts of a division still be estimated. Scale and telescope may be placed in the usual relative position, as appears from fig. 1. When desired the source of light may be put close to the reading-arrangement. And lastly the heat given out by the source of light need not be feared as the latter may be of very small size.

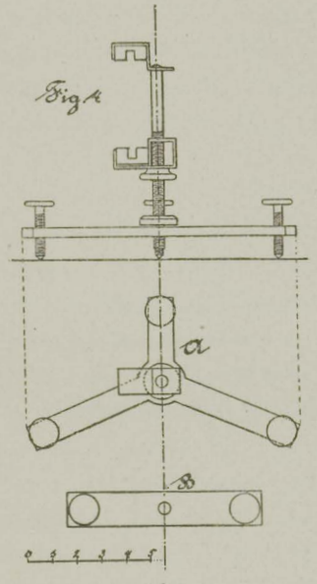
2. If concave strips of arbitrary ellipsoïdal curvature were available, instead of the spherical strips which are so much easier to obtain one could place the light at any distance and illuminate the whole scale by a single source of light of extremely small breadth, placed in one of the two foci of an ellips, the reading mirror occupying the other. The elliptic arc may be imitated to some degree by a certain number of spherical strips. To this end each strip is to be brought into such a position that the image of the source of light which it gives, is thrown in the direction of the reading mirror, and that the strip looks uniformly and brilliantly illuminated as seen from the place of the reading mirror. It is only possible to fulfil the latter condition for each of the spherical strips when the source of light has a certain breadth.

In investigating the connection between the breadth of the source of light required, the angle subtended by each mirror strip and the position and the radius of curvature of this strip, the following points have to be born in mind. A point of the scale is illuminated if the extreme rays, drawn from that point towards the reading mirror when produced backwards towards the illuminating mirror strip, after reflexion on the strip fall within the light emitting surface. If the radiation of the flame is known the intensity of the illumination may be deduced from a consideration of the cones formed by those extreme rays.

Without going into details however it is evident that the broader the source of light the less it matters whether the mirror strip when placed in a certain position has the correct radius of curvature and whether its normal points in the true direction. Moreover it is found that, if the source of light is not too narrow and if the angle subtended by each of the strips not too large, a proper adjustment may be obtained even with strips all of which have the same curvature and that the illumination even in that case becomes very nearly uniform throughout the scale. The radius of curvature of the strips which are used at Leiden is about 96 cMs.; the longest strips subtend an angle of 28° and the length of the chord is therefore 45 cMs. With two strips of that kind at a distance of 3 metres from the

mirror a scale of 80 cMs. may be illuminated by means of an Argand gasburner; for a scale of 100 cMs. three strips are required or two long ones and two smaller ones.

Fig. 2 shows in plan the arrangement of fig. 1 and gives an idea of the relative position of mirror, strips, scale and source of light. In this case a scale of one metre at a distance of 3.2 metres from the galvanometer-mirror was beautifully illuminated by means of a colza-oil flame. This light is suspended at a distance of 54.3 cms. from the mirror strips and of 30 cMs. in front of the scale. Some of the light rays are traced in the diagram. For the sake of comparison the elliptic arc with flame and mirror as foci has been added to the diagram as a dotte curve. The width of the flame is 3.5 cMs., the height of its luminous part from 1.5 to 2 cMs.

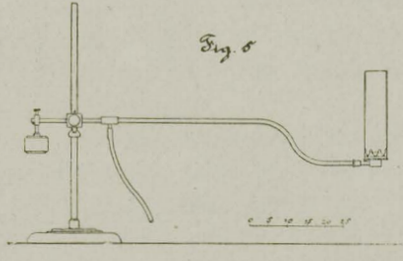


3. In considering some further details we may restrict ourselves to the case in which the scale is horizontal [1]).

It is in general desirable to put the source of light

[1]) For mirror readings on vertical scales and for illuminating cales for other purposes the same method is applied.

above the scale to avoid parts of the scale being covered or the sharpness of the images being spoiled by currents of hot air. The path of the rays will then be according to the diagram in fig 3 (see plate) and the mirror strips must be inclined somewhat backwards. To effect this the mirror strips are clamped between pieces of cork in light, adjustable holders. The construction of these holders is shown in fig 4; fig. 4*a* showing the pattern which is used when a mirror strip is supported by one holder only, fig. 4*b* that which serves when two holders carry one strip. The vertical distances between telescope, illuminating mirror and flame (fig. 3) should be kept as small as possible. In order to prevent the edges of the scale from spoiling the uniform illumination of the background by their dark images the strips should not be placed too near the scale. It is not desirable to use very long strips, firstly, because they are expensive, secondly, because of the inclined position which has to be given to them, and lastly for the sake of the approximation to the elliptic shape which is aimed at. Yet it is advisable to take as few strips as

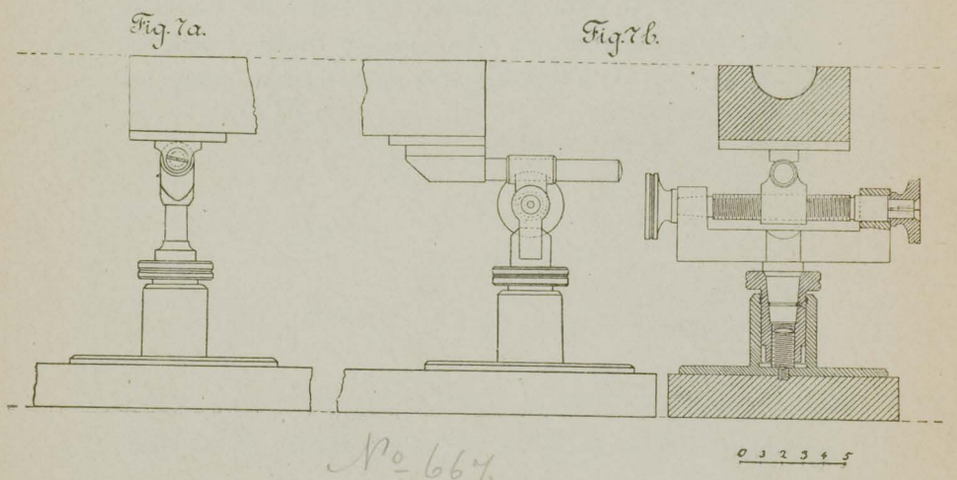


possible, as the edges of the strips disturb by a dark

line the uniform illumination of the background on which the scale divisions are seen.

4. If the radiation from the source of light is of no consequence it is convenient to use an Argand gas-burner. The gas is supplied to the burner through a long arm which reaches over the reading apparatus (fig. 5) in order that the luminous part of the flame may be brought as low as possible above the scale. The foot of the lamp is clamped to the reading table on the side of the observer.

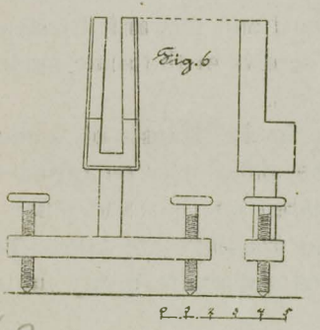
In cases where it is not indifferent, how much heat is given off by the source of light the special advantage of the method is obvious. In fact the inevitable production of heat can be limited to a minimum as only one single luminous surface of very small dimensions is required. To actually reach this minimum may sometimes be of great importance, e. g. for magnetic observations in cellars, where the temperature should



be kept constant. As a source of light which gives little heat I use as a rule a colza-oil lamp consisting of a broad wick in a long trough (fig. 1). There has at yet been no inducement to get rid of the little heat, which this source of light gives off[1]).

According to a personal communication Prof. HAGA of Groningen, in applying the method makes use of an incandescent lamp and does not find this gives any trouble, not even with very accurate galvanometric observations.

5. The reading telescope is mounted on a support which is provided with the necessary adjustments. These supports are also used for the other experiments. The telescope lies in a groove in a wooden beam, which rests on two adjustable brass feet (fig. 7). The foot in front (see fig. 7*a*) allows of an upward and downward motion, a rotation round a vertical axis and a rotation round a hinge with horizontal axis. This hinge which is carefully constructed is fixed underneath the front of the beam. To the end of the beam a brass rod is



[1]) There was no more inducement to investigate the advantages that according to § 2 might be derived from using strips of another radius of curvature and giving another place and shape to the light source. In all the cases of reading scales with mirror and telescope that occurred, the means present proved sufficient.

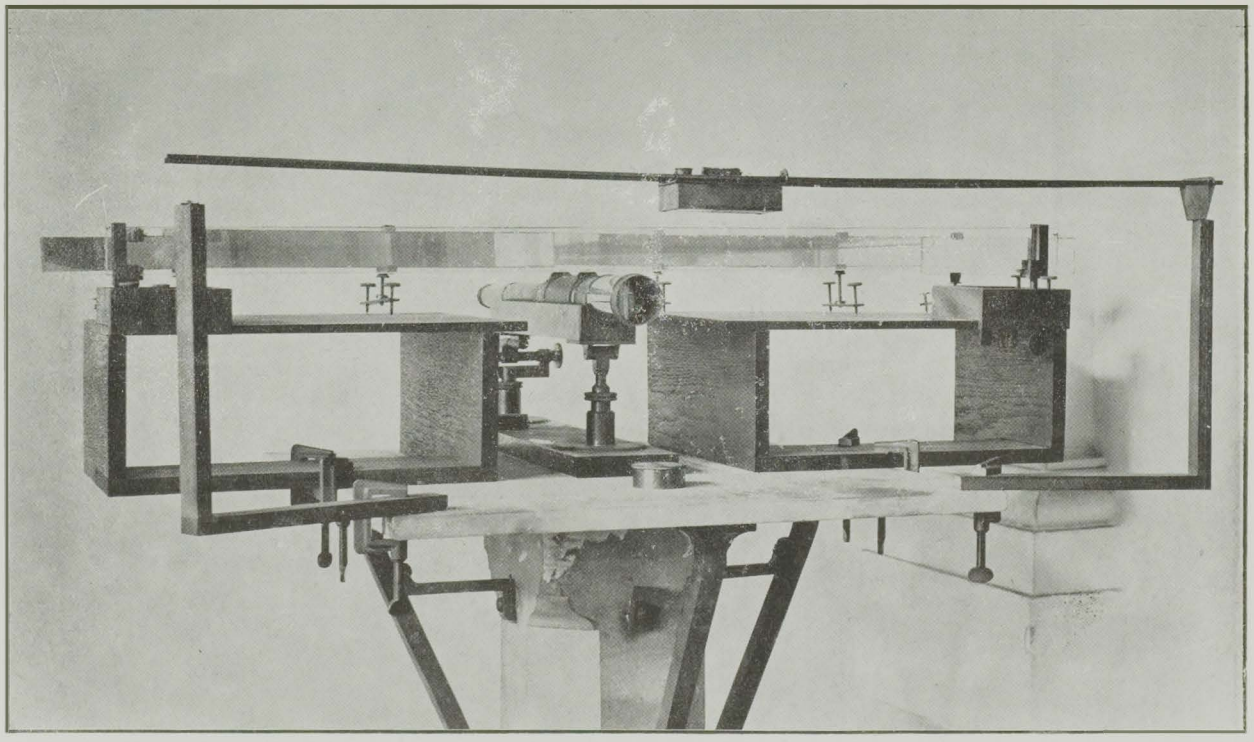


Fig. 1.

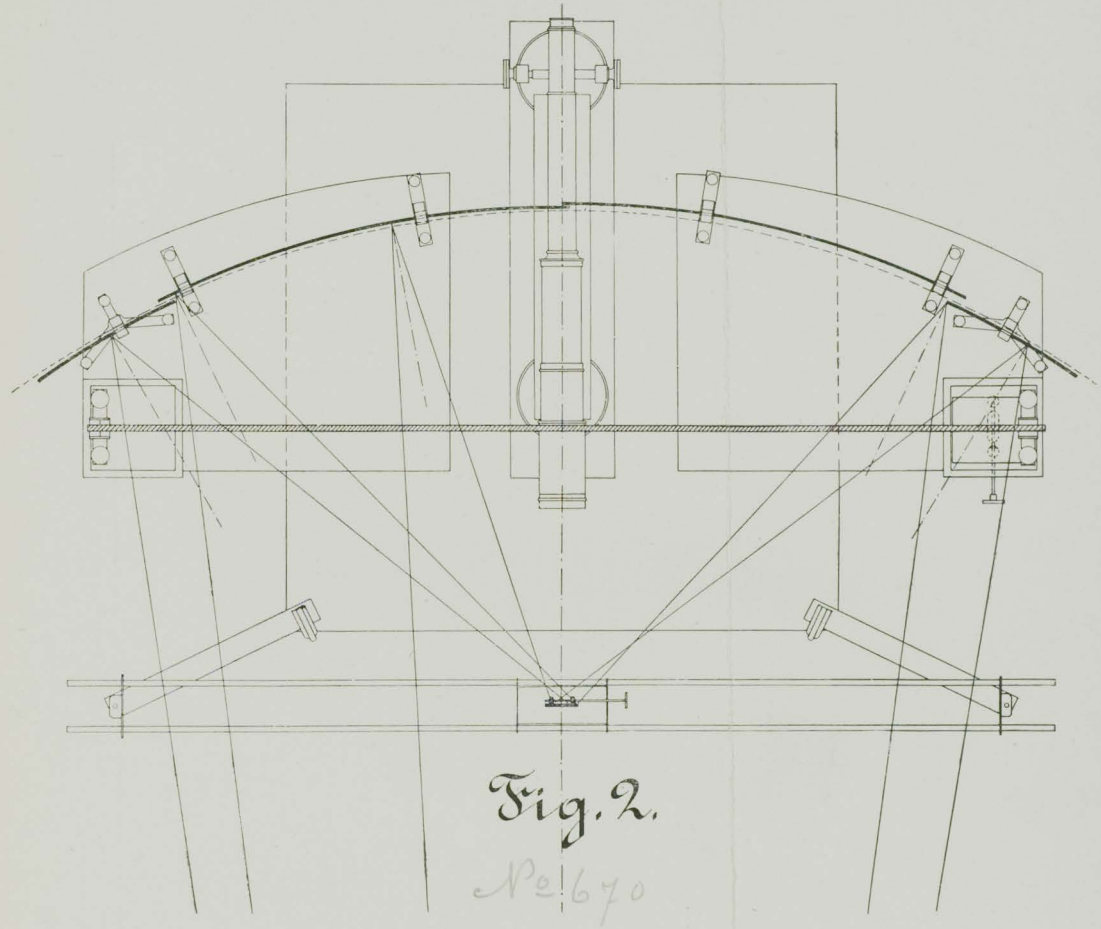


Fig. 2.

№ 670

Fig. 3

№ 671

Communication N°. 25.

attached which slides through an annular brass piece: this piece forms part of the second foot, is movable round a vertical and a horizontal axis and may be adjusted by means of two screws from left to right and up and down.

Only a few more things are required to complete the apparatus. The glass scale is held by two scaleholders which consist of two flexible metal strips lined with cork and are fitted with levelling screws. Both these scale holders with the scale and the mirrorholders with the mirror strips rest on wooden supports of the shape most convenient for the scale length used. These supports as well as the telescope-support are firmly clamped on to a marble slab. The holders are prevented from slipping by being placed in small wooden trays which are fixed on the top of the supports. The bottom of each tray has a fine screw adjustment backward and forward. By means of these screws and the fine screws of the scale holders the scale may be accurately set.

The marble slab as shown in fig. 1 rests on a movable pillar of freestone: the pillar may be raised on appropriate blocks and is placed on the fixed pillar of the magnetic room when the readings are taken. The slab, pillar, blocks and fixed pillar are firmly united by plaster of Paris.

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E. VAN EVERDINGEN Jr., *Remarks on the method for the observation of the* HALL-*effect.*

In all the researches, that will be spoken of in the following communications, the method is used, described by Dr. A. LEBRET in Chapter VIII, § 6 of his inaugural dissertation [1]).

At first I used also the »by-current" [2]) to annul the secondary current in a zero magnetic field, caused by fastening the secondary electrodes not wholly in the right places. This however is liable to various difficulties.

1. It is often found, especially at high temperatures, that when the resistance in the by-current is well-chosen *before* an observation, *after* the observation in a zero magnetic field again a secondary current arises when closing the primary current. Nothing else remains then but rejecting the whole observation, since a continual change in the by-current causes an error as well in the result for the mean HALL-current, as in that for the dissymmetry, and the error cannot be calculated.

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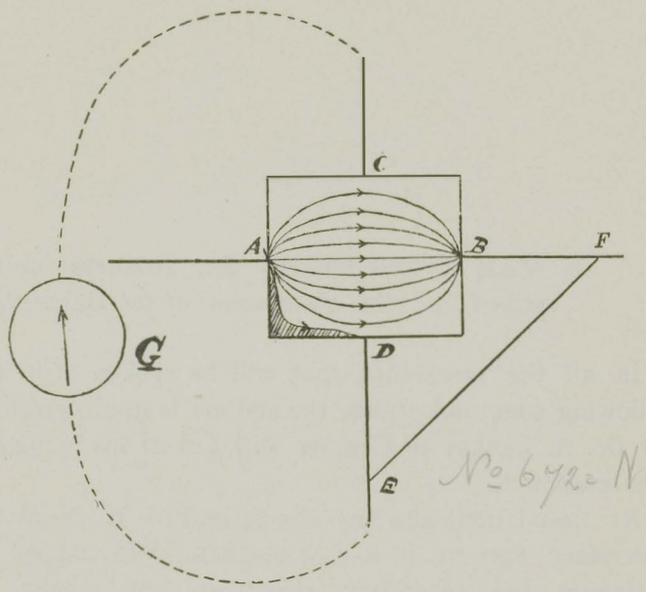
[1]) See also LEBRET, these Communications N°. 19, p. 24.

[2]) LEBRET, Dissert. Chapt. III § 1γ. Comm. N°. 19, p. 4 and 14.



2. Even if before and after the observation the by current has the right value, it is not certain that the observed secondary currents are due to the HALL-effect *only* [1]).

Let us suppose that the primary current travels through the plate from A to B, that the secondary electrodes C and D are connected through the galvanometer G, and the by-current is flowing along EF. If the resistance in this branch be so chosen, that the galvanometer shows no deflection on closing the primary current, C and E are æquipotential points.

[1]) The remarks of LEBRET on this source of errors on p. 85 and 86 of his inaug. diss. (Comm. N°. 19 p. 18) are not wholly correct.

The part of the primary current below the line of flow A D continues its way along D E F.

The whole may now be considered a Wheatstone's bridge; the four resistances are then ADE, ACD, CDF and EF. In the magnetic field, the resistance of bismuth increases. Only if the proportionality between the four resistances remained, the galvanometer would again be without current; as however the resistance of D E, E F and B F does not change, this is not likely to happen. So, already without the existence of a HALL-effect there might arise a secondary current in a magnetic field, which would show the same direction for both directions of magnetisation. In the presence of the HALL-effect this error causes dissymmetry, the amount of which cannot be calculated.

If, on the contrary, the secondary current in the zero magnetic field (which we will indicate henceforth by S) is measured by means of the compensative current, the error appears as well, but a correction for the increase of resistance is possible. Indeed, as the resistance of the plate may be neglected against that of the whole primary circuit, the primary current does not change, and, for an increase of resistance of p perCt, all differences of potential in the plate depending on the resistance, also the difference at the secondary electrodes, increase with p perCt. So S_0 should be also increased with p perCt in order to obtain the secundary current, which would appear if the HALL-effect did not exist.

If we take moreover for S_0 the mean of the values, obtained before and after the observation, the first difficulty is also eliminated for the greater part.

The increase of the resistance of bismuth in the magnetic field differs for pure and impure bismuth and depends also on the treatment of this metal in casting it, etc. [1]) There remains therefore some uncertainty in the correction. I have always used the numbers, given by HENDERSON [2]), but at the same time taken care as much as possible, that S_0 should be small, and so the whole correction did not grow too large.

By means of the thus altered method a number of round plates are tested at different temperatures and in different magnetic fields. Here arises the question, if we are allowed to draw conclusions from these observations concerning mean HALL-current and dissymmetry in the same manner as with quadratic plates, and so to write for the difference of potential at the secondary electrodes: [3])

$$e = \left\{ H + \tfrac{1}{2} \sin 2\alpha (K_{11} - K_{22}) \right\} \frac{I}{d}.$$

In practice, some objections may indeed be made against this assumption.

Firstly, for equal values of magnetic force, temperature, strength of (primary) current (I) and thickness of the plate (d) the secondary currents are much weaker in the round plates than in the quadratic ones.

Then, it is difficult to reconcile with this formula an observed variation of the mean HALL-current after turning circular plates round the axis of the magnet.

[1]) See RIGHI, Atti della R. Acc. dei Lincei, Serie III, Memorie XIX, p. 576, 1884.

[2]) Wied. Ann. 53, p. 912, 1894.

[3]) See LEBRET, Diss. p. 108; Comm. N°. 19, p. 24.

Already during the experiments for the determination of the axes of symmetry on plate N° 1 it struck me, that for the mean HALL-current always larger values were found, if the secondary electrodes were at the ends of an axis of symmetry. Afterwards the plates N°. 2 and 3 were tested in 8 positions, differing by 45°, and also then always the same was observed.

Round plate N°. 2. M [1]) = 8600.

Position	1	2	3	4	5	6	7	8.
S [2])	9,89;	11,30;	10,12;	12,39;	11,16⁵;	12,96;	11,35;	11,36.

Mean of 1, 3, 5, 7 = 10,63

„ „ 2, 4, 6, 8 = 12,00

Round plate N°. 3.

Position		1	2	3	4	5	6	7	8.
M = 8500	S	40,88;	50,70;	40,40;	44,55;	36,10;	47,27;	37,97⁵;	51,40.
M = 5450	S	28,80;	33,93;	25,74;	31,23;	25,83;	33,12;	29,34;	35,82.

	8500	5450.
Mean of 1, 3, 5, 7	38,84	27,43
„ „ 2, 4, 6, 8	48,48	33,52⁵.

In positions, situated between two consecutive ones of these series, always numbers were obtained between the corresponding values.

During the experiments of the last series a resistance of 10 Ohms was added to the secondary circuit, so that the possible variations of resistance in that circuit could not have much influence.

The observations may be represented tolerably well by an empirical formula, containing A $\cos n\pi$ and

$$B \cos\left(n\frac{\pi}{4} - \Phi\right) \qquad (n = 1, 2 \ldots 8)$$

[1]) Magnetic force.

[2]) Mean secondary current.

Since there cannot yet be given theoretical meaning to such terms, it was not thought necessary to mention the calculated values. The amplitudines were

N°. 2	A = 0,73	B = 0,84
N°. 3	A = 4,6	B = 2,96 [1]

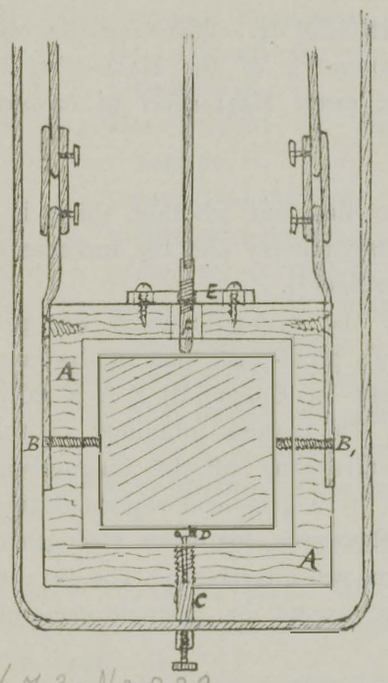
In order to obtain more certainty in this question, it would be necessary, for instance, to construct two differently situated quadratic plates out of the same round one, after determining the axes of symmetry; generally it will be desirable to repeat the experiments with quadratic plates.

For the present, however, I have continued to use the above mentioned formula.

The electrodes are fastened to the plate by means of a carrier, which, after several changes, is now composed as follows:

To the sides of a small wooden frame A copper strips are attached, which at the upper end are connected by means of screw-clamps with the wires of the primary current. At the middle of the sides screws B, B_1 penetrate the copper strips and the wood, between which the quadratic or round plate is clamped. At the middle of the lower side a screw C is placed, pierced at the lower end horizontally to admit the double wire of the secondary circuit, which is fastened in it by means of a small screw, and provided at the upper end

[1]) Obtained from a series, composed of the means of corresponding terms in the series for 8500 and that for 5450, reduced to a mean of 43,7.



with a vertical bore, in which moves the screw D, serving as lower secondary electrode. To the upper side is fastened by means of two woodscrews the little plate E, which, together with the upper secondary electrode F, may be moved to and fro some millimeters in horizontal direction, allowing us to select a position, where S_0 is as small as possible.

This apparatus is convenient especially for the examination of round plates in different positions with respect to the primary or secondary electrodes. If the various screws are well pressed, the resistance, measured from the ends of the primary or secondary wires, remains always short of 0,05 Ohm.

E. VAN EVERDINGEN Jr., *Measurements concerning the dissymmetry of the* HALL-*effect in bismuth, and the mean* HALL-*effect in bismuth and antimony.*

1. Firstly, I tried to find out whether there is a relation between the dissymmetry and the increase of resistance in a magnetic field [1]).

The measurement of the magnetic force was performed with a ROWLAND's proof-plate, just in the manner described by LEBRET in Chapt III § 1, p. 26 of his inaug. dissertation. The values, obtained in this way for five different strengths of the magnetising current were gathered in a graphical representation, by means of which afterwards for each strength of current the magnetic force was interpolated. The distance between the poles was, except on a single occasion, left constant = 13 mM.

With a distance of 16,5 mM. between the poles and a current of 9 ampères I found instead of 3000 c. g. s., 6000 c. g. s. for the magnetic force.

(If we calculate with this value the constant R of HALL from the experiments of LEBRET, we find for bismuth I and II instead of 7 and 13 [2]) respectively 3,5 and 6,5.

In a magnetic field of 6000 c. g. s. von ETTINGSHAUSEN and NERNST [3]) found for pure bismuth R = 7,3)

[1]) See LEBRET, Dis. p. 92; Comm. N°. 19, p. 25.
[2]) „ „ „ p. 102; „ N°. 19, p. 31.
[3]) Wied. Ann. 33. p. 474, 1888.

In the following tables we mean by:
M : the magnetic force in c. g. s. units.
S_A : „ HALL-current for the direction of magnetisation A.
S_B : „ HALL-current for the direction of magnetisation B.
S : „ mean HALL-current.
D : „ difference between S_A and S_B.
S_0 : „ secondary current in a zero magnetic field.
$S_{0(c)}$: „ same, corrected for the increase of resistance by magnetisation.

The numbers for the secondary currents are obtained by dividing 1000 by the compensative resistances.

The value of S_0 is added here to allow a judgment of the accuracy in the results for the dissymmetry.

Round plate N°. 1.

This is the same plate, which has served in the investigation of LEBRET, described Chapt. VIII § 5 and 6 of his inaug. diss. [1]).

M	S_A	S_B	S	D	S_0	$S_{0(c)}$	Q
1350	3,44	3,94	**3,69**	**0,50**	0,66[5]	0,68	0,17
2700	5,78	7,34	**6,56**	**1,56**	0,66	0,71	0,21
5050	6,94	10,66	**8,80**	**3,72**	0,38	0,45	0,20
6800	6,69	12,23	**9,46**	**5,54**	0,35	0,45	0,19
8600	6,22	13,75	**9,98**[5]	**7,53**	0,34[5]	0,48	0,19

S_0 was in the same direction as S_A.

[1]) Communications N°. 19. p. 24.

The last column contains the quotients of D and the numbers, which express according to HENDERSON the increase of resistance in percents.

According to the theory, exposed by LEBRET in Chapt. X § 2 [1]), in the position where the dissymmetry has its maximum:

$$e_A = \left\{ H + \tfrac{1}{2}(K_{11} - K_{22}) \right\} \frac{I}{d}$$

$$e_B = \left\{ -H + \tfrac{1}{2}(K_{11} - K_{22}) \right\} \frac{I}{d} \quad \text{or}$$

$$= \left\{ H - \tfrac{1}{2}(K_{11} - K_{22}) \right\} \frac{I}{d}$$

if we reckon here positive a difference of potential in opposite direction, hence

$$e_A - e_B = (K_{11} - K_{22}) \frac{I}{d} \qquad e_A + e_B = 2H \frac{I}{d}.$$

So D is proportional to $K_{11} - K_{22}$.

With quadratic plates we would be able to determine $K_{11} - K_{22}$ in absolute measure from the above data, where $\frac{e}{I}$ should be calculated as indicated on p. 102 of LEBRET's dissertation.

With round plates it is not allowed to assume the streamlines to run everywhere in the direction of the line uniting the primary electrodes. In order to get nevertheless an idea about the values of K_{11} and K_{22}, we will derive from the experiments the ratio of $(K_{11} - K_{22})$ to H on the supposition, that the HALL-currents remain at least proportional to the terms

[1]) Communications N°. 19. p. 21.

$$\left\{ H + \tfrac{1}{2}(K_{11} - K_{22}) \right\} \frac{I}{d}$$

and $$\left\{ H - \tfrac{1}{2}(K_{11} - K_{22}) \right\} \frac{I}{d}.$$

Then, we take $H = RM$, taking into account the change of R with the magnetisation. From the above formulae follows immediately:

$$\frac{K_{11} - K_{22}}{H} = 2 \frac{S_A - S_B}{S_A + S_B} - \frac{D}{S}.$$

In a magnetic field of 8600 we find $\frac{D}{S} = 0{,}754$.

In a field of 6000, for bismuth I, $R = 3{,}5$, so $RM = 21000$.

For S we find here by interpolation 9,20.

RM is proportional to S, so we find in the field of 8600:

$$H = RM = 21000 \times \frac{9{,}98^5}{9{,}20} = 22792$$

and

$$K_{11} - K_{22} = 0{,}754 \times 22792 = 17185.$$

The resistance of bismuth, which perhaps may be represented best by $\frac{K_{11} + K_{22}}{2}$, is at 20° in a zero magnetic field $1{,}36.10^5$ in c. g. s. units, in a field of 8600 c. g. s. $1{,}88.10^5$.

So we would find here

$$K_{11} = \pm\ 1{,}97.10^5 \qquad K_{22} = \pm\ 1{,}79.10^5.$$

These experiments confirm therefore the results of LEBRET and indicate, that in this plate the dissymmetry varies indeed in the same manner with the magnetic force as the increase of resistance of bismuth does.

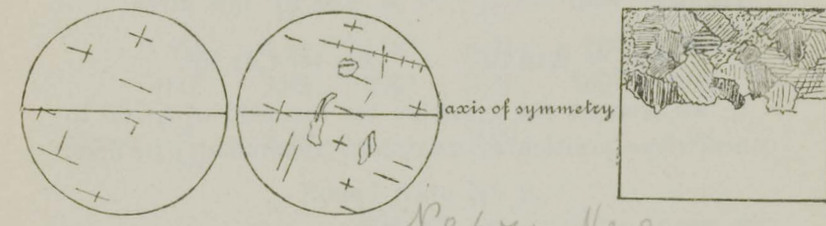
Relation between the dissymmetry of the HALL-*effect in bismuth and the state of crystallization.*

2. Already LEBRET has observed, that the dissymmetry may be connected with the planes of slit found in the plates, and tried to obtain a big crystal of bismuth, or crystals regularly grown together, from dealers in minerals. These efforts were continued by me, but without success [1]). In the meantime I have tried to construct plates myself, wherein the crystals should be placed more uniformly side by side, in order to come thus a little nearer to the wished-for end. I succeeded best with the round plate N°. 2, which showed a very considerable dissymmetry.

This plate was made out of a larger round plate of bismuth, cast of bismuth I in a mould of glass, heated beforehand to ± 300°, which was placed in a sand-bath and after the casting was covered with sand of the same temperature, in order to cause the cooling to take place very slowly.

For the sake of comparing the degree of regularity in the grouping of the crystals, this plate and a quadratic one were polished and heated a moment in dilute nitric acid; the round one looked then wholly equal of tint, except in three little spots, whilst on the quadratic one, that had shown almost no dissymmetry, after the treatment some hundred irregularly shaped and very differently tinted spots were to be seen. Under the microscope the round plate showed in several places parallel lines in two directions almost at right angles, the same over all the plate and on front and back, which directions did not wholly coincide with the axes of symmetry, found afterwards upon it (see the engravings).

[1]) On a request, directed then to the „Königliches Blaufarbenwerk Oberschlema," this has been so very kind as to send me very fine specimens of crystals grown together, which were received while printing this communication, and give rise to the hope, that a suitable plate may be constructed from them.



With this round plate I found then in a position of the secondary electrodes near one of the directions of maximum dissymmetry:

M	S_A	S_B	S	D	S_0	$S_{0\,(c)}$	Q
1200	1,79	1,66	**1,72⁵**	**0,13**	1,47	1,51	0,05
5400	9,29	3,67	**6,48**	**5,62**	1,44	1,74	0,27
7400	15,39	2,87	**9,13**	**12,52**	1,28	1,71	0,38
± 13700	29,53	— 8,39	**10,57**	**37,92**	1,10	1,92	0,52

S_0 was again in the same direction as S_A.

The sign — before S_B in the last line indicates, that here the direction of the HALL-current did *not* change sign with the reversal of the magnetic field.

In the position of maximum dissymmetry was found:

8600 21,14 — 1,36 **9,89** **22,50** 2,15 3,01.

As shown by the last column, the dissymmetry increases here much faster with magnetisation than the increase of resistance does.

The last experiment gives $\frac{D}{S} = 2{,}27$.

If we take for H also here 22792, $K_{11} - K_{22}$ becomes 51737.

Together with $\frac{K_{11} + K_{22}}{} = 1{,}88.10^5$ this gives:

$$K_{11} = 2{,}14.10^5. \qquad K_{22} = 1{,}61.10^5.$$

If we assume the numbers for the field of 13700 to concern the position of maximum dissymmetry we find:

$$\frac{D}{S} = 3{,}59.$$

$$H = 22792 \times \frac{10.57}{9{,}89} = 24359. \qquad K_{11} - K_{22} = 0{,}874.10^5.$$

$$\frac{K_{11} + K_{22}}{2} = 2{,}35.10^5, \quad \text{so}$$

$$K_{11} = 2{,}79.10^5 \qquad K_{22} = 1{,}91.10^5.$$

Several more plates were by means of the same method constructed of bismuth, ordered from MERCK as „purissimum". None of these was formed as regularly as the plate just spoken of, and accordingly some showed no, others only moderate dissymmetry. So from these experiments may be concluded without doubt, that the dissymmetry is related to the state of crystallization.

Variation of the dissymmetry of the HALL*-effect in bismuth with temperature.*

3. Round plate N°. 1.

All observations show that the dissymmetry decreases rapidly with rise of temperature, more rapidly than the HALL-effect itself.

The most trustworthy ones (large dissymmetry and small S_0) give for a magnetic field of 8600:

	D		D		D
17°	6,44;	12°	8,31;	16°	3,90
100°	2,92;	100°	3,31;	100°	1,08
Hence $\frac{D_{20}}{D_{100}} =$	2,13		2,29		1,98

Round plate N°. 2.

M = 5500

T	D	Q	(Q has the same signification as formerly.)
16°	10,59	0,49	$\frac{D_{20}}{D_{100}} = 3{,}52$
± 63°	4,86	0,46	
100°	2,87	0,48	

Round plate N°. 3.

M = 7400

T	D	Q	
17°	8,88	0,27	$\frac{D_{20}}{D_{100}} = 3{,}01.$
100°	2,84	0,31	

With plate N°. 2 also a series of observations was made in a magnetic field of 6000 c. g. s. between — 70° and + 20°. Since the increase of resistance with magnetisation at temperatures below 0° is not known, S_0 was made as little as possible and it was ascertained, that this current remained always weak. In order to reduce the importance of little variations in the resistance of

the secondary circuit, 3 Ohms were added to this circuit, the usual resistance being ± 1 Ohm.

The addition of a correction to S_0 for the increase of resistance would tend to increase the values for the dissymmetry. Here follow some numbers obtained by interpolation.

M	T	S_A	S_B	S	D
6000	— 70°	— 3,60	8,50	**2,45**	**12,10**
6000	— 50°	— 1,94	8,06	**3,06**	**10,00**
6000	— 30°	— 0,47	6,97	**3,25**	**7,44**
6000	— 10°	+ 0,73⁵	5,67	**3,20**	**4,93⁶**
6000	+ 17°	+ 1.59	4,69	**3,14**	**3,10**

At the lower temperatures both HALL-currents are again in the same direction. S_0 was about 0,2.

$$\frac{D}{S} \text{ at } -70^0 \text{ almost } = 5.$$

H is there $21000 \times \frac{2,45}{3,14} = 16380$, so $K_{11} - K_{22} = 0,819.10^5$.

In a zero magnetic field the resistance $\frac{K_{11} + K_{22}}{2}$ at $-70^0 = 1,22\,10^5$.

If we estimate the increase of resistance in a field of 6000 c. g. s. at — 70° 50 perCt. (at 100° it is 7 perCt., at 16° 24 perCt.) $\frac{K_{11} - K_{22}}{2}$ becomes $1,83\,10^5$ and we would find:

$$K_{11} = 2,24.10^5. \qquad K_{22} = 1,42.10^5.$$

For the plates Nos. 2 and 3 the proportionality between

dissymmetry and increase of resistance, as we see above, remains true tolerably well between 16° and 100°.

In order to display still better the variation of the dissymmetry with temperature in plate N°. 2, we multiply the numbers of the last series by 4, to allow for the larger resistance of the secondary circuit, and reduce the numbers 10,59 and 2,87, in a field of 5500, to 12,40 and 3,36 in a field of 6000.

We find then for a same magnetic field of 6000:

T	100°	16°	— 70°
D	3,36	12.40	48,40.

Variation of the mean HALL-*current in bismuth with temperature for different values of magnetic force.*

4. During the experiments on variation of dissymmetry with temperature in plate N°. 1 it was observed at the same time, that between the limits of temperature occurring thereby (10° and 100°) the mean HALL-current did not decrease sensibly with heating. This looked contrary to the results of LEBRET, who found $\frac{S_{100}}{S_{20}} = 0,668$ in bismuth I and $= 0,656$ in bismuth II, always with quadratic plates; the round ones he used for the experiments on dissymmetry only.

The experiments spoken of were made in a field of 8600.

Later experiments in a field of 5500 however *gave*

a decrease of mean HALL-current with rise of temperature.

I have investigated then whether the strength of the magnetic field exercised influence on the variation with temperature of the HALL-effect and obtained the following results.

Round plate N°. 1.		N°. 2.		N°. 3.	
M	$\frac{S_{100}}{S_{20}}$	M	$\frac{S_{100}}{S_{20}}$	M	$\frac{S_{100}}{S_{20}}$
7600	0,893	8600	0,728	7600	0,706
5500	0,783	5500	0,680	4800	0,700
500	0,658	1400	0,582	1400	0,575

These numbers are calculated from single observations, so the accuracy is not great. If we are allowed to draw conclusions from experiments with round plates in the same manner as with quadratic plates, they show clearly, that the relative *increment* of the HALL-effect with *fall* of temperature *increases*, when the magnetic field *grows weaker*. This may be expressed also as follows: At higher temperatures the HALL-current *increases more rapidly* with magnetisation than at lower temperatures.

Almost the same we observe with the electric conductivity of bismuth.

The *increment* of conductivity with *fall* of temperature *increases*, when the magnetic field *grows weaker*.

At higher temperatures the conductivity *decreases less rapidly* with magnetisation than at lower temperatures.

In the fields of 7600 and 1400 the ratio $\frac{r_{100}}{r_{20}}$ (r = resistance) is 1,10 and 1,34 respectively.

If we derive from the above tables the values in the fields of 7600 and 1400 for the three plates, and multiply the thus obtained numbers by 1,10 and 1,34 we find:

M	N°. 1.	N°. 2.	N°. 3.
	$\frac{S_{100}}{S_{20}} \times \frac{r_{100}}{r_{20}}$		
7600	0,982	0,781	0,777
1400	0,904	0,780	0,770

It appears from the experiments of HENDERSON, that in strong magnetic fields the conductivity of bismuth has a maximum value between 0° and 100°, which maximum for weaker fields wanders to lower temperatures. Very likely there will be also a maximum conductivity in a field of 6000 c. g. s., but at temperatures *below* 0°. Also here then the similarity with the HALL-effect reveals itself; this too reaches a maximum value at low temperatures [1]).

A variation in the rate of decrease with temperature

[1]) LEBRET reached this maximum with bismuth II only. My experiments at low temperatures however indicate for bismuth I a maximum at about — 30°.

of the mean HALL-current with altered magnetisation is observed also by CLOUGH and HALL in nickel [1]).

If this variation is found back also in quadratic plates, it will be possible perhaps to state a relation between the conductivity and the variation of the HALL-effect in bismuth [2]). To this and other questions I hope to revert at a later period.

Preliminary communication on the HALL-*effect in antimony.*

5. The preliminary results of the experiments on a round and a quadratic plate, cast by means of the same method as used for bismuth, but at a temperature of the mould still far below the melting point of antimony ($\pm$ 450°), are:

Coefficient of HALL R = 0,22 in a field of 5450.

Dissymmetry too small to be stated without doubt.

Decrease of R from 13° to 200° in the ratio of 1 to 0,66.

Observations during or soon after a cooling give always a too small value for R. The same phenomenon

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[1]) LEBRET, Diss. p. 35, 36. CLOUGH and HALL, Proceed. of the Amer. Acad. 20, p. 189, 1893.

[2]) A later experiment with the quadratic plate spoken of in 2 has given for $\frac{S_{100}}{S_{20}}$ in fields of 950, 1500 and 4600 resp. 0,528, 0,578 and 0,654.

occurred in a very high degree in the experiments of DRUDE and NERNST [1]).

The small dissymmetry may be a consequence of the structure of the tested plates, which was still rather irregular, may however also be caused by a very low value of $K_{11} - K_{22}$. For the whole increase of resistance was found by VON ETTINGSHAUSEN in a field of 10600 only 1,1 perCt., by LENARD in a field of 6600 1.2 perCt.

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[1]) LEBRET, Diss. p. 39. Wied. Ann. 42 p. 568. 1891.

COMMUNICATIONS

FROM THE

PHYSICAL LABORATORY

AT THE

UNIVERSITY OF LEIDEN

BY

PROF. DR. H. KAMERLINGH ONNES.
Director of the Laboratory.

No. 27.

Dr. H. KAMERLINGH ONNES. On the measurement of very low temperatures.

Translated from: Zittingsverslagen van de Afdeeling Natuurkunde der Kon. Akademie van Wetenschappen te Amsterdam, **30 Mei 1896,** *p.* 37– 46 *en* **27 Juni 1896,** *p.* 79—93.

EDUARD IJDO — PRINTER — LEIDEN.

H. KAMERLINGH ONNES. *On the measurement of very low temperatures.* **Part. I.** *(30 May 1896).*

1. *Hydrogen-thermometers for very low temperatures.* As the measuring of very low temperatures is becoming more and more important it appears desirable to me to describe the apparatus which during the last few years have been used in the cryogenic laboratory at Leiden. (Compare my Comm. of 29 December 1894) [1].

Under my direction they have been constructed by Mr. H. A. BLOM under-instrumentmaker at the laboratory whose ingenuity has been of great value to me.

As a basis for the determinations of temperature the hydrogen-thermometer at constant volume was chosen.

A gas-thermometer which is to be used for accurate determinations at very low temperatures must fulfil other conditions than ordinary standard-thermometers.

As it is difficult to obtain large quantities of liquid gas, the thermometer vessel must be so small that it can be put entirely into the glasses which are generally used for collecting the liquid gases.

The special precautions which must be taken when working with liquid gases make it very desirable that the apparatus should be easily handled, so that we need not shrink from carrying it from one place to another and that the thermometer-bulb can be placed in the

[1]) Communication from the Labor. of Phys. at Leiden, no. 14.

vessel of liquid gas and taken out again without difficulty. It is easy to see the reason why. In order to obtain the liquid gas or to keep it, different apparatus are required, which generally are very difficult to move and must also remain available for other experiments. Moreover it is a great advantage to be able to immerse different thermometers successively in the same liquid.

In the following pages two models of hydrogen-thermometers will be described, with which very accurate measurements are possible and which fulfil the conditions for very low temperatures. In the one model the smaller volume, in the other the greater accuracy have been specially aimed at. First we shall describe the smaller model and then show in what respect the larger model differs from this. The whole construction of the thermometer and the manner of using it may be seen without further comments from Pl. I, fig. 1 and 2 which represents the comparing of WROBLEWSKI's thermo-element with the hydrogen-thermometer as already mentioned in the Proceedings of 29 Dec. 1894, pag. 179 [1]) and as will be fully treated in a following communication. The hydrogen-thermometer and thermoelement are immersed in the bath of liquid oxygen, collected in the boilingglass O [2]) described above. The figure on Pl. I is in some parts semi diagrammetic especially as regards the connections; moreover the

[1]) Commun. from the Labor. of Phys. at Leiden no. 14. p. 27.

[2]) A drawing on a larger scale of the boilingglass is found in the paper of E. MATHIAS: Le laboratoire cryogène de Leyde. Revue Générale des Sciences, 1896 p. 387, fig. 3.

way in which some parts of the apparatus are protected from change of temperature is not represented here, nor are the thermometers on which the temperatures required for correction purposes are read.

2. *Description of the small hydrogen-thermometer.* The thermometer-part consists of a bulb, a, fig. 1, Pl. II of about 30 cc. blown on to a capillary tube, b, of 0.25 c.m. inner diameter. The capillary has been chosen so narrow because it forms the transition between the parts of the thermometer at a low temperature and those at the temperature of the room; therefore the temperature of the gas at different places in the capillary is rather uncertain.

The little bulb is made of Jena-glass and is tested to a pressure of 2,5 atmospheres above the outside pressure. The volume of the capillary tube is determined with mercury and the volume of the whole bulb with water; the change of volume with pressure is also measured.

The thermometer is connected with the space, where the adjustment at constant volume is made, (the volumenometer part) [1]) by means of a steel capillary tube d: this kind of tube has been used for many years for experimental work at Leiden and may be obtained from P. J. KIPP & SON successor J. W. GILTAY at Delft (comp. Proceed. Amsterdam Dec. 1894, pag. 168: capillary connection of pump body and compression tube). [2]) The

[1]) CHAPPUIS, Mém. d. Bur. Intern. T. VI, p. 28. WIEBE u. BÖTTCHER, Zeitschrift f. Instr. 1890, p. 17 u. 19.

[2]) Commun. no. 14 p. 8. A drawing of this is found in MATHIAS l. c., p. 389, fig. 5.

capillary is on the outside covered with oil which is then converted by heating into an adhering varnish and thereby protected from rusting.

The diameter of the capillary is about 0.8 m.m., its length about 180 c.m., its volume therefore nearly 1 c.c. If the temperature of it is determined carefully, a volume of that size is permissible. The connection of the steel capillary to the glass capillary on one side and to the volumenometer-part on the other is made by steel caps *c* and *e*, fastened to the steel capillary into which the glass capillary and the volumenometer-tube are cemented. For the precautions taken in fastening the caps to the capillary and in cementing the capillary and the volumenometer-tube into these caps see below (§ 3.)

The lower surface of the steel cap *e* is provided with a small point *f* exactly at the centre which has been carefully turned out of the same piece of steel. The adjustment at constant volume is obtained by making the surface of the mercury in the volumenometer touch this point *f*.

As the cap fits accurately in the glass tube *g* its surface forms a perfectly flat top [1]) to it and the volume of the volumenometer-part above the mercury-meniscus may be accurately calculated. The volumenometer-tube is made of a perfectly cylindrical tube and is accurately calibrated. Its width is 9 m.m. In choosing this dimension it was taken into account that by measuring the height of the mercury-meniscus the correction for the capillary depression is to be obtained; this correction however leaves

[1]) Pernet according to Chappuis. Mém. Bur. Intern. VI. p. 31.

an accidental error which becomes smaller as the diameter of the tube is larger. With the dimension chosen this error agrees approximately with the accidental error in the adjustment of the top of the meniscus on the steel point: which latter error would increase with a wider tube. The volumenometer-tube is [1]) connected with the open manometer by an india-rubber tube *l*, and as usual carries a three-way stopcock. The tube *k* ends in a fine capillary tube which in the calibration of the volumenometer-tubes serves to run out known quantities of mercury for the computation of which the position of the mercury in the capillary point is taken into account. The manometer-tube is of the same kind and diameter as the volumenometer-tube.

When no observations are made with the thermometer the three-way stopcock remains closed. Above the stopcock is a space *i*, which serves to receive particles of dust or gas-bubbles which might issue from the india-rubber tube. Moreover the volumenometer-tube is provided with a bulb *h* which in the process of filling the thermometer with hydrogen (comp. § 6) serves to admit so much gas into the apparatus under a little less than the usual pressure that on reducing the volume to its normal value a tension of about 1100 m.m. at 0° is obtained i. e. of about 1500 m.m. at 100°, and of about 300 m.m at −200°.

3. *Difference in the measurements of the larger pattern.*

The larger pattern differs from the small one only in having a longer bulb of the same diameter (volume

[1]) Jolly Pogg. Ann. Jubelband.

about 90 c.c. (Pl. II, fig. 2). Like the small one it can be plunged entirely into the boiling-glasses (Proceed. Dec. '94 l. c. § 9. Commun. no. 14. p. 27.)

The volumenometer-tube is taken wider (12 m.m.) and the volumenometer-bulb larger in proportion to the larger volume of the thermometer part.

4. *Some precautions and auxiliary apparatus for the construction and use of the thermometers.* The glass is cleaned first with boiling concentrated nitric acid, then with 25 % alkali solution, finally with distilled water. Further it is dried with a current of air that has passed over caustic soda, sulphuric acid and phosphorus pentoxide, the tube being heated to a high temperature and if possible exhausted by means of a mercury-pump. In order to subject the thermometer part to these operations the bulb is originally provided with a tube (comp. Pl. II, fig. 3*a*) through which the liquids can be sucked up and the upper part of the capillary is originally lengthened by a wider piece, provided with a ground joint (comp. Pl. II, fig. 3*b*.) by means of which it can be fastened to the air-pump. After the operations described in this and the preceding paragraph the first tube is sealed off, the second lengthening piece is cut off carefully so as to get a perfectly even crack, which is necessary in order not to leave any additional air-space when the steel cap is cemented on.

The steel capillary is annealed by glowing by means of an electric current furnished by a battery of accumulators; after this air at 100 atmospheres is pressed into it, in order to discover any small leaks, which are often

found in these capillaries, and then dry air is blown through it under high pressure. Finally pure mercury is pressed through and in order to determine the volume it is exhausted and filled with mercury. Both ends of the tube are provided with extremely fine screw threads; after the above described operations the tube is screwed into the two steel caps with marine glue in order to make it tight; after which the capillary is soldered to the upper part of the caps. Another reason why marine glue is used, especially in the cap which is cemented on to the volumenometer tube, is to prevent the mercury from coming in contact with the solder if by accident it should rise in the capillary.

In order to cement the volumenometer tube into its steel cap (comp. Pl. II fig. 4*a*) the latter is turned upside down, the groove is filled with sealing wax, and into this after being properly heated and covered with a thin layer of sealing wax on the outside the volumenometer part is pressed down with closed stopcock. Care must be taken that air is supplied or sucked out through the steel capillary, so that in consequence the sealing wax may flow out on the outside when the glass piece is being slowly pressed down, while on the inside only a thin line of sealing wax remains round the steel when cooled down. Only when these precautions are taken an air space is obtained which may be accurately calculated. If these precautions were neglected sealing wax might spread over the interior of the glass and would in that way hinder the adjustment of the mercury on the steel point.

In cementing the glass thermometer-capillary into its cap (comp. Pl. II fig. 4*b*) a thin layer of sealing wax

is spread over the glass and also over the inside of the upper rim of the cap.

The glass capillary is then heated carefully and slowly (5 minutes) pushed into the cap; care being taken that the air can escape from the steel capillary. In this way we get a perfect connection without any air space being left except the top of the tube itself. In order to acquire the necessary skill in this operation, it is first tried several times on a trial piece which is then coupled to a cylinder with liquid carbonic acid in order to see whether a good fit is obtained.

The india-rubber tube is treated with pure mercury for a long time before using it and while hot is cemented on to the tubes with sealing-wax. The manometer-tube and india-rubbertube are filled with mercury, which has been distilled in vacuo in a mercury distiller into which only mercury is put which is free from admixture with other metals.

The volumenometer- and manometer-tubes are fastened in a very light but carefully worked stand by means of clips which have a very true motion. They are adjustable with a micrometer. This expensive stand is generally only used for the thermometer with which the observations are made. Thermometers temporarily out of use are clamped in ordinary stands by means of ordinary clips. The manometer- and the volumenometer-tubes when taken out of the clipstands can be held in one hand and the thermometer proper in the other hand and in that way the thermometer may without difficulty be transferred into the apparatus where the temperature is to be observed, the volumenometer part

being fixed in the stand which is placed near these apparatus. Also if the apparatus is fastened in the above described stand, we can easily remove the whole by holding the stand in one hand and the thermometer-tube in the other.

It has proved a great advantage that the thermometer with which we can reach the accuracy of a standard instrument is so easy to handle. This is owing especially to the steel capillary being used.

In order to bring the thermometer into apparatus which have to be usually closed and must often be subjected to exhaustion it is provided with a stopper as represented in Pl. II, fig. 5.

The copper ring *c* may be slid over the bulb. In the ring, supported by the rim *e*, a copper plate is placed consisting of two parts which serves as a bottom, the cylindrical part is filled with a cork cut in two and finally in the conical part an india-rubber stopper cut in two is put and made tight with dissolved india-rubber. The cylindrical part of the ring is fastened to the mouth-piece of the apparatus, in which the temperature is to be measured, by means of india-rubber rings, cement and tightening strips of brass.

5, *Apparatus for the preparation of pure hydrogen-gas.* For measurements at very low temperatures it is of great importance that the thermometer should be filled with absolutely pure hydrogen gas. For this purpose the gas is prepared electrolytically and following COOKE and RICHARDS [1]), who have made ample researches about

[1]) Proceed. Amer. Acad. 1887—88.

the preparation of hydrogen for determinations of atomic weights, I have chosen diluted hydrochloric acid as the electrolyte. Plate I, fig. 3, shows the apparatus used for this method. The figure is again diagrammetical: e.g. the glass tubes which act as a glass spring and connect together the generating apparatus, the mercury-air-pump and the apparatus which is to be filled have been left out.

The voltameter V, (comp. Pl. I, fig. 3) consists of a glass vessel *a* (comp. Pl. II, fig. 6) with an ebonite cover *b*, containing a bell jar *c*, with a stopcock *d*. In this jar *c*, hangs a piece of platinum foil *e*, from a properly insulated wire which serves as one of the electrodes; the hydrogen generated at the electrode collects in *c*. The neck of the jar is fastened in the same india-rubber stopper *g*, as the wire through which the current is led off. The bottom of the vessel is covered with a layer of zinc-amalgam, *h*, obtained by dissolving chemically pure zinc in chemically pure mercury. The current is supplied by a bundle of twisted platinum wire soldered to a copper wire, and together of a section sufficient to carry the current. The juncture is protected by a surrounding tube *l*, sealed on to the lower end *k* of the platinum by means of enamel. The electrode (the upper part of which is covered in the way described) is fixed into the apparatus by means of a second tube *m*, which is cemented in the cover and reaches below the surface of the amalgam. In this tube the electrode is fastened airtight by means of india-rubber.

Only few gas-bubbles are formed on the surface of the amalgam. But in order to prevent even these from

finding their way into the jar which receives the hydrogen, a glass dish *n* floats on the amalgam the upper edge of which reaches above the bottom of the jar. The dish is prevented from rising by three rods of enamel *p*, sealed together, and hooked over the edge of the dish and resting against the bottom rim of the jar. The india-rubber stopper in the neck of the jar, the neck of the jar itself, and the india-rubber stopper which carries the jar, are joined airtight to each other and to the ebonite cover by means of dissolved india-rubber. For drawing off and running in the electrolytic liquid two tubes *q* and *r*, the former being provided with a cock *s*, are cemented into the cover.

The cover is made airtight by means of a washer *u* and tightening rods *t*, which press it down on to the ground rim of the vessel.

The hydrogen passes through the stopcock at the top of the bell jar into two wash-bottles (comp. Pl. I, fig. 3*a* and *b*) filled with a chemically pure 25% alkali-solution. The first of these *a*, is really a sloping tube which contains glass beads in order to divide the slowly rising hydrogen bubbles; the second is a Woulf's bottle, filled also with glass beads and alkali-solution. After the gas had passed through the wash-tube alone a trace of chlorine could still be detected in it by means of silver nitrate. The solution is brought into the alkali-apparatus and if necessary renewed by the aid of the taps *d*, *c* and *k* and the tap-bottles.

The 20% hydrochloric acid, which serves as electrolytic liquid is previously boiled in a flask as was also

done by COOKE and RICHARDS [1]). During this operation as well as during the cooling a current of hydrogen is led through. For this hydrogen was used from a pressure cylinder supplied by JOHN ORCHARD, which might have been washed beforehand in hydrochloric acid and caustic potash. According to the rules of the Leiden laboratory, while the boiling is going on, the cylinder *f*, containing the compressed gas is not brought into the room where the flame is burning.

In the same way the air is previously expelled from the alkali with hydrogen from the cylinder and the whole apparatus before and after the introduction of the solutions is also filled with hydrogen. When the solutions are put into *a* and V there is only very little space left which is filled with almost pure gas. The generation of electrolytic hydrogen is now set in motion and the hydrogen is not used until the whole space has been once more swept out with it: in the meantime it escapes through the safety-tube.

While thus the air is expelled from *a* and V by a constant gas-current, the same object is reached with the drying battery by repeated exhaustion by means of the mercury-air-pump. The partition between the two parts of the apparatus one only of which has to be exhausted is formed by the regulating-cock *R*, of the kind that are used at Leiden in operations with compressed gases [2]).

[1]) l. c. pag. 167.

[2]) Compare STOEL. Measurements on the influence of the temperature on the inner friction of liquids between the boiling-point and the critical state. Diss. Leiden, 1891, p. 12.

MATHIAS l. c. pg. 383. fig. 1, n°. 8.

The drying battery consists in the first place of a double or safety drying tube *g* [1]) with strong sulphuric acid [2]). In a tube of that kind the reversal of the air current has no other effect than that of carrying the liquid from the one half into the other; it cannot flow back into the rest of the apparatus. For exhausting purposes this drying tube is provided with a connecting tube carrying a cock *h*, which is shut when we want gas to pass through the liquid but is opened during exhaustion. When the apparatus is exhausted the fine regulating cock *R* must be opened with the utmost care in order to let the first gas-bubbles pass through the sulphuric acid. The gas afterwards goes through two drying towers *i*, *i*, filled with chemically pure phosphoruspentoxide spread on glass-wool.

As preparation of pure hydrogen has been repeatedly necessary for some years, the generating- and washing-apparatus are permanently mounted together on a board fastened on the table by means of clamps, the connection with the mercury-air-pump *L* and the apparatus which has to be filled *Th*, being obtained by means of springs

[1]) Comp. DE VRIES. Measurements on the influence of temperature on the capillary elevation of ether. Diss. Leiden, 1893, Pl. I.

[2]) An action of the gas obtained by electrolysis on the sulphuric acid as noticed by CHAPPUIS (Mém. Bur. Intern. VI p. 105), who filled his apparatus with hydrogen, obtained by electrolysis of orthophosphoric acid was neither detected by COOKE (in his determiuations) nor by me.

of glass tubes which are joined to these by ground joints, and are sealed together, when connected to the apparatus, with the hand blow-pipe.

6. *Filling the thermometer.* In order to fill the thermometer with the pure gas, the volumenometer part is provided with a thickwalled side-tube, originally ending in a ground joint (comp. Pl. II, fig. 7), which forms the connection with the mercury-air-pump and the apparatus furnishing the gas.

After mercury from the manometer and india-rubber-tubes has flown out for some time through the three-way stopcock of the volumenometer part, and a layer of about 1 c.m. has been let in above the cock, it is shut. The thermometer- and the volumenometer part on the one hand and the drying battery of the hydrogen-apparatus on the other are now exhausted to a few thousandths of a m.m. by means of the mercury-air-pump through the tube mentioned above; when everything is found to be absolutely tight, the current is closed, and is regulated by means of a resistance to a strength of 2 or 3 Ampères; the regulating cock is then opened so wide that the same quantity of gas as is generated in the apparatus flows into the drying battery; the progress of the filling process is followed on the manometer. Special care must be taken not to open the regulating cock too far, in order to prevent liquid from flowing over into the washbottle at *a* Pl. I, fig. 3. The operation of exhausting and filling, which takes about 2 hours each time, is repeated several times. After the lats exhaustion so much mercury is admitted in the volu-

menometer part that the space *i* is not only shut off but becomes completely filled with mercury when the gas is admitted. The side tube is then sealed off and the mercury in the volumenometer part is pressed up.

7. *The zero.* For the determination of the zero planings of [1]) pure ice are used. Pl. II, fig. 8 shows the apparatus for planing the ice [2]). The block of ice is slid to and fro along the fixed plane; the fine shavings of ice fall into a vessel under the knife and when soaked with distilled water form a grainy mass of icy particles covered with a thin layer of water. In this condition they do not freeze together and even after hours the mass is as powdery as in the beginning. The knife *a* is of iron and has a steel edge; by adjusting it by means of adjusting screws, *b* the planings become coarser or finer. If the ice planings are very fine there is no objection to placing the hydrogen thermometer, although it is fragile, into the ice, which for this purpose is put into a large flower-pot. At some distance from the thermometer the ice may even be pressed tightly together if done carefully. The melted ice flows off through several holes into an earthen-ware plate from which it flows off again through a small tube on a level with the bottom of the flower-pot.

It is intended to cover the top of the plane with two plates of marble-glass in order to protect the ice even better from becoming dirty.

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[1]) Pernet, Meteorol. Zeitschr. 1879, p. 131.

[2]) Guillaume, Mém. Bur. Intern. T. V. p. 41.

Part. II. *(27 June '96).*

8. *Boiling point.* For heating the hydrogen thermometer in steam, the copper boiling point apparatus represented in Pl. III, fig. 1 is used. The section of the vessel *a*, which contains the water is so large that the walls above the water cannot be heated above the boiling point by ascending currents of hot air. The steam therefore cannot become superheated. (For the same purpose a similar apparatus of smaller size (Pl. I, fig. 2) was provided with an asbestos ring below which the water surface is not allowed to fall. As the outlet of the steam is on one side the steam might be too much in that direction, to prevent which the annular space above the outlet is fitted (comp. fig. 1) with a perforated ring, the holes being closer together the further away they are from the side of the outlet). The apparatus is carefully cleaned and only distilled water is used in order that the water may always boil from a pure metal-surface. The cover of the double jacket *b* is convex so that water which condenses on the cover flows off to the exterior jacket. Moreover the upper part of the apparatus is covered with felt and the part of the stem which projects above the cork in the cover, is tightly packed in fine sheepswool, so that the space above the cover is heated to near the boiling point. In this manner the flowing down of drops along the stem is prevented and the thermometer remains enveloped in the thin water-layer which condenses on it in the beginning and gradually assumes the temperature of the steam. From the thermometer a funnel-shaped glass-screen, *c*, is suspended which protects the thermometer from splashed drops. A floater, *d*, indicates the position of the surface of the liquid, which does not fall more than 3 cm. per hour when boiling at the greatest rate. The velocity of the steam in the outlet of the jacket is at least 40 cm. Under these circumstances cold air currents cannot flow back into the apparatus. The velocity of the steam in the inner space is from 3 to 10 cm.

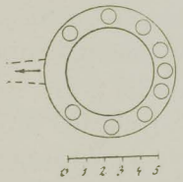

Fig. 1.

At this rate of boiling the excess of pressure inside is also nearly constant and without fluctuations, (the diameter of the outlet is 2,5 cm.) and may be measured by means of a water-manometer, which is in communication with the inner space, in which the thermometer is suspended. Following PERNET, JAEGER and GUMLICH, [1]) the one level of this water-manometer is heated to 100° and exposed alternately to the pressure of the steam and to that of the atmosphere, the other level being maintained constant and always at ordinary temperature and at the pressure of the atmosphere. The excess of pressure of the steam however never exceeds 1 m.m. of water pressure, corresponding to about $^1/_{300}{}^{\text{th}}$ of a degree in the boiling point, if the outlet is not narrowed on purpose, which would occasion fluctuations of pressure.

In the operation represented in Pl. III, fig. 1 of the calibration of a thermoelement for low temperatures, the determination of the slight differences, that may yet occur in the pressure of the steam with a given

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[1]) PERNET, Bureau Intern. I. B. 16. 1881.
PERNET, JAEGER und GUMLICH, Zeitschr. f. Instr. XV, p. 124, 1895.

velocity of boiling, may as a rule be omitted and it will be more than sufficient to use for this end a value determined once for all. For in comparison with the disturbances and uncertainties, which are as yet not to be avoided in determining low temperatures, the above deviation may be neglected.

In order to carry off the steam, without giving rise to differences of pressure or fluctuations, Mr. BLOM has constructed the apparatus represented Pl. III, fig. 1 and on a larger scale Pl. III, fig. 3. In this apparatus the steam which flows in from the top is condensed by fine jets of water, which issue from holes in the tube, *b*. This tube may revolve round the supply-tube of the condensing water, or axis and when lifted up by the water pressure it rests against a glass plate with a sharp point in the direction of the axis. A fine hole having been made in the side at each end, the tube is set in motion as a reactionwheel as soon as the supply-cock is opened. With an apparatus of this kind which may easily be placed anywhere, the steam is sufficiently condensed. If there is no objection to employ a wide tube leading to a draught chimney in the manner indicated by BUNSEN [1]), this method is simpler.

The values for the coefficient of expansion of hydrogen, deduced from the measurements at the boiling point and the zero point, will be treated of in a following communication.

9. *Thermoelement, German silver-copper.* With this element the first measurements of low temperatures were

[1]) Wied. Ann. XXXI p. 6. 1887.

made by WROBLEWSKI. I have chosen it as being at the time the most suitable, partly also because, taking into account that the resistance of German silver varies little with the temperature. In the observations made at Leiden a method has been followed, closely resembling that of WROBLEWSKI. We had occasion to investigate in this way what difficulties may have presented themselves in his determinations. It appeared that different sources of error may endanger the accuracy of the measurements and that special precautions are necessary to make a long series of temperature readings with the same thermoelement.

CHASSAGNY and ABRAHAM [1]) in their investigation on accurate determinations of temperatures between 0° and 100° by means of a thermoelement have arrived at experiences and precautions which are in different respects similar to ours. The research described in the following pages was undertaken quite independently of theirs with a view to measurements of low temperatures in the cryogenic laboratory. In former investigations on this subject these precautions have never been specially considered nor is attention drawn to them by DEWAR and FLEMING in their investigation on the thermo-electric forces of metallic combinations at very low tempe ratures.

First of all we shall consider the construction of two similar thermoelements which were to be used for accurate temperature determination, the one as observation element, the other as comparison element. The commercial covered galvanoplastic copper wire has as a rule a sufficiently homogeneous structure to be used without further precautions as part of a thermoelement. If a

[1]) Ann. de Ch. et de Phys. 6 Sér. t. 27, 355, 1892.

galvanometer is closed by a wire of this kind and the wire is heated with a BUNSEN flame at a sufficient distance from the galvanometer, it is not probable that one shall find a greater deviation than corresponds to a fraction of a microvolt. Not so with commercial German silver (even the best kinds) especially not if it has been strongly bent. Differences which give disturbances of some microvolts when part of the wire is heated to 130° are very common; in heating with a BUNSEN flame they become so capricious that one would be inclined to reject the wire altogether. When annealed with a BUNSEN flame the wire generally did not yet prove fit for use. Nor was this the case after it had been shut up in an asbestos box, put on a blacksmith's fire and afterwards cooled slowly. The German silver wires (1 m.m. section) which were used by me for the construction of thermoelements, were therefore carefully drawn straight, annealed with the aid of a battery of accumulators (28 Volt, 20 Amp.) [1]) and cooled carefully by slowly diminishing the current. Both ends of the German silver wire are then connected with the galvanometer by means of copper conductors and avoiding sharp curves. The wire is immersed by parts in an oilbath heated to 130°, or stretched through a T tube, into which steam is led sideways which flows towards the two ends, by which the wire enters and leaves. In those parts of the wire which were near the connecting-screws supplying the current during the annealing, considerable irregularities are left. But apart from this, no greater

[1]) Also NOLL, Wied. Ann. 53, p. 583 has annealed his wires in this manner.

differences than some tenth parts of a microvolt should remain. It the wire after repeated annealing and if possible after yet more careful cooling does not fulfil this condition, it is totally unfit for use.

If one succeeds in finding a piece of sufficient length, where the electromotive force by the aforesaid heating rises nowhere above $^1/_2$ microvolt, and which does not show any appreciable potential difference (less than 0,05 microvolt) at the therminals over a length of 50 to 60 cm., this piece is cut off the wire, and both terminals serve as the legs of the thermoelement. If there is some difference the worst terminal is chosen as the zero-leg, i. e. that which will be placed in ice. Along this part of the wire the differences of temperature will as a rule be much smaller than along the part which is used as measuring needle.

Further we must prevent the wire from being bent into sharp curves. The wire is to that end (see Pl. III fig. 5) enclosed in a thick-walled india-rubber tube *C*, through which it can be drawn, by soldering it to a thin auxiliary wire, which is worked through the rubber tube by means of a small ball. This rubber tube serves at the same time to isolate the German silver wire completely and to dry the element.

10. *The protecting of thermoelements.* Thermoelements consisting of bare wires, which are plunged into the bath, the temperature of which one wishes to determine, have the great advantage of indicating the temperature almost immediately. To be able to use those bare elements in each special case it is necessary to make sure

beforehand that they can be plunged into the liquid gas, without any danger for chemical action or without generation of chemical potential differences.

As moreover bare wires cannot be brought into ice or steam (because of this action) the calibration of the element must be performed by plunging the element into suitable liquid baths (petroleum at 0°, oil at 100°) which are brought to the required temperature by ice or steam.

A bath of that kind is represented in fig. 1 and 2 Pl. III. The edges, *g*, which project from the steam or ice, must be protected as much as possible from exchange of heat with the air and by filling up with wool and closing with a non-conducting lid; air currents above the liquid must be prevented. Stirring appears to be indispensable, and there remain small differences of temperature which have to be determined separately. In calibrating the element by comparing with the hydrogen thermometer it is preferable to immerse the thermoelement and the fragile hydrogen-thermometer together in the one liquid bath, in that case, in order to protect the latter, a cylinder *f* is placed in the bath and the stirrer *r* (wooden rod through glass tube) is moved up and down in the exterior space.

As a rule however it will be necessary to insulate the wires of the thermoelement completely from the bath. This cannot be avoided even if one wishes to plunge the measuring needle immediately into steam or ice, or if the wires or the junctures run the risk of being attacked in the long run, either by the liquid gas or by the moisture of the atmosphere which condenses on the cooled element, when it is removed from the apparatus.

Nor can it be avoided where in using bare wires chemical potential differences may arise which will endanger the accuracy of the observations even in shunt-circuits of considerable resistance, because they are so much larger than the thermoelectric potential differences.

Finally by enclosing the thermoelement in a separate covering one prevents the wires from being exposed to sudden great cooling, sometimes in one place, sometimes in another, which might have some influence on it. A priori it is not to be taken for impossible, that such irregular sudden coolings may influence the thermoelement, which is prevented by the envelope.

For these reasons it seemed desirable in the measurements hitherto performed to use protected elements.

If the wires of the thermoelement are protected special attention has to be paid to the heat, which is carried away from the juncture by the conduction of the wires. In precise determinations it must be negligible in comparison with the heat-supply from the bath in which the thermoelement is immersed. To increase the heat-supply the contact of both wires is soldered into a small block of copper *a*.

In several respects it is desirable to keep the resistance of the element small and it is therefore not allowed to diminish the conduction by taking the newsilver wire very thin: it has to go straight to the juncture in order to avoid sharp bents. Wire of 1 mm. section caused no difficulties.

The German silver wire is insulated by a glass tube *b*, (Pl. III, fig. 4) which is put into the india-rubber tube *c*; 50 c.m. of the copper wire of $^1/_2$ m.m. diameter

covered with silk is wound spirally round this tube [1]). If the spiral was omitted a variation of temperature of the juncture could be detected in some cases. The glass tube b_2 insulates the whole element, with the exception of the small copper block a, from the bath into which it is plunged. In previous constructions an attempt was made to replace the glass tube by a very thin-walled brass tube, which was insulated from the apparatus, containing the bath. Such a tube would have had the advantage of being able to be soldered to the copper block and therefore tightly connected to it. But the temperature of the juncture proved then to be no longer certain to $^1/_{50}{}'$ degree, even when the thermometer was plunged very deeply into the bath.

It is very difficult to join copper and glass, especially if the joint has to resist steam and stand exhaustion. If the joint is not absolutely tight, moisture may penetrate into the element during the calibrating in steam and there cause disturbing potential differences. India-rubber tube firmly tied round the glass is not sufficient. Ordinary cements melt at 100° or are difficult to remove. It is intended to try if a juncture of glass and metal according to CAILLETET's method can resist the considerable and repeated variations of temperature which the thermoelement undergoes; also a stuffing-box with screw-fastening on a glass ring might be tried.

But provisionally I have resorted to cementing with

[1]) The mutual insulation of the turns of the spiral might be obtained by sealing a spiral of thin glass thread round the glass tube.

sulphur, because it is not necessary to heat the element above 100°. On the copper block a small piece of tinned tubing is slid; tubing, block and contact place are soldered together, the copper block being heated with a pointed flame and resin serving as melting agent. The groove g between copper and glass is filled with melted sulphur and care is taken that the sulphur in the interior does not come in contact with the copper wire, which would soon be dissolved.

If such a tube cemented with sulphur has been heated in steam several times, as is often necessary for testing purposes, the sulphur is gradually pulverized. To prevent this interaction the joint is protected by a piece of vulcanized india-rubber which is renewed from time to time. When the copper tubing was used untinned, it was attacked by the sulphur especially at 100°. It is still to be proved whether tinning (or blackening) prevents this action sufficiently. But there is no difficulty in replacing the copper tubing or even the copper block and glass cover by a new one. The element itself need not undergo any change thereby.

When the glass covers are fastened to both the legs, A and B, fig. 5, of the thermoelement, they are connected hermetically by an india-rubber tube and rubber-cement to the rubber tube which covers the newsilver wire. The copper wires K are allowed to remain free. The element is thereby enclosed in a completely closed space, communicating with the atmosphere by the tubes s_a, s_b only. By means of these tubes, dry air is sucked through this space under proper heating, entering by the first tube s_a, passing by the juncture c_a, through

the interior insulating tube into the tube C and further through the tube of the juncture c_l, to the side-tube s_b. After the operation the side-tubes are sealed off.

The envelopes of the thermoneedles being of glass, one can always easily make sure between the experiments that no moisture has found its way into the apparatus. If this should be the case it can be removed by means of the side-tubes.

In order to bring the themoneedle into the apparatus in which the temperature is to be determined it is provided with a stopper as represented Pl. II, fig. 5, and if necessary the copper block a and the joint with the glass are protected in the way required by each special bath.

11. *The testing of thermoelements.* When completed the thermoelement is submitted to the ordinary test, that no current arises, when both junctures are at the same temperature. For this purpose it is connected to the galvanometer (§ 13) by suitable "current-make and break's" and "current-reverser" (see § 12) and as is usual, put into ice with both junctures.

In general this single test in considered sufficient. However in this test the temperature of the contact-places differs only little from the surrounding temperature and defects might be overlooked which become considerable at greater differences of temperature. For this reason an obvious improvement consists in putting the two contacts into steam also and immersing them to different depths in order to see if by the method followed one has actually succeeded in avoiding the defects, which can

be detected by this experiment. To this end the two needles of the thermoelement are put into two identical boiling apparatus as described in § 8.

When a needle of the thermoelement is heated above the surrounding temperature and the little block turned downwards a convection current of more or less importance will begin to flow in this needle. By observations with a third boiling-apparatus, in which the needle is placed with the block turned upwards we may ascertain whether possibly such a convection current can lower the temperature of the juncture somewhat.

Pl. III fig. 6 shows this steamcap. It can be drawn out to different lengths and the steam is supplied from a separate boiler through a tube wrapped up in wool. The whole steamcap is covered with felt. The water condensed in the interior space flows off by a capillary tube c, in the bottom. This capillary is closed by the drop hanging from it. The remarks made in § 8 with respect to the temperature of the steam are also applicable here. Moreover as an additional test the contacts are interchanged in both the boiling apparatus.

The thermoelement is finally subjected to a third test by plunging both contacts in a mixture of solid carbonic acid and alcohol, which to this end is poured into a DEWAR-vacuum-glass. After having been subjected to these tests the thermoelement is calibrated by comparing it with the hydrogen thermometer at different temperatures. Plate I represents the comparison in liquid oxygen, Pl. III, fig. 1 and 2 show the thermoelement together with the hydrogen thermometer immersed in steam and ice.

12. *"Current-make and break's", commutators and "current-reserver's" with contact surfaces of mercury.* In making measurements of the small thermoelectric potential differences it is necessary to avoid with the utmost care disturbing potential differences at the contacts in the apparatus, which serve for making or breaking the current. The method that presents itself at first is to use commutators of pure copper. In that case only small thermoelectric forces are possible as the circuit including the galvanometer wire usually consists of pure copper. But if the copper commutators are not made with the greatest care and packed in cotton-wool these disturbances are not to be neglected.

If the contact is made by causing mercury to flow together, potential differences at the place of contact are excluded. But the use of these contacts is necessarily accompanied with the introduction into the circuit of the considerable potential differences mercury-copper. These are bound to give rise to disturbing forces, if the contacts are not kept exactly at the same temperature. It is just the same difficulty as that caused by resistance-boxes wound with German silver wire. If the terminals of the separate resistances are not exactly at the same temperature disturbances must arise, which may influence the results especially in zero methods.

If on the other hand we succeed in keeping the fixed contacts of circuit and commutators at temperatures sufficiently near [1]) (as is also the case in resistance-boxes) these mercury-commutators have the great advantage

[1]) Chassagny and Abraham l. c. p. 361.

that with pure mercury one is perfectly sure of the contact, and that by a suitable construction of the apparatus the flowing together may be obtained by a slight motion of the hand as with a Poggendorf-switch, while on the other hand it is well known that perfect contacts with plug-commutators are very difficult to obtain. When using an aperiodic galvanometer and a mercury-commutator the observer seated before the reading-telescope may take one reading immediately after the other.

For a long time therefore I have tried to make use of contacts obtained by causing mercury to flow together. At first I constructed very simple apparatus for this purpose, the principle of which may be seen from Pl. IV, fig. 1 and 2. By lifting the weight g the mercury in the tube b fig. 1 rises and causes contact between c_3 and c_2; by putting an additional weight the mercury rises in a and causes contact between c_2 and c_1. Similarly in the apparatus fig. 2, a rise of the mercury in the tubes a and b causes contact between c_1 and c_3, c_2 and c_4, by a rise in the tubes d and c contact is obtained between c_1 and c_2, c_3 and c_4. The motion of the mercury is controlled by the strap f (a piece of the belt of a lathe) Several difficulties, however, remained in the use of these apparatus, viz. uncertainty of adjustment, splashing of mercury, necessity of cleaning the apparatus from time to time, difficulties of properly protecting the contacts against changes of temperature.

These difficulties were removed by constructing completely closed glass apparatus. Plate IV, fig. 3 and 4,

and Plate V represent the very handy patterns invented by the technical attendant of the laboratory Mr. BLOM.

In the first place in the "commutator" fig. 3, the contacts of the circuit and the mercury in the glass apparatus are obtained by platinum wires sealed into the tubes C_1, C_2, C_3. Further the apparatus is mounted on a little board p, which can turn easily round an axis x. When turned to the left c_1 and c_2, when turned to the right c_2 and c_3 are in contact. In the position represented in the figure the circuit is broken. The platinum contacts consist of interwoven thin platinum wire, the connecting pieces of the tube C and the small brass blocks b, being therefore very flexible. The latter are insulated by ebonite, and joined to the rest of the circuit. If the juncture of the platinum, enamel and glass is not absolutely tight, as appears at the mercury pump, this is remedied by putting on some shellac. The glass is previously cleaned in the manner described in § 4 and is exhausted with the mercury-pump and then filled with mercury; afterwards dry air is admitted, which serves as cushion for the mercury when the commutator is reversed; the auxiliary tubes which are used in this operation are afterwards sealed off. Owing to these precautions the mercury-surfaces are now after several years just as clean as at the moment the apparatus was delivered. Care has been taken that the resistance along c_1, c_2 is the same as along c_2, c_3; the small difference which remains is constant and may be measured and taken into account if necessary.

The "current-reverser" fig. 4 is constructed on the same principle as the commutator. In the bent tube a,

in the position A fig. 5, the mercury makes contact between the mercury tubes d_1 and d_4, and between d_2 and d_3 which open out into a; in the position C there is contact between d_1 and d_2 and between d_3 and d_4. In the position B the contact is broken. The apparatus is again mounted with cork fig. 4, k, on a little board p, which can be turned round the axis x by a slight movement of the finger. The tube l (fig. 4 and separately fig. 6 in the original state when the tubes for cleaning, exhausting and filling are not yet sealed off), forms the connection between the different branches of the tube a, and in this manner forms a continuous space above the different mercury levels. Again care is taken that the resistances along C_1 C_2 and C_3 C_4 are the same as those along C_1 C_3 and C_2 C_4 and here also the difference that might still exist is constant and can be measured.

The apparatus including the wires is packed in cotton-wool in a large box, which is further carefully protected against heat radiation and conduction; it is placed in the room for magnetic measurements, where of course care is taken to assure a constant temperature.

13. *Galvanometer.* In the measurements a galvanometer made by BRAUN (cat. 1894 nº. 371) was used. As suspending wire a beautiful quartz fibre is used. I have to thank this fibre to the special kindness of Prof. V. A. JULIUS, who at my request made this fibre in the desired dimensions and a number of similar ones as a reserve.

The suspensive contrivance of the galvanometer was modified as represented in fig. 2 and requires no further explanation. To fasten the fibre its terminal, supported by

suitable small clipstands, is brought near the little cylinder which is also held in a small stand; with a needle the fibre is pointed into the fine groove, which has been drawn on the flat side of the halfcylinder in the direction of the axis; a piece of shellac is laid on it and the end of the small cylinder is heated until the shellac melts. [1]) At the other end in the same manner a small eye is attached, on which also a groove has been drawn. This is done by laying it on a mica plate and heating on the other side with a small soldering bolt. When the shellac is completely hardened the bellmagnet is suspended from the fibre; it has never happened that the connection gave way. The considerable and often irregular changes which occur when silk fibres are used in consequence of change, of moisture and temperature are excluded by the use of the quartz fibre.

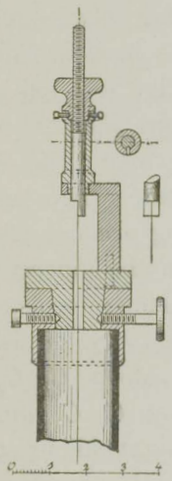


Fig. 2.

The middle ring of the galvanometer is easily adjusted in the meridian by a little mirror leaning against this ring and a magnetic theodolite. The galvanometer is provided with the bobbins the coil of which has a resistance of 3 ohms. The four coils are connected in two parallel sets of two in series. [2]) They are pressed against the middle

[1]) Comp. Boys, Phil. Mag. 37. 463. 1894.

[2]) The soft iron rings are not applied. The galvanometer therefore is not aperiodic but the changes of zero and sensibility may be better studied, also in reference to the changes of terrestrial magnetism.

ring and in order to attain a good fit they are surrounded by india-rubber rings; little corks are thrust into the openings of the bobbins. Over the top of the galvanometer a glass bell is fixed (see fig. 3) by means of a cork cut in two pieces. In this manner the galvanometer is totally closed and disturbances by dust, small spiders, etc., that might otherwise be found cannot occur. The instrument with wires is packed in cotton-wool and protected by a pasteboard cap, in which a hole is made through which the readings are taken. In this manner the conditions are fulfilled, under which the galvanometer in a permanent position may be used for a long time without any change, a calibration of the sensibility in its dependance upon the deviation once made, remains correct for a long time and accidental disturbances are excluded. A very

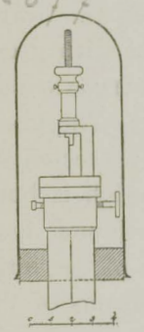


Fig. 3.

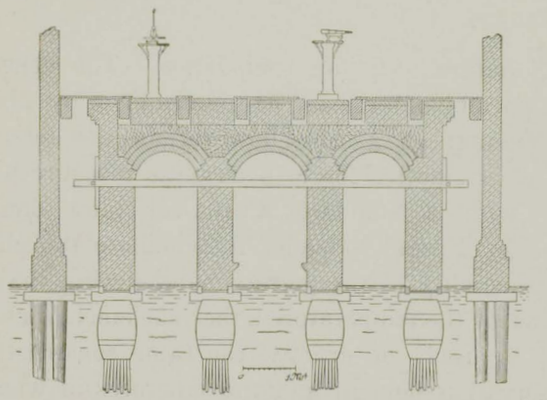


Fig. 4.

firm disposition was obtained by placing the galvanometer on a marble slab fixed with plaster to an observation pillar, which in its turn was firmly fixed with plaster to the great pillar of the magnetic room. On this same pillar is placed the reading-apparatus, described Proceed. April '96. Comm. n°. 25, by which the deviations of the galvanometer mirror are read with great accuracy on a glass scale made by HARTMANN and BRAUN (compared with the standard metre). The great pillar (fig. 4) consists of a block of masonry erected on three rows of barrels with perch poles, which is built out sideways at the top and forms a continuous floor of great stability under the ordinary floor of the magnetic room. On this floor between the beams of the ordinary floor the proper observation pillars are erected. Galvanometer and scale are therefore mounted on the same mass of brickwork in an absolutely fixed relative position. The stability of the zero is excellent in consequence.

14. *Standard cell for comparison.* The sensibility of the galvanometer can be determined by absolute measurements; as a rule however for the determination of electromotive force in absolute measure a comparison with a standard CLARK-cell (now one constructed by FUESS) is made. To eliminate the changes in the sensibility of the galvanometer during the determinations of the temperature a thermoelement (newsilver copper) is used, of exactly the same construction as the observation-element and the junctures of which are placed in steam and in ice. It is a great convenience

especially in this case that this comparison-cell can itself be placed in ice and steam and stay there as long as is desired. The boiling point apparatus serving for this purpose is described in § 8 (Pl. I, fig. 2.) The velocity of evaporation is kept as constant as possible, so that only a correction for the change of the boiling point with the barometer has to be applied, for which correction use is made of a calibration performed with this element. The element for comparison is compared from time to time with the CLARK-cell. If it should appear by absolute measurement that the thermoelement, when treated always in the same manner, always gives the same electromotive force at the same temperature, one would be certain of the electromotive force within $^1/_{10000}$ after having made the aforesaid correction and the element itself might then be used as a standard for the measurement of electromotive forces. Although this constancy is highly probable, I have not been able to put it beyond all doubt so far.

In determining a temperature the reading of the deviation of the galvanometer produced by the observation element, is immediately followed by a reading with the comparison element. Pl. I fig. 2 shows the way in which use is made of the mercury-commutators for this purpose. Two commutators and one "current-reverser" are used to send the current of the observation element (*oa*, *gh* or *if*, *od*) and the comparison element successively (*ob*, *gf* or *ih*, *od*) through the galvanometer in two directions. The measurements are repeated with a second resistance.

Further by turning the commutator to the right

PLATE I.

№ 679.

Fig: 1.

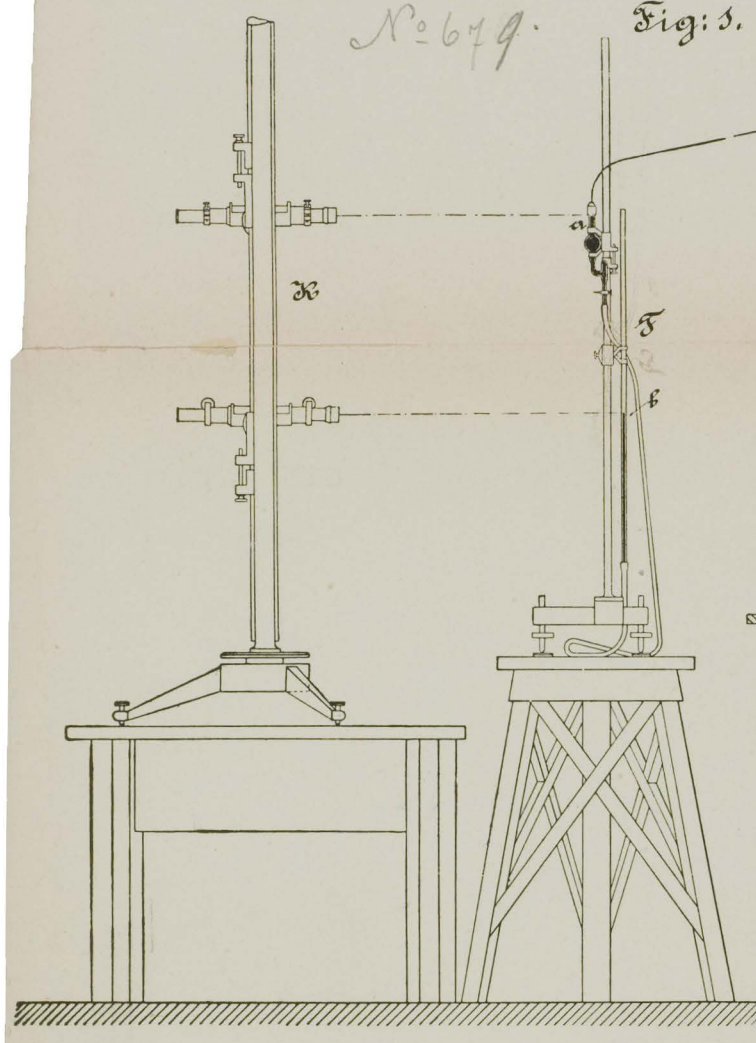


Fig: 2.

№ 680.

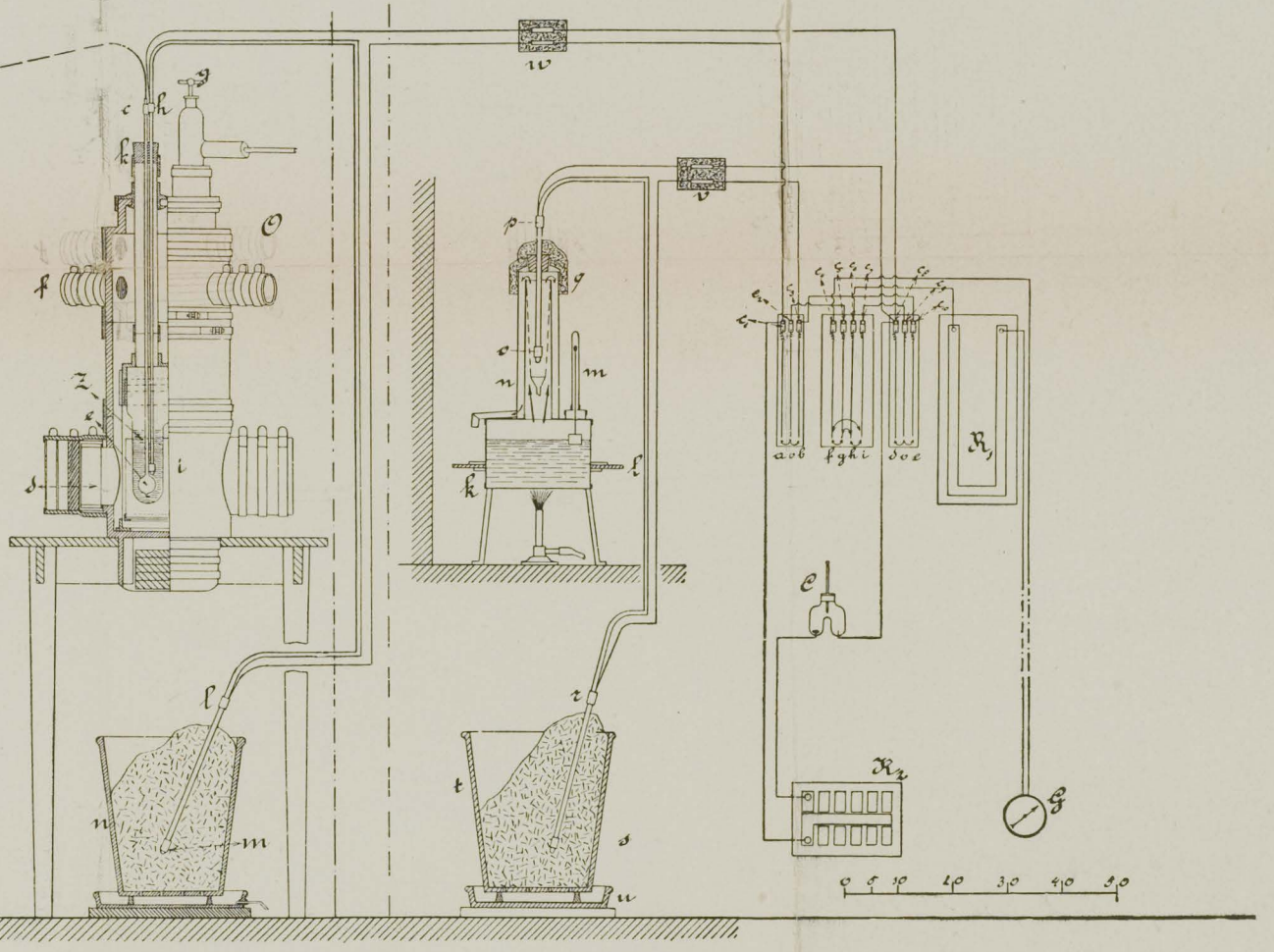
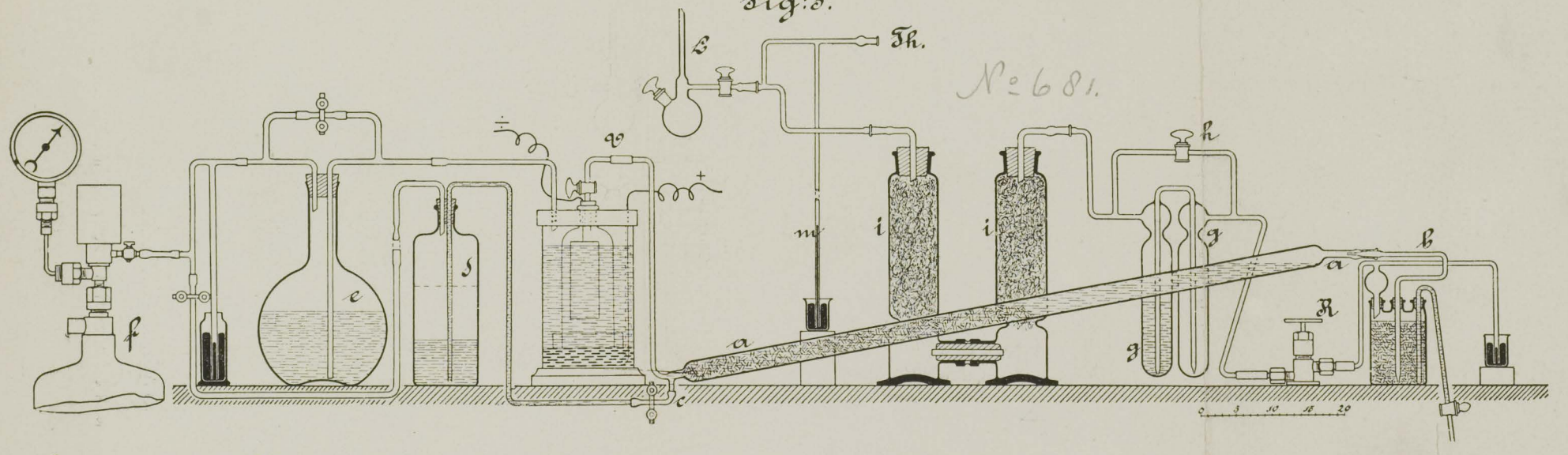


Fig: 3.

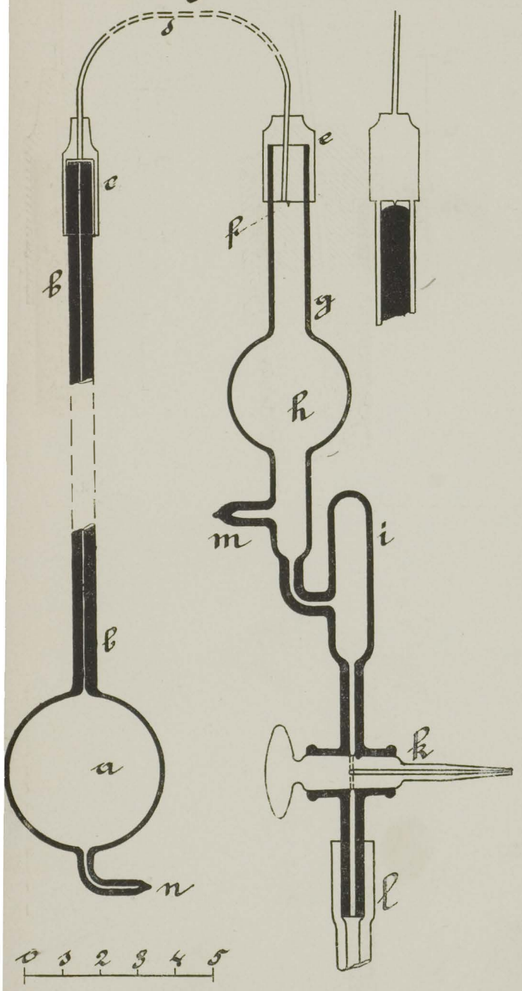
№ 681.



Communication N°. 27.

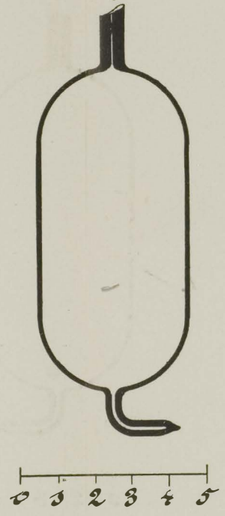
№ 682.

Fig: 1.



№ 683

Fig: 2.



№ 684

Fig: 3a.

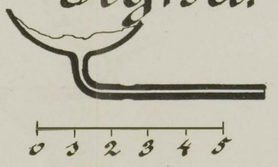
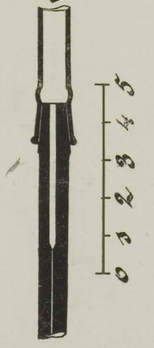


Fig: 3b.



№ 685

Fig 4a

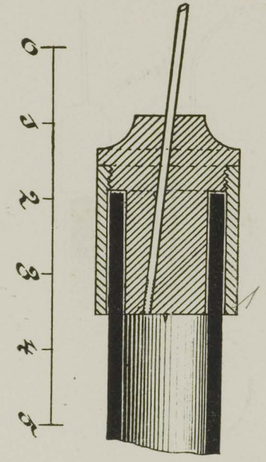
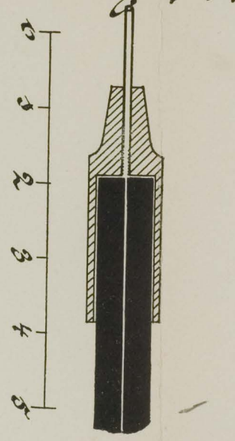
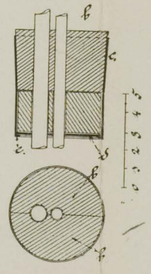


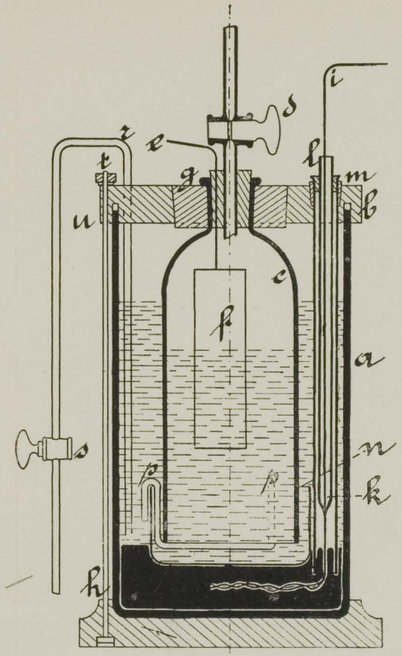
Fig: 4b.



№ 686.

Fig: 5.





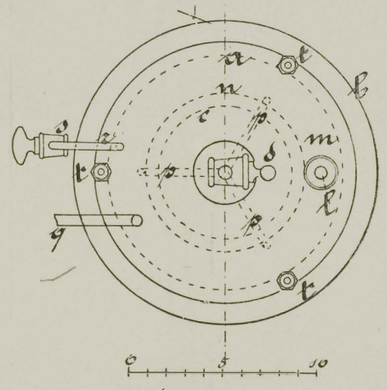
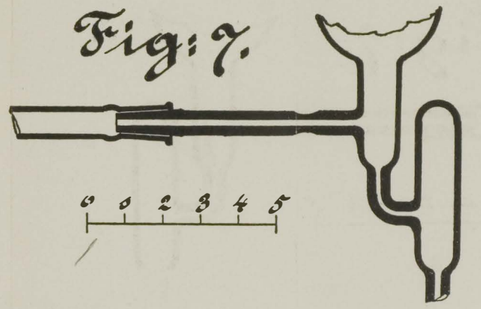


Fig: 6

№ 687

№ 688.

Fig: 7.



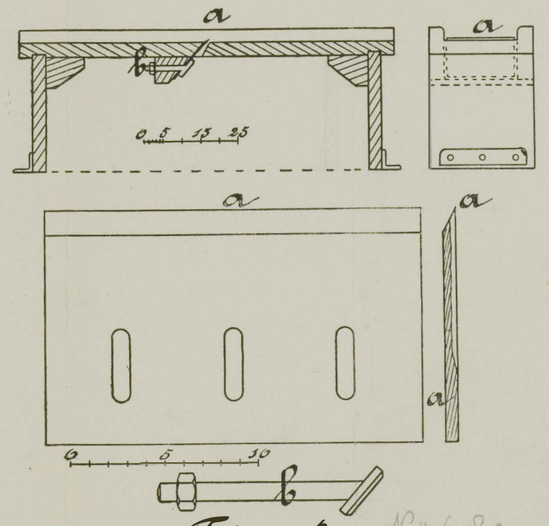


Fig: 8.

№ 689.

Communication №. 27.

PLATE III.

Fig: 1. N° 690

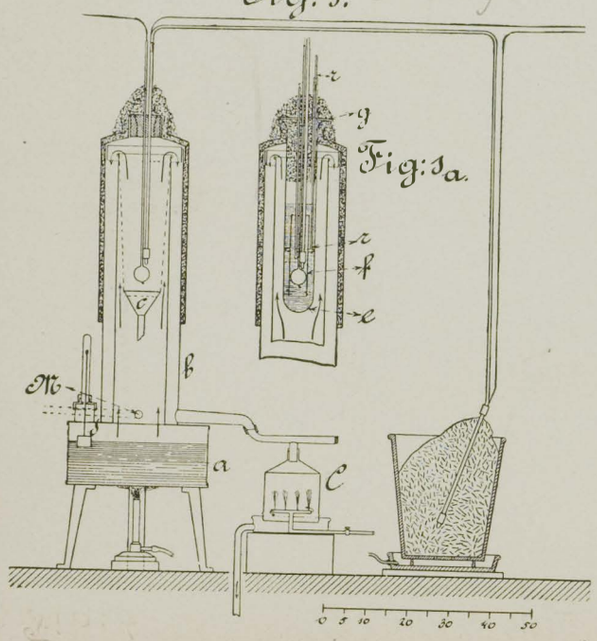


Fig: 1a.

Fig: 2a. Fig: 2b. N° 691

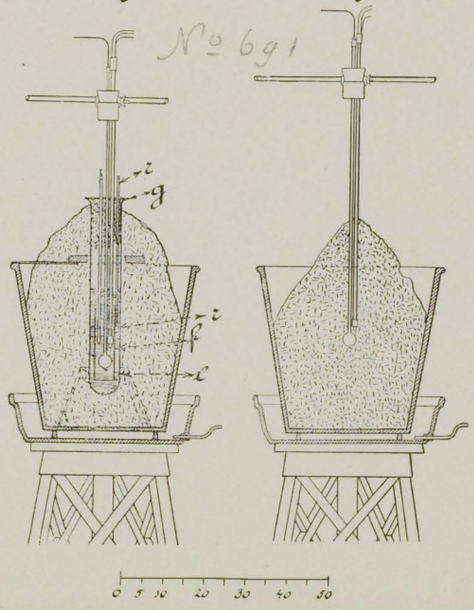


Fig: 4. N° 693

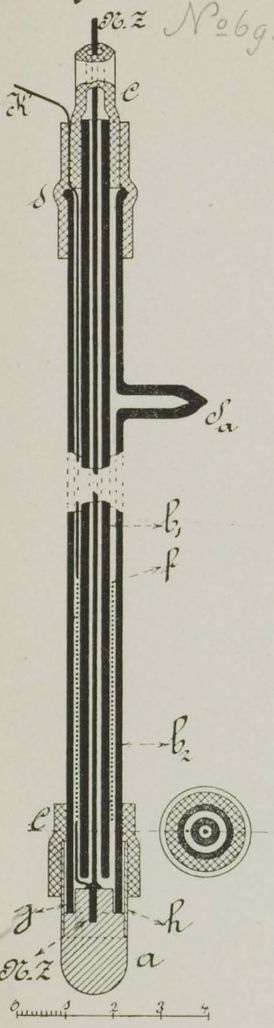


Fig: 3. N° 692

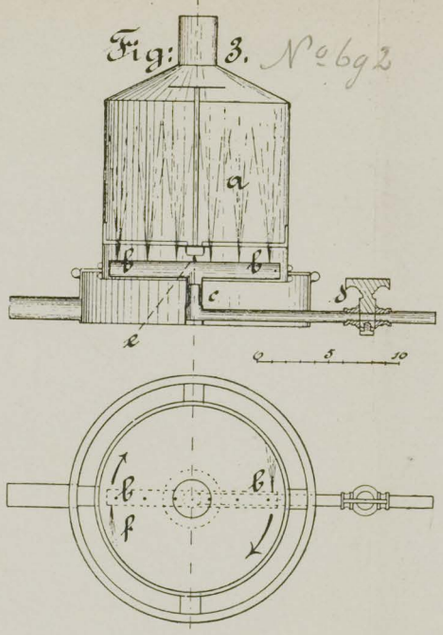
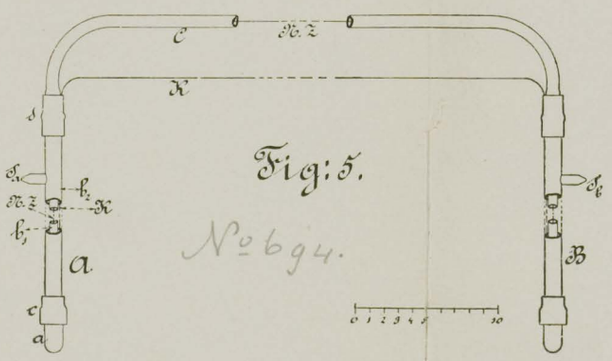
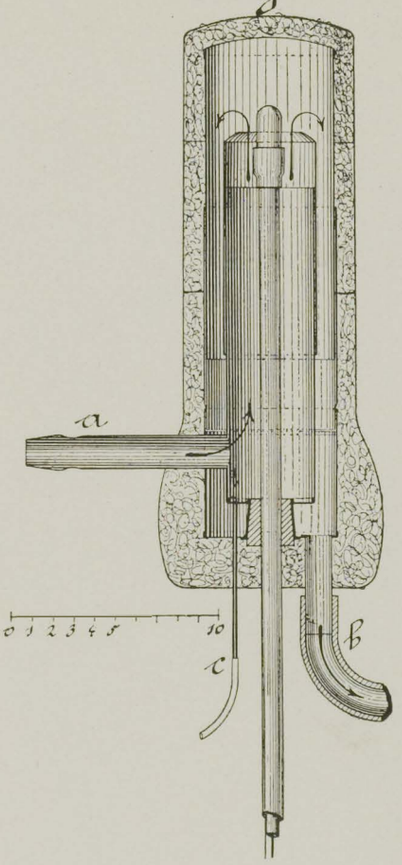


Fig: 5. N° 694

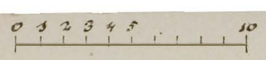


N° 695.

Fig: 6.

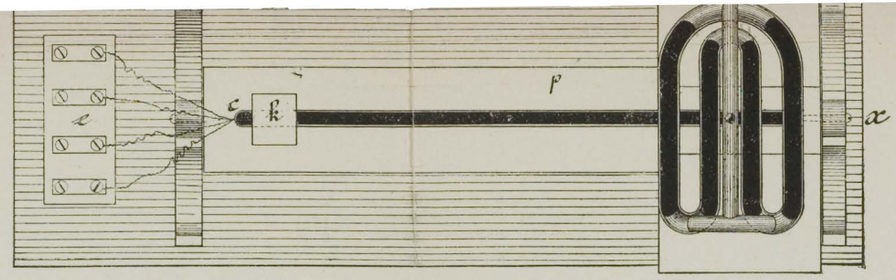


Communication N°. 27.



№ 699

Fig: 4.



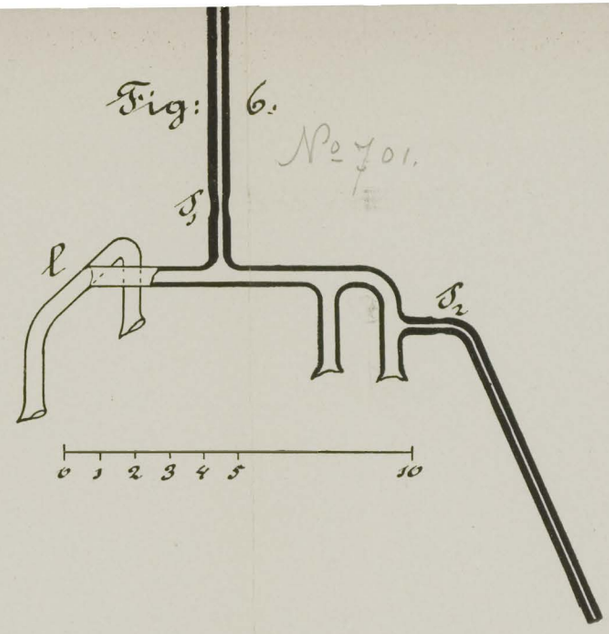
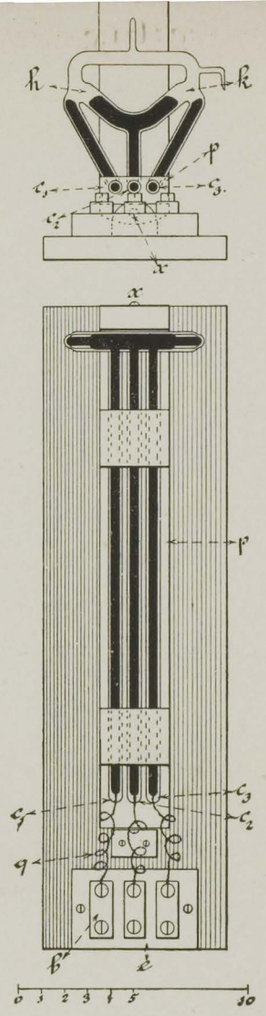


Fig: 6.

№ 701.



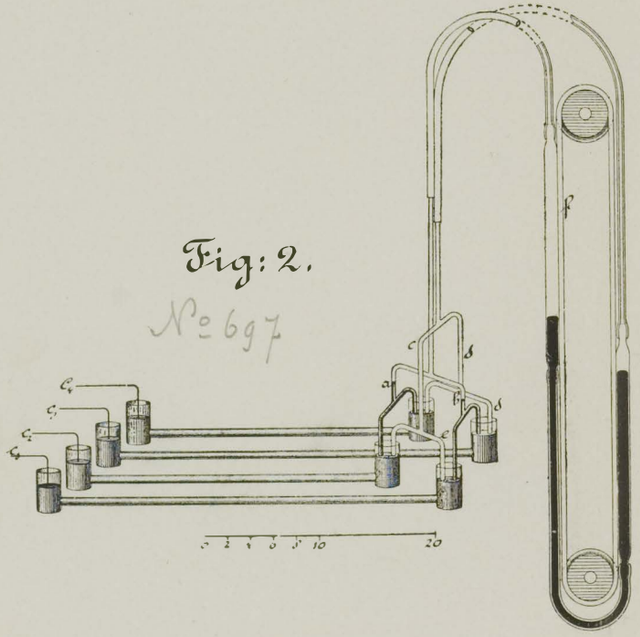


Fig: 2.

№ 697

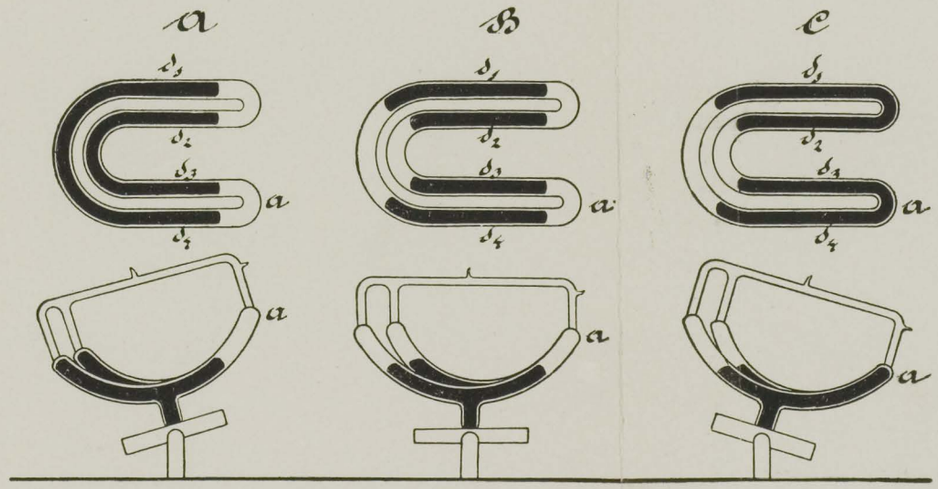
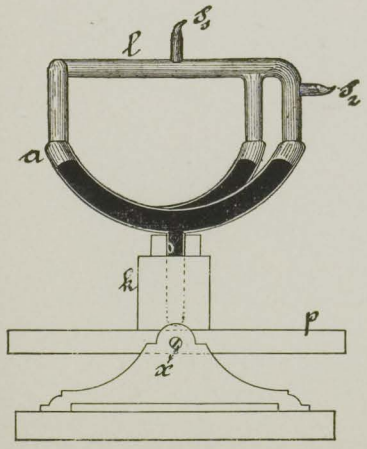
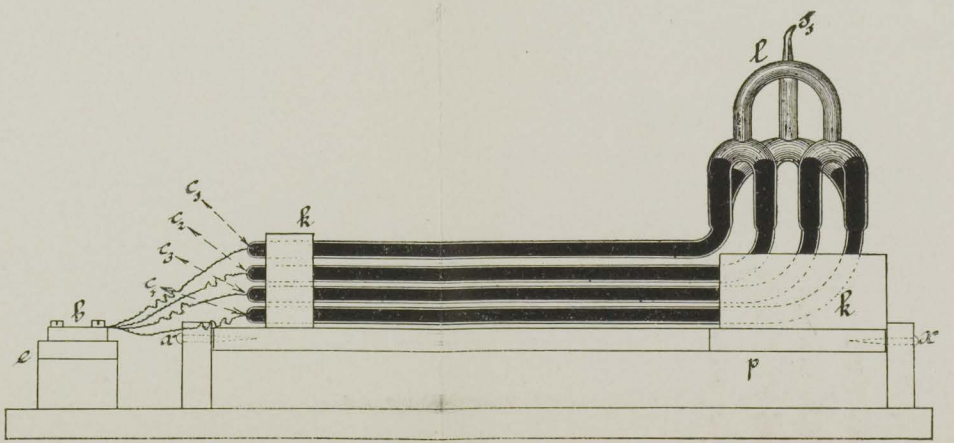


Fig: 5. № 700.

Communication N°. 27.


l
δ_3
δ_2
a
k
p
d

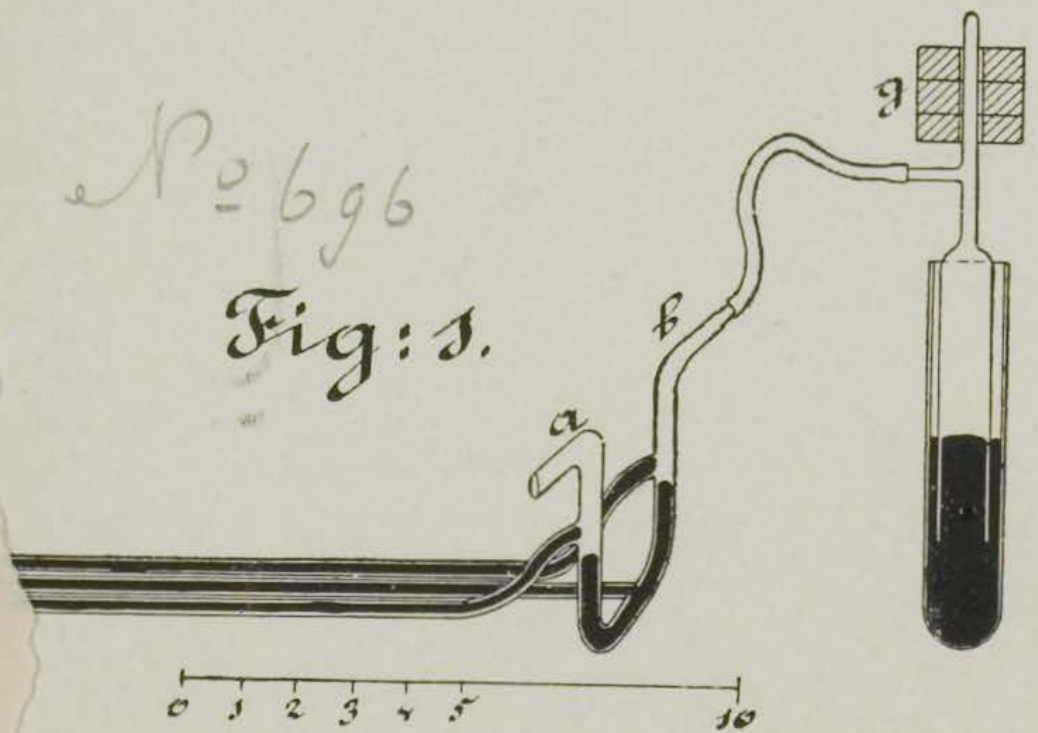

l
δ_3
ζ_1
ζ_2
ζ_3
ζ_4
k
b
e
d
p


p
δ_2

PLATE IV.

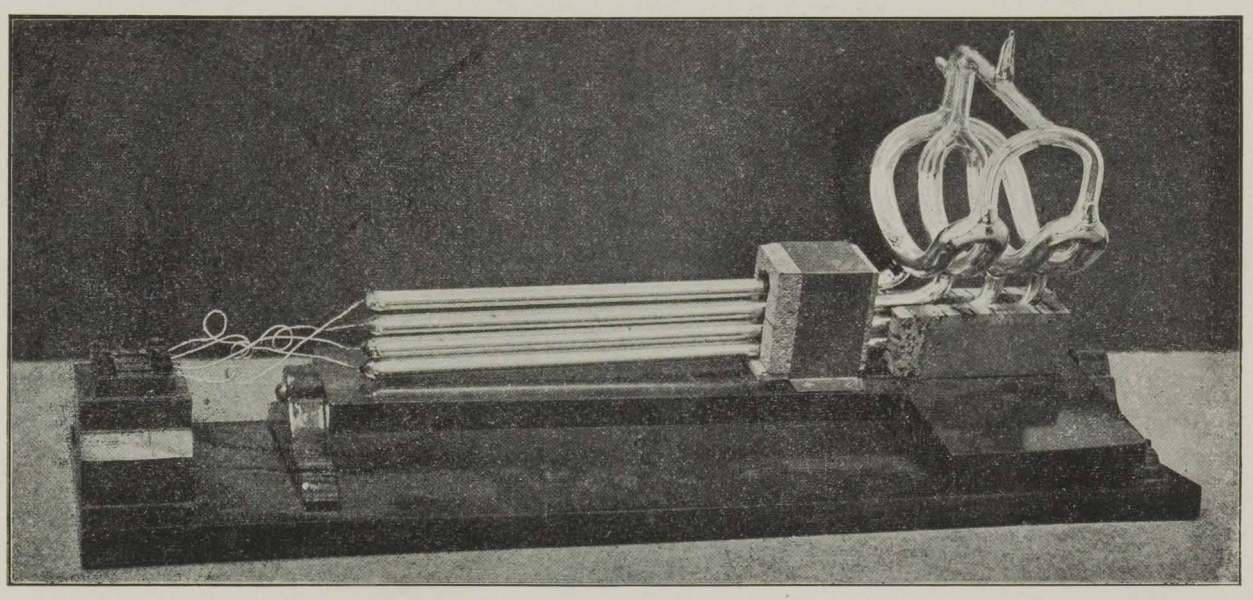
No 696

Fig: 1.



No 698

Fig: 3.



Communication N°. 27.

N°7702.

(*oa, gh* or *if, oe*) the connection of the wires being as given in the figure, the current of the two elements in series is sent through the galvanometer in two directions. If the copper connection at *w* is then reversed, the elements are placed in opposition (*oa, gh* or *if, oe*), the same result being obtained independently by changing the copper contacts at *v* (*oa, gh* and *if, oe*). In each of these measurements again two resistances are used successively.

Ultimately by shortcircuiting the wires, connected to the commutators at *v* and *w*, it may be tested whether the commutators are free of current.

In this manner data are obtained for different independent determinations of the same thermoelectric force, expressed in parts of the *E M F* of the comparison element between 0° and 100°.

By means of observations, made with the element at 0° and 100° it is possible to express the measured electromotive force in the *E M F* of the observation element at 0° and 100°.

Finally by replacing the observation element by the CLARK-cell at C_1 and C_8 it becomes possible to express the comparison element in terms of the standard cell and so, independently of the supposition as to the constancy of the comparison element, to compare different series of readings with the comparison element. The results of these measurements will be published in a subsequent communication.

COMMUNICATIONS

FROM THE

PHYSICAL LABORATORY

AT THE

UNIVERSITY OF LEIDEN

BY

PROF. DR. H. KAMERLINGH ONNES,

Director of the Laboratory.

No. 28.

(REPRINT).

Dr. J. VERSCHAFFELT. Measurements on capillary ascension of liquefied carbonic acid near the critical temperature. (*With a plate*).
(Translated from: Verslagen en Mededeelingen der Kon. Akad. v. Wetenschappen te Amsterdam, **27 Juni 1896,** *p.* 94—103).

EDUARD IJDO — PRINTER — LEIDEN.

Dr. J. VERSCHAFFELT. *Measurements on capillary ascension of liquefied carbonic acid near the critical temperature.*

In his thermodynamical theory of capillarity [1]), VAN DER WAALS has calculated the value of the surface-energy of a liquid near the critical temperature. He has arrived at the conclusion that, at least at a very short distance from this point, the surface-energy may be represented by a formula

$$\sigma = A\ (1-m)^{3/2}$$

in which A is a constant, and m the reduced temperature.

The values of σ, calculated by means of the experiments of DE VRIES [2]), very well may be represented by a formula $\sigma = A\,(1-m)^B$; B however is not equal to $^3/_2$, but smaller, with a mean value of 1,23, and increases slowly when m comes nearer to 1.

Observations made by RAMSAY and SHIELDS [3]) with a rather great number of liquids, have been used for such calculations [4]). It appeared again that B is never equal

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[1]) Verh. d. Koninkl. Akad. v. Wetensch. te Amsterdam (Eerste Sectie), Deel I, N°. 8.

Zeitschr. f. physik. Chem., 13, 1894.

Arch. Néerl., 1894.

[2]) Metingen over den invloed der temperatuur etc., Proefschrift, Leiden, 1893. Communications, n°. 6.

[3]) Zeitschr. f. physik. Chem., 13, 1894.

[4]) See v. d. WAALS, loc. cit.

to $^3/_2$, but has a nearly constant value, the same for all liquids.

Many liquids however, investigated till 7° from the critical temperature, give a value of B that increases and even reaches 1,37; this seems to indicate a tendancy to reach the theoretical limit value of 1,5. Others on the contrary, investigated also till a very short distance from the critical point, show no increase of B at all.

In a former communication [1]), in which I have given capillary ascensions of liquid carbonic acid till 10° from the critical temperature, I found B = 1,311; again a high value. It appeared consequently not uninteresting to pursue these observations nearer to the critical temperature, in order to decide wether the exponent still increases or not.

For such a research carbonic acid seemed to be the most conveniant substance; for its critical temperature may easily be reached, while neighbouring temperatures easily may be kept constant. The desirability of this research has already been pointed out by DE VRIES [2]).

Apparatus. The apparatus used in these researches is substantially the same as used in my preceeding experiments. Some modifications however were rendered necessary by the higher temperature, and the great variability of level of the liquid-surface in the neighbourhood of the critical temperature.

In order to get a suitable temperature a warm waterbath was used. The water of the supply was directed through a copper spiral placed in a waterbath, the

[1]) Comm., n°. 18.

[2]) Comm., n°. 6, p. 7.

temperature of which was kept constant; in this spiral the streaming water reached a temperature that could be raised to the wanted degree, by regulating either the velocity of the circulation or the temperature of the surrounding bath.

The thus heated water entered at the top of a cylindrical space, left between two glass tubes, the inner of which contained the observation tube, and was filled with water in rest. This water became accordingly heated, and when it had reached the state of equilibrium of temperature with the warm water streaming along, it rendered quite insensible the still possible small variations of temperature thereof. It appeared indeed possible to obtain, even above 30°, a temperature varying less than 0°,1. I have moreover observed that the temperature was the same in all sections of the bath.

In order to maintain the surface of the liquid in the middle of the observation tube, this tube was left open at the inferior end, and connected with an hydraulic pump through a pressure-cylinder filled with mercury. The mercury could be driven up through a narrow manometer tube, connected with the narrow manometer tube at the lower end of the observation tube by means of a steel capillary, and two brass pieces in each of which the glass was fastened with sealing wax. The steel capillary was divided in two pieces, joined again by a steel cock, which closed the observation tube when the surface of the liquid had been raised to a convenient niveau; in this way any leakage of pressure-cylinder or pump was rendered harmless. To the inferior part of the observation tube was joined another reservoir the

volume of which was so chosen, that for a suitable mass of fluid, at all temperatures, the niveau of the mercury could remain in it.

There was still a difficulty to be eliminated, concerning the connection of observation-tube with purifying apparatus. In order to prevent the liquid, distilled over in the apparatus by cooling it beneath the temperature of the room, from distilling over again when raising the temperature, the glass apparatus was shut, immediately above the water-bath, by means of a brass cock, again connected with the glass-apparatus by means of a steel capillary. The connection of this capillary with the glass was not formed as formerly by means of a brass piece fastened with sulphur: for the carbonic acid dissolves the sulphur, as appears from a colouring of the mercury, soon followed by a passing of gasbubbles between glass and metal. The connection was at last made as follows (Fig. 2): at the upper end of the glass-tube a brass piece with screw was fastened with sulphur, glass and metal then rendered quite level, and on the so obtained plane a ring of leather was strongly applied by putting on the screw-nut to which the steel capillary was soldered; this leather ring separated the carbonic acid at the same time from sulphur and free air.

Experiments. — In calculating the surface energy from the capillary ascension, we suppose that in the capillary tube the meniscus is spherical; it is known that this supposition holds only as long as the capillary ascension is at least 20 times as great as the radius. At the critical point the capillary ascension is zero; in order to allow trustworthy values of the surface energy to be obtained as near as possible to this temperature,

among a series of capillary-tubes delivered by GEISSLER, I have chosen the narrowest one, the radius of which was 0,0441 mM.

The pure carbonic acid was obtained in a way already described. A first quantity of this purified gas showed following ascensions:

t	h	d
12°,8	15,36 mM.	0,78 mM.
14°,5	13,92	0,74
16°,5	12,30	0,70
18°,7	10,50	0,64
21°,5	8,24	0,54
25°,3	5,18	0,42
27°,3	3,52	0,34
28°,9	2,20	0,24

d represents the height of the annular meniscus in the wide tube; this height will be taken into account for correcting the ascensions.

In order to ascertain how far these ascensions are to be trusted, I purified a second quantity of gas, and made with it a second series of observations.

t	h	d
15°,9	12,78 mM.	0,74 mM.
20°,3	9,22	0,60
22°,5	7,40	0,52
24°,7	5,68	0,48
26°,6	4,06	0,38
28°,4	2,60	0,28

This second series agrees perfectly with the first. All ascensions lie almost exactly on a straight line, which intersects the axis of temperatures at 31°,6. If therefore the ascensions continued to decrease, with rising temperature, according to a linear law, the critical point, that is the temperature at which $h = 0$, would be

about 31°,6. Direct experiments however had shown that the liquid surface vanished at 31°,0. In the immediate neighbourhood of this temperature, this line must therefore be curved towards the axis of temperatures[1]).

In order to determine where the curvature nearly begins, I made a third series of experiments in the

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[1]) The perfect agreement between these two series of observations may not yet be considered as a proof of the purity of the gas. For in both cases the process by which the pure gas has been obtained was quite the same; the two specimens were therefore necessarily of the same degree of purity and should show the same capillary ascension.

That however the gas became more pure by the treatment to which it was subjected, I have proved in making experiments with carbonic acid, directly taken from the box. The capillary ascension was much lower, but different specimens gave also values agreeing with each other:

$t = 13°,3$	$h = 13,40$ mM.
15°,9	11,60
17°,4	10,50
20°,3	8,48
22°,4	7,04
23°,2	6,44
26°,1	4,36
27°,5	3,40

Also these ascensions lie upon a straight line, the ordinates of which however are not only smaller than those of the former, but even the point of intersection, 32°,3 is situated further from the true critical one. The purifying of the gas has not only the effect to increase the ascensions, but causes also the straight line upon which they lie to come nearer to the point $h = 0$, $t = 31°,0$. For absolutely pure carbonic acid the line would perhaps meet this point, and the ascensions decrease according to a linear law, till reaching the critical temperature itself.

immediate neighbourhood of the critical point. I succeeded in obtaining a constant temperature at 29°,3, and from that point I made it rise very slowly (nearly 0°,1 in 30 minutes)

$t = 29°,3$	$h = 1,88$ mM.	$d = 0,20$ mM.
29°,7	1,48	0,16
30°,2	1,00	0,12
30°,4	0,74	0,10
30°,5	0,62	0,08
30°,6	0,52	0,06

Though till 30°,9 both menisci could be observed well, yet above 30°,6 observations were rendered impossible by the instability of the meniscus in the capillary; this meniscus was continually in motion, now rising, then falling, even descending beneath the surface of the liquid outside the capillary.

This last series shows clearly a curvature, beginning at $\pm$ 29°,5, towards the axis of temperatures.

Applying the correction already used in the former communication:

$$h' = \left(h + \frac{r_1}{3}\right)\frac{\frac{2d}{(r_3 - r_2)^2}}{\frac{1}{r_1} - \frac{2d}{(r_3 - r_2)^2}} = \left(h + \frac{r_1}{3}\right)\frac{2d}{\frac{(r_3 - r_2)^2}{r_1} - 2d}$$

the real ascension is found to be

$$H = h + h' + \frac{r_1}{3}$$

I found $r_3 = 3,25$ mM., and $r_2 = 0,280$ mM. Hence

$t = 12°,8$	$h' = 0,120$mM.	H = 15,50mM.	H(*calc.*) = 15,48mM.
14°,5	0,104	14,04	14,08
15°,9	0,095	12,89	12,92
16°,5	0,086	12,40	12,43

t = 18°,7	h'=0,067mM.	H =10,58mM.	H (*calc.*)=10,61mM.
20°,3	0,055	9,29	9,29
21°,5	0,045	8,30	8,30
22°,5	0,039	7,46	7,48
24°,7	0,029	5,72	5,66
25°,3	0,023	5,22	5,17
26°,6	0 016	4,09	4,10
27°,3	0,013	3,55	3,52
28°,4	0,007	2,62	2,61
28°,9	0,006	2,22	2,20
29°,3		1,88	1,87
29°,7		1,48	1,54
30°,2		1,00	1,13
30°,4		0,74	0,96
30°,5		0,62	0,88
30°,6		0,52	0,79

Till 30° the observations may be represented by the formula

$$H = 26{,}04 - 0{,}825\,t$$

by means of which the fourth column has been calculated. As the temperatures have not been read further than in tenths of a degree, differences of $\frac{0{,}0825}{2}$ between the observed and calculated H are possible; in two cases only this difference has been exceeded below 30°, what we may however ascribe to an experimental error.

Above 29° the corrections become incertain; as they are however very small and reach zero at the critical point, I have put there $H = h$. It would perhaps not be quite unnecessary to make out whether a correction due to compression by gravity must be taken into account or not.

Surface-energy. — We calculated this energy by means of the formula

$$\sigma = \frac{1}{2}\,g\,H\,(\rho_l - \rho_v)\,r_1.$$

AMAGAT [1]) has determined the densities ρ_l and ρ_v till at a distance, from the critical temperature, of some tenths of a degree only. His critical point 31°,35, is not quite the same as mine; this however does not cause the least difficulty when we compare densities and capillary ascensions observed, not at the same absolute temperature, but at the same distance of the corresponding critical one; or, what is identical, at the same *reduced* temperature [2]).

CAILLETET and MATHIAS [3]) have given parabolic interpolation-formulae for the densities of liquids and saturated vapours; according to these formulae we should have

$$\rho_l - \rho_v = k\sqrt{1-m}.$$

And VAN DER WAALS [4]) has shown that this relation

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[1]) Journ. de Phys., 3e sér., 1, p. 297, 1892.

[2]) If τ represents the distance from the critical point, and m the reduced temperature, we have between τ and m the relation

$$1 - m = 1 - \frac{T}{T_c} = \frac{\tau}{T_c}$$

T_c being the absolute temperature: 273°,3 + 31°,35 as found by AMAGAT, 273°,3 + 31°,0 in my experiments. The values of m corresponding to a same value of τ, are thus for densities and ascensions nearly quite indentical.

[3]) Journ. de Phys., 2e sér., 5, 1886.

[4]) *Thermodynamical theory.* The very thing demonstrated by VAN DER WAALS reads thus

$$V_v - V_c = \alpha\,(1-m) + \beta\sqrt{1-m}$$
$$V_l - V_c = \alpha\,(1-m) - \beta\sqrt{1-m}$$

must be satisfied at least very near the critical temperature.

In order to submit this first theoretical result to experimental verification I have calculated, from AMAGAT's densities, the values of the quotient $\frac{\triangle log(\rho_l - \rho_v)}{\triangle log(1-m)}$, which, according to CAILLETET and MATHIAS must be constant and equal to 0,5, and would reach this value for $m = 1$, according to VAN DER WAALS.

$\tau = 0^0,1$	$1-m = 0,00033$	$\rho_l - \rho_a = 0,075$	$\frac{\triangle log(\rho_l - \rho_v)}{\triangle log(1-m)} = 0,521$
0°,35	0,00115	0,144	0,468
0°,85	0,00279	0,218	0,414
1°,35	0,0044	0,264	0,386
2°,35	0,0077	0,327	0,357
3°,35	0,0110	0,371	0,336
4°,35	0,0143	0,405	0,356
5°,35	0,0176	0,436	0,356
6°,35	0,0209	0,463	0,351
7°,35	0,0241	0,489	0,374
8°,35	0,0274	0,514	0,391
9°,35	0.0307	0,535	0,354
10°,35	0,0340	0,556	0,379
11°,35	0,0373	0,575	0,383

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and accordingly

$$V_v - V_l = 2\beta\sqrt{1-m};$$

but thence

$$\rho_v = \frac{1}{V_c + \alpha(1-m) + \beta\sqrt{1-m}} = \rho_c - \rho_c^2(\alpha - \rho_c\beta^2)(1-m) - \beta\rho_c^2\sqrt{1-m}$$

$$\rho_l = \frac{1}{V_c + \alpha(1-m) - \beta\sqrt{1-m}} = \rho_c - \rho_c^2(\alpha - \rho_c\beta^2)(1-m) + \beta\rho_c^2\sqrt{1-m}$$

so

$$\rho_l - \rho_v = 2\beta\rho_c^2\sqrt{1-m}.$$

Column 4 shows immediately that the formula of CAILLETET and MATHIAS does not well represent the experiments, for carbonic acid at least. Till 1° from the critical temperature the quotient $\frac{d log(\rho_l - \rho_v)}{d log(1-m)}$ remains sensibly constant, mean value = 0,367, and the experiments can therefore be represented well by

$$\rho_l - \rho_v = A(1-m)^{0,367}$$

A being a constant.

From $\tau = \pm 1^0$ begins a sensible increase of the quotient $\frac{d log(\rho_l - \rho_v)}{d log(1-m)}$, as required by the theory of VAN DER WAALS, and the last value 0,521 comes very near the theoretical one.

Let us now calculate the surface energy; the H has been deduced by graphical interpolation from my observations.

$1-m = 0,00033$	$H = 0,13$ mM.	$\sigma = 0,0021$	$\frac{\triangle log\,\sigma}{\triangle log(1-m)} = 1,512$
0,00115	0,45	0,014	1,413
0,00279	1,04	0,049	1,248
0,0044	1,53	0,087	1,216
0,0077	2,42	0,171	1,187
0,0110	3,25	0,261	1,198
0,0143	4,07	0,357	1,242
0,0176	4,89	0,461	1,256
0,0209	5,71	0,572	1,291
0,0241	6,53	0,691	1,290
0,0274	7,35	0,817	1,318
0,0307	8,17	0,946	1,321
0,0340	8,99	1,082	1,341
0,0373	9,82	1,224	

We see that the quotient $\frac{d\log\sigma}{d\log(1-m)}$ decreases till $\tau = \pm 2^0$ and then increases again. The last value 1,512 agrees again with the theoretical limit value 1,5.

Critical remarks. — When we examine sharply the conclusions arrived at, we must confess that the agreement found between theory and experiments has indeed not yet been proved.

For when we consider this first result, that the value of $\frac{d\log(\rho_l-\rho_v)}{d\log(1-m)}$, obtained very near the critical temperature, is 0,521, while VAN DER WAALS deduced 0,5 from his equation of state, we see in it the expression of the fact that the density-curves have in the critical point a contact of the second order with their common tangent. The densities however used in our calculations, are not those observed by AMAGAT, but values deduced by graphical interpolation from the observed ones, in such a way that a curve was traced as near as possible to the observed points, and tangent to the ordinate of the critical point. And as we know that generally a curve has with every tangent a contact of the second order, we must confess that the obtained result, though it may really be contained in the experiments, is nevertheless a consequence of the manner of interpolating.

As for $\frac{d\log\sigma}{d\log(1-m)}$, here also we could almost assert that agreement between theory and experiments would be found.

$$\frac{d\log\sigma}{d\log(1-m)} = \frac{d\log H}{d\log(1-m)} + \frac{d\log(\rho_l-\rho_v)}{d\log(1-m)}.$$

We have already seen that the limit-value of the last

term is 0,5; and it appears from all experiments that the tangent to the H-curve in the critical point, may be represented by an equation $y = a(1-m)$. As now $\frac{d\log y}{d\log(1-m)} = 1$, we have also $\frac{d\log H}{d\log(1-m)} = 1$ at the critical temperature. So the limit-value 1,5 is a consequence of it.

This verification would therefore be satisfying only then when it was founded quite on observed densities; these observed values however I have nowhere found.

COMMUNICATIONS

FROM THE

PHYSICAL LABORATORY

AT THE

UNIVERSITY OF LEIDEN

BY

PROF. DR. H. KAMERLINGH ONNES,

Director of the Laboratory.

No. 29.

(REPRINT).

Dr. P. ZEEMAN. Measurements concerning the influence of a magnetization, perpendicular to the plane of incidence on the light reflected from an iron mirror.

(*Translated from: Verslagen der Afdeeling Natuurkunde der Kon. Akad. van Wetenschappen te Amsterdam van* **27 Juni 1896**, p. 103—110.)

EDUARD IJDO - PRINTER - LEIDEN.

Dr. P. ZEEMAN. *Measurements concerning the influence of a magnetization, perpendicular to the plane of incidence on the light reflected from an iron mirror.*

1. Dr. C. H. WIND in his paper on KERR'S phenomenon has derived from theory the very interesting and unexpected result, that also a magnetization perpendicular to the plane of incidence must influence the light reflected from a magnetized mirror. This influence however is only exercised, if the incident light is polarized perpendicular to the plane of incidence. The phenomenon may be described in this manner: in the case mentioned and with a magnetization, perpendicular to the plane of incidence, a magnetical component is originated also perpendicular to the plane of incidence. Mr. WIND calculated magnitude and sign of the action to be expected and hence numerically has described also the new phenomenon.

In the academical report concerning Mr. WIND'S paper it is mentioned that a non-occurrence of the phenomenon would make necessary a thorough revision of the theory of magneto-optic phenomena. Therefore it was of a particular interest to test Dr. WIND'S pre-

diction. I have done this by means of BABINET'S compensator. It appears from the calculations kindly furnished me by Mr. WIND (his paper not yet being published) that the phenomenon is so exceedingly small, that it may easily remain unobserved.

The amount of the variations of the re-established azimuth and difference of phase is of the same order of magnitude as the errors of measurement. It is only with the utmost care and from long series of measurements that one may hope to detect the effect.

I have made 2 complete series (series I and II) of measurements by means of BABINET'S compensator. In both series are measured the variation of the difference of phase and that of the re-established azimuth with reversal of magnetization.

In taking these measurements, I was not subjected to the well-known unconscious temptation to see the thing we desire to see (extremely dangerous when such small quantities were to be measured). After having determined sign and amount of the variations in both series in phase and azimuth I perceived from the results then sent to me by Mr. WIND, that the sign and amount of the variations were in good agreement with theory. Also as to the expectations, both series are wholly independent of each other. The disposition of series II sufficiently differs from that of series I to make it difficult, without further considerations, to foresee the result to be expected. This consideration concerning the relation of the results of series I and II was made only after their termination.

2. *Method.* It has already been said that the measurements were made by means of BABINET'S compensator. In making the observations, the precautions were taken and the auxiliary apparatus used, which have been described on various occasions [1]). The light used was of the mean refrangibility of the sodium lines. Using this light about 14.3 revolutions of the head of the screw correspond to a phase-difference of half a wavelength. The head has been divided into 50 parts. The mirror (see 3) was adjusted for an angle of incidence $i=75^0$. The polarizer was placed in an azimuth of 45^0 and in the 4 possible positions. The observations were made with each of the 4×2 possible positions of the analyser. The determination of the variation of the phase-difference was made in the following manner. The black band in the compensator was made as dark as possible by turning the analyser. With positive and negative magnetizations the band was brought as accurately as possible between the wires, the analyser remaining in the same position. The determination of the variation of the azimuth was also observed in the 8 positions of the analyser, the position of the compensator now being unchanged. With the successive alternately directed magnetizations, the central part of the black band in the compensator was made as dark as possible, by turning the analyser. The position of the analyser commonly is read on a graduated circle, fixed

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[1]) SISSINGH, Dissertation 1885.
„ Archiv. Néerland. T. 20.
ZEEMAN. Archiv. Néerland. T. 27, p. 259, 1893.

to the analyser. I have however considerably increased the accuracy of this reading, by determining with a mirror and a vertical scale the angles over which the analyser is turned.

3. *Mirror.* Dr. SISSINGH used in his investigation of aequatorial reflexion, mirrors ground on iron rings. I have now used one of these rings, preserved since that time under a clock with chloride of calcium. The length of the mirror is 28 m.M. and the breadth of the middle part 2.8 mM. The ring was easily placed in a vertical plane, being fastened on a wooden board, which itself was clamped to the plate of copper [1]), used in my investigation of polar reflexion to support the magnet. The copperplate being fastened to an adjustable platform, it was also possible to put the mirror accurately into the correct position.

4. *Arrangement of the observations.* In order to give a clear survey of the measurements, I will give a complete set of observations in one position of the analyser. At the same time it will be possible to consider the degree of accuracy obtained. In one position of the analyser 12 observations were always taken. In the table below, the readings on the head of the compensator screw are entered. The magnetization is called positive for lines of force going vertically upwards.

[1]) ZEEMAN, l. c. p. 258.

Reading polarizer 173.7, Reading analyser 246.
Readings compensator (position 45,..)

—	+ magnetization	difference
36	34	— 2 parts of screwhead
33	33	+ 0
33	30	— 3
34	32	— 2
39	34	— 5
36	34	— 2
35	35	— 0
35	37	+ 2
39	37	— 2
37	36	— 1
42	38	— 4
34	35	+ 1

mean — 1.5 (0.5)

The mean error of the mean is entered in brackets.

The following table may serve as an instance of the observation relating to the variation of the re-established azimuth The figures are the readings on the vertical scale; 1′ corresponds to 1.4 divisions of the scale.

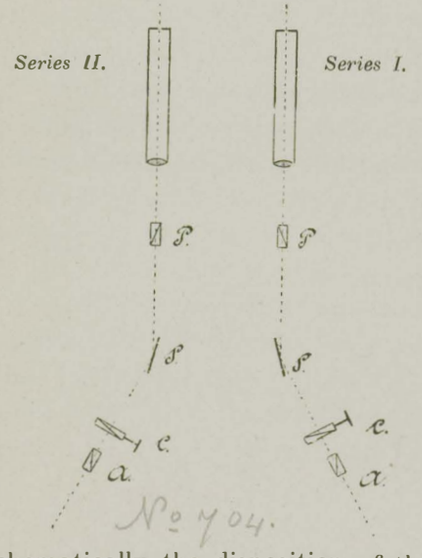
Reading polarizer 353.7, Reading analyser 246.
Readings analyser.

—	+ magnetization	difference.
262	238	—24 divisions of scale.
265	259	— 6
243	226	—17
201	193	— 8
241	185	—56
249	218	—31
250	195	—55
228	214	—14
218	204	—14
268	294	+26
269	204	—65
166	235	+69

mean. —16 div. = 12′ (8′).

Also here the mean error is entered in brackets. It follows from the now given figures that the mean error of one determination of the analyser amounts to about 28′, of one determination of the compensator to about 1.8 [1]) divisions of the screwhead. The expected theoretical variations (with reversal of magnetization) are about 14′, viz. 2.6 divisions of the head, hence of the order of magnitude of the errors of measurement.

The position of the compensator was in both series that for which the difference of phase is rendered half a wave length.



5. *The two series of observations.* The two series taken in order to secure mutual control (see 1) differ from each other by the direction of the normal to the mirror. In Series I it was directed to the S. W., in Series II to the S. E. The joined figure represents, as seen from above, schematically the disposition of the apparatus, a further

[1]) Perchance this value is very small, the mean is 3.1 divisions of the screwhead.

elucidation of which seems unnecessary. However it should be observed that P is the polarizer, S the mirror, C the compensator and A the analyser.

6. *Results of Series I.* Polarizer and analyser have graduated circles divided in degrees. Rotations are negative if in the same direction as the motion of the hands of a watch for an observer placed in the mirror. Negative rotations gave lower readings on the divided circles. The magnetization is positive if the lines of force run vertically upwards. The readings 128.7 and 308.7 on the circle of the polarizer correspond to incident light polarized parallel to the plane of incidence, the readings 38.7 and 218.7 to light polarized perpendicular to that plane. The light emergent from the polarizer, is quenched at the positions 128.7 and 308.7, the position of the analyser being given by 93 and 273, to the other two positions of the polarizer correspond the analyser positions 3 and 183. During the experiments the polarizer was placed in the azimuth 45°, corresponding to the readings 83.7, 173.7, 263.7, 353.7.

The difference of phase produced by the compensator is 0 for the reading 38.78 of the index which is fixed to the movable plate of the compensator. The difference in phase was $+\frac{\lambda}{2}$ and $-\frac{\lambda}{2}$ for the readings 53.08 and 24.48 During the measurements the reading of the index was about 45,..... Rotations of the head (divided in 50 parts) in the direction of the higher readings, gave also higher readings on the index-scale.

The results of the measurements concerning the re-established azimuth are entered in the following table;

the rotations are given in minutes. Each separate number is the result of a set of twelve observations. The angle of incidence was 75°. The analyser was approximately in the position given by the cipher behind A.

Position I. Variation of the re-established azimuth with + magnetization.

Pol. 83.7	A. 120	+	9′(5′)
	300	+	2(5)
Pol. 173.7	A. 66	—	11(7)
	246	—	28(11)
Pol. 263.7	A. 300	+	5(8)
	120	+	15(6)
Pol. 353.7	A. 246	—	5(9)
	66	—	12(3)
		mean	10.9′

With + magnetization the compensator was to be displaced to higher readings. In this case I give the positive sign to the variations of the phase (double variations) given in divisions of the head. The results are entered in the following table. Also the mean error of each series of 12 observations is given.

Position I. Variation of the phase with + magnetization.

P. 83.7	A. 300	+ 4.4(1.0)
	120	+ 0.9(1.5)
P. 173.7	A. 66	+ 0.4(1.1)
	246	+ 1.7(1.3)
P. 263.7	A. 300	+ 2.6(0.5)
	120	+ 1.1(0.7)
P. 353.7	A. 246	+ 1.1(0.5)
	66	— 0.2(1.0)
	mean	+ 1.5

Hence from the 2 tables it follows: with + magnetization the re-established azimuth increases, the difference of phase diminishes.

7. *Results of Series II.* The results of the measurements in Position II I have entered in the following table.

Position II. — Variation of the re-established azimuth with + magnetization.

P. 83.7	A. 120	—	14′(9′)
	300	—	7(7)
P. 173.7	A. 66	+	20(10)
	246	+	1(6)
P. 263.7	A. 300	—	12(6)
	120	—	10(10)
P. 353.7	A. 246	+	12(8)
	66	+	16(8)
		mean	11.2′

Position II. Variation of the phase with + magnetization.

P. 83.7	A. 120	— 0.3(1.0)
	300	+ 0.5(0.5)
P. 173.7	A. 246	— 1.5(0.5)
	66	+ 0.8(0.6)
P. 263.7	A. 300	+ 0.6(0.5)
	120	— 1.8(0.7)
P. 353.7	A. 66	— 3.4(0.8)
	246	— 1.8(0.8)
	mean	— 0.8

Hence it follows: In Series II with + magnetization the re-established azimuth diminishes, the difference of phase increases.

8. *Agreement of Series I and II.* If there exists a phenomenon causing in Series I the then observed phenomena, it must manifest itself in series II in the manner given in (7). If in the first mentioned case with a positive magnetization the re-established azimuth increases, it must be diminished in the second case. Prof. LORENTZ was so kind as to point out to me how this is most easily seen in applying the theorem of reciprocity.

9. *Accuracy of the observations.* Now it is still the question what is the value to be attributed to the results of the two series, viz. what is the mean error of the final result? Dr. E. F. VAN DE SANDE BAKHUIJZEN kindly informed me of the best manner of calculation in this case. The manner in which the 2 series are to be combined follows from (8). Taking this into account I have found the result, the disposition being as in series II, that the diminution (double) of the phase is + 1.14 divisions of the head, the mean error being 0.38 divisions. For the final result of the increment (double) of the re-established azimuth I find 11.2′, the mean error being 1.9′ [1]). (The mean error being calculated from the degree of agreement of the 16 separate results.)

10. *Result.* Mr. WIND'S theoretical result is:

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[1]) The accuracy of the phase determinations is given by SISSINGH to be $0.005 \frac{\lambda}{4} = 1.8$ divisions of the head and that of the determination of the re-established azimuth to 0.1°. Hence since his investigation one has succeeded in increasing not unconsiderably the precision of the observations.

Angle of incidence.	*Magnetization.*	*Difference of phase*	*Re-established azimuth.*
75°	+ 1400 C. G. S.	$0.004 \times \frac{\lambda}{4} = 1.4$	
71°	+ 1400 C. G. S.		8.5′

Taking into account that in my experiments the magnetization was somewhat above 1100 C. G. S. and paying attention to the well known result, that a real error equal to 3 or 4 times the mean error is not at all so improbable as is stated in the calculus of probabilities, one finds this result: The general conclusion to be drawn from the observations, is that a variation of the re-established azimuth and of the phase occurs, agreeing with the magnetic component polarized perpendicular to the plane of incidence predicted by Mr. WIND.

Herewith the phenomenon is, at least qualitatively, sufficiently confirmed. Hence there is no reason in this case for constructing a theory of the KERR-phenomenon on an entirely new basis.

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PROF. DR. H. KAMERLINGH ONNES.
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No. 30.

(REPRINT).

Dr. P. ZEEMAN. Measurements concerning the Variation of the Absorption of Electrical Vibrations with the Wave length and with the Concentration of the Electrolyte (*with 3 diagrams*).
(Translated from: Verslagen der Afdeeling natuurkunde der Kon. Akademie te Amsterdam van **26 September 1896**, p. 133—140.)

EDUARD IJDO — PRINTER — LEIDEN.

Dr. P. **ZEEMAN**. *Measurements concerning the Variation of the Absorption of Electrical Vibrations with the Wave length and with the Concentration of the Electrolyte* (with 3 diagrams).

1. On a previous occasion [1]) was communicated to the Academy a determination, the first ever made, of the absorption-coefficient of electrical vibrations travelling through an electrolyte. The wave length of the vibrations used was 6.6 M. [2]) in air, and the conductivity of the solution 3340.10^{-10} that of mercury. In continuing this inquiry, I wished to investigate what function the absorption is of the concentration of the solution and of the wave length of the incident vibrations. I have now measured the absorption coefficient for two different vibrations travelling through solutions the conductivity of which lies between the limits 3500.10^{-10} and 40000.10^{-10}. I take the liberty to offer in this paper the results of these measurements.

2. *Apparatus.* Except some small changes of inferior

[1]) ZEEMAN, Communications n°. 22.

[2]) I have given in my former communications 6.5 M. However 6.6 is more accurate.

importance, the apparatus used were the same as used with the former experiments and were then described.

3. *The double-wire circuit.* With the waves of 6.6 M. also the circuit, transferring the vibrations of the oscillator to the basin was the same as the one formerly used. That circuit having run through a corridor returned with a great curve into the room where the apparatus are placed. Using this circuit with waves greater than 8 M. perturbations appeared of the kind examined by v. GEITLER [1]), manifesting themselves in my experiments in irregularities of BJERKNES' curve of interference. I was therefore obliged to construct a new circuit. A first endeavour for that purpose failed. The circuit was made with much care in the garden of the laboratory, returning with a great arch parallel to the original direction in the room and continuing in a corridor with a second arch horizontally in a direction perpendicular to the original one. The horizontal part serves for the displacement of the bridge in order to get the interference-curve. One end of the horizontal part was relatively near to the vibrator. The two variations of the direction of the circuit and the last named circumstance were probably the reason that also on this occasion no satisfactory results were obtained. I obtained these only after disposing the oscillator in a separate building, the circuit now running in *one* straight line. The 2 parallel wires are continued straight forward in a horizontal plane to the extreme place intended for the bridge in the

[1]) v. GEITLER. Wied. Ann. Bd. 49. p. 184. 1893.

measurement of the curve of interference. The then following part of the double circuit, only destined for amortizing the waves passing the bridge in the mentioned measurement, naturally need not be arranged with so much care.

4. The observations concerning the absorption were made with the basin used on former occasions and also the method of measurement is the same. The diminution of the energy of the vibrations in the electrolyte was always determined by moving along the two parallel wires in the interior of the fluid, the two little Leyden jars, transferring the energy to the bolometer. The distance over which the jars are moved is entered in cM. in the following tables.

The final result of one series generally depends upon 3×4, sometimes upon 4×4 separate sets of measurements.

I have called coefficient of absorption the value of p in the expression Ae^{-2pz}, A being the incident energy, z the length of the traversed layer.

5. *Measurements with the wave of 6.6 M.* (logarithmic decrement $\gamma = 0.34$); diameter of the wire 0.70 mM. The diminution of the energy in the fluid is given by »observed deflection." Under »calculated deflection" have been entered the values following from the exponential function, by means of the value of p representing as well as possible the observations. That value of p is entered at the bottom of the table. In the fourth column the differences between the observed and calculated values are given.

$\lambda = 3480.10^{-10}$				$\lambda = 8100.10^{-10}$		
Traversed layer	Observed Deflection.	Calculated Deflection.	Difference.	Observed Deflection.	Calculated Deflection.	Difference.
0	46.3	46.3	0.0	35.0	35.0	0.0
2.5	28.5	29.3	− 0.8	14.7	15.3	− 0.6
5	17.7	18.5	− 0.8	6.8	6.7	+ 0.1
7.5	11.7	11.7	0.0	2.3	3.0	− 0.7
10	8.5	7.4	+ 1.1	0.7	1.3	− 0.4
15	4.5	3.0	+ 1.5			
20	1.6	1.1	+ 0.5	0.3	0	+ 0.3
47	0	0	0.0	0	0	0
$p =$	0.091			=	0.165	

$\lambda = 14600.10^{-10}$				$\lambda = 28000.10^{-10}$		
Traversed layer	Observed Deflection.	Calculated Deflection.	Difference.	Observed Deflection.	Calculated Deflection.	Difference.
0	22.6	22.6	0.0	13.9	13.9	0.0
1	14.3	14.3	0.0	7.8	7.6	+ 0.2
2	8.1	8.9	− 0.8	4.2	4.2	0.0
3	4.8	5.6	− 0.8	1.9	2.3	− 0.4
4	3.6	3.5	+ 0.1			
6	0.9	1.4	− 0.5			
10	0.1	0.2	− 0.1	0.5	0.6	− 0.1
30	0	0	0	0	0.0	0
$p =$	0.231			=	0.300	

6. *Measurements with the wave of 11.8 M.* [1]) ($\gamma = 0.38$) diameter of wire 0.83 mM.

$\lambda = 11400.10^{-10}$				$\lambda = 16000.10^{-10}$		
Traversed layer	Observed Deflection.	Calculated Deflection.	Difference.	Observed Deflection.	Calculated Deflection.	Difference.
0	43.0	43.0	0.0	33.0	33.0	0 0
1	34.5	32.8	+ 1.7	22.5	23.6	− 1.1
2	25.5	25.1	+ 0.4	19.0	16.8	+ 2.2
3	19.7	19.1	+ 0.6	11.5	12.0	− 0.5
6	7.2	8.5	− 1.3	4.5	4.4	+ 0.1
9	3.0	3.8	− 0.8	3.5	1.6	+ 1.9
19	0.3	0 3	0.0	0	0.1	− 0.1
$p =$	0.135			=	0.170	

$\lambda = 20600.10^{-10}$				$\lambda = 29800.10^{-10}$		
Traversed layer	Observed Deflection.	Calculated Deflection	Difference.	Observed Deflection.	Calculated Deflection.	Difference.
0	51.0	51.0	0.0	47.7	46.5	+ 1.1
1	33.7	34.0	− 0.3	28.0	28.7	− 0.7
2	22.7	22.7	0.0	18.0	17.7	+ 0.3
3	19.3	15.1	+ 4.2	14.3	10.9	+ 3.4
6	7.3	4.5	+ 2.8	5.7	2.6	+ 3.1
9	3.3	1.3	+ 2.0	1.4	0.6	+ 0.8
19	0	0	0	0	0	0
$p =$	0.200			=	0.240	

[1]) Determined according to BJERKNES' method. This value of the wavelength was still verified by calculating the frequency from the dimensions of the oscillator by means of the formula $2\pi\sqrt{L.C}$. The value of the inductance L I determined by means of a formula given by MASCART. The capacity C I measured directly.

$\lambda = 40000.10^{-10}$

Traversed layer	Observed Deflection.	Calculated Deflection.	Difference.
0	27.1	27.1	0.0
1	15.9	15.3	+ 0.6
2	7.9	8.7	− 0.8
3	4.9	4.9	0.0
6	1.5	0.9	+ 0.6
9	0	0.2	− 0.2
19	0	0	0

$p =$ 0.285

7. *Influence of galvanic resistance of the wires.* From the theory of the propagation of electrical waves it follows, that in some cases, e. g. with great resistance of the parallel wires, the electrical lines of force are no longer perpendicular to the wire surface. Herewith is intimately connected that in these cases part of the propagated energy is converted into heat and hence that then the measured coefficient of absorption no longer is that of the electrolyte. However in our case one may convince oneself by actual calculation that this source of error lies entirely between the limits of the errors of observation. Hence it is to be expected that in my experiments a change in the diameter of the wire does not interfere with the measured coefficient of absorption. In one case I have verified experimentally that the diameter of the wire has no perceptible

influence on the measured coefficient of absorption [1]). For that purpose the wires of diameter 0.70 mM in the basin and about 60 cM. at the head of the basin were substituted by thicker ones. The wave length was 6.6M. as given above and the conductivity $\lambda = 3800.10^{-10}$. The following was found:

Measurements with thick wires.

$\lambda = 3800.10^{-10}$

Traversed layer	Observed Deflection.	Calculated Deflection.	Difference.
0	56.8	57.2	− 0.4
2.5	37.2	36.0	+ 1.2
5	22.4	22.7	− 0.3
7.5	14.0	14.3	− 0.3
10	7.3	9.0	− 1.7
15	2.5	3.6	− 1.1
31	0.0	0.2	− 0.2
40	0.0	0.0	0

$p =$ 0.093

[1]) These measurements were already made when DRUDE published measurements (Berichte der Sächs. Gesellsch. d. Wiss. p. 318, 320 1896) whence it follows that the refractive index of electrical vibrations remains the same if the diameter of the wires is changed from 1 mM. to $^1/_2$ mM. (distance of the wires 18 mM.) Direct experiments concerning the influence of the diameter on the absorption however, as far as I know, have not been made by DRUDE.

From the graphical representation of the measurements with the same wave and *thin* wires (5) it follows that to a conductivity 3800.10^{-10} corresponds 0,096. Within the limits of the errors of measurement this is the same value as found above with *thick* wires. Hence the results arrived at (5,6) cannot contain a considerable systematical error, dependent on the diameter of the wire.

8. *Results.* Respecting a few of the results that may be derived from the observations, I have something more to say.

a. We ma yask: how does, the wave length being given, the coefficient of absorption change with the conductivity? It seems to me, with regard to theory, of some importance to see how far p is proportional to $\sqrt{\lambda}$. In the following tables the quotients $\frac{p}{10^3\sqrt{\lambda}}$ have been entered, as derived from the results of 5 en 6.

Wave length 6,6 M.

p	$\lambda . 10^{10}$	$\frac{p}{10^3.\sqrt{\lambda}}$
0.091	3480	0.154
0.165	8100	0.183
0.231	14600	0.191
0.300	28000	0.179

Wave length 11,8 M.

p	$\lambda . 10^{10}$	$\frac{p}{10^3.\sqrt{\lambda}}$
0.135	11400	0.126
0.170	16000	0.134
0.200	20600	0.139
0.240	29800	0.139
0.285	40000	0.142

The numbers of the last column in each of the tables, appear to be nearly constant with greater concentration and hence within the explored region *with given wave length, the coefficient of absorption is approximately proportional to the square root of the conductivity.*

The graphical representation of the observations is given in fig. I.

b. Another question is this: suppose that for a given wave length and given conductivity, the coefficient of absorption is known, how is with increasing wave length the conductivity to be changed, if the absorption is to remain unchanged? The answer to this question is to be found in the following table. From the diagram I for 6 different values of p the corresponding λ's have been taken and the quotients $\frac{\lambda 10^{10}}{l}$ evaluated for the two waves.

	Wave length 6,6 M.		Wave length 11,8 M.	
p	$\lambda . 10^{10}$	$\frac{\lambda}{l} . 10^{10}$	$\lambda . 10^{10}$	$\frac{\lambda}{l} . 10^{10}$
0.120	5000	758	10000	847
0.150	7000	1060	13000	1100
0.180	9300	1410	17400	1470
0.210	12200	1850	23200	1970
0.240	16200	2450	29800	2520
0.270	21600	3270	37400	3170

Taking into account the possible sources of error we have with greater concentration the law: *If the wave length is increased and in the same ratio the conductivity*

of the solution, the absorption remains unchanged. In a diagram with the coordinates l and λ the points belonging to the same p are nearly in a straight line through the origin, cf. fig. II.

c. The variation of the absorption of the energy with the conductivity appears from the graphical representation III. For $\lambda.10^{10} = 5000$ till $= 25000$ and $l = 6.6$ M. lines representing the intensity at different depth in the fluid are given.

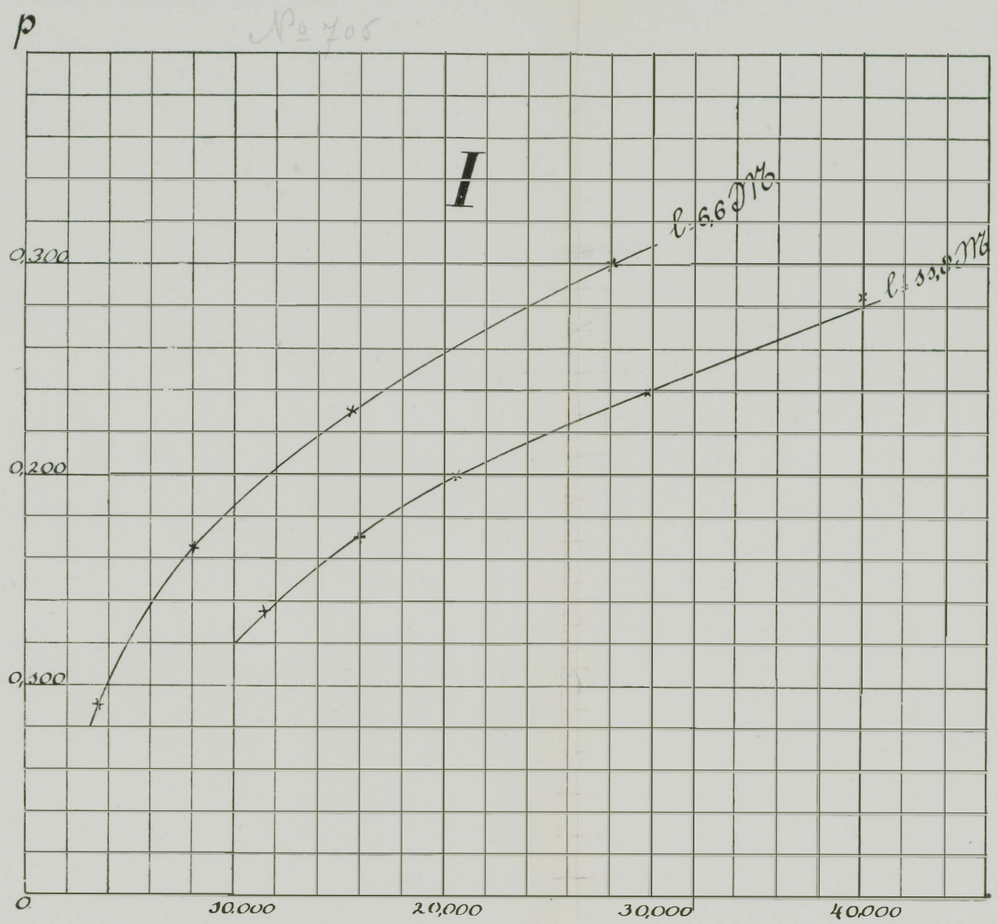
d. From *a* and *b* together it follows within the same limits, that with different wave length the intensity sinks at $^1/_e$ of the original value at distances proportional to the square root of the wave length.

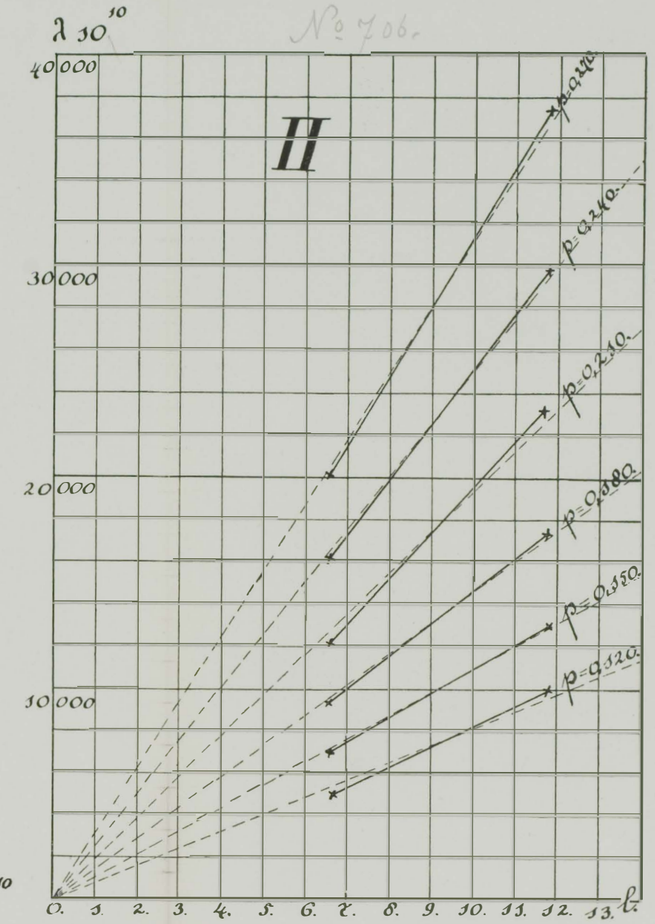
9. *Conclusion.* Though I have allowed myself to deduce some conclusions from my measurements, I think I must expressly declare that I believe it still necessary to confirm these conclusions by another method. For it must he conceded, that many perturbating circumstances can influence the measurements. Especially other electrical motions can have been superposed on the one defined by $l = 1.18, \gamma = 0.30$, and supposed by us to be quite alone. We may even go so far as to suppose that in the course of the experiments the superposed movements have changed. Perhaps even the differences between the last lines of the columns headed »calculated deflection" and »observed deflection" in 6 concerning $\lambda = 20000.10^{-10}$ and $\lambda = 29800.10^{-10}$ are to be ascribed to this cause. It is certainly remarkable that there »differences" attain a higher value than with higher and lower concentrations, though the differences *can* yet be attributed to accidental errors. The manner in which I intend to make

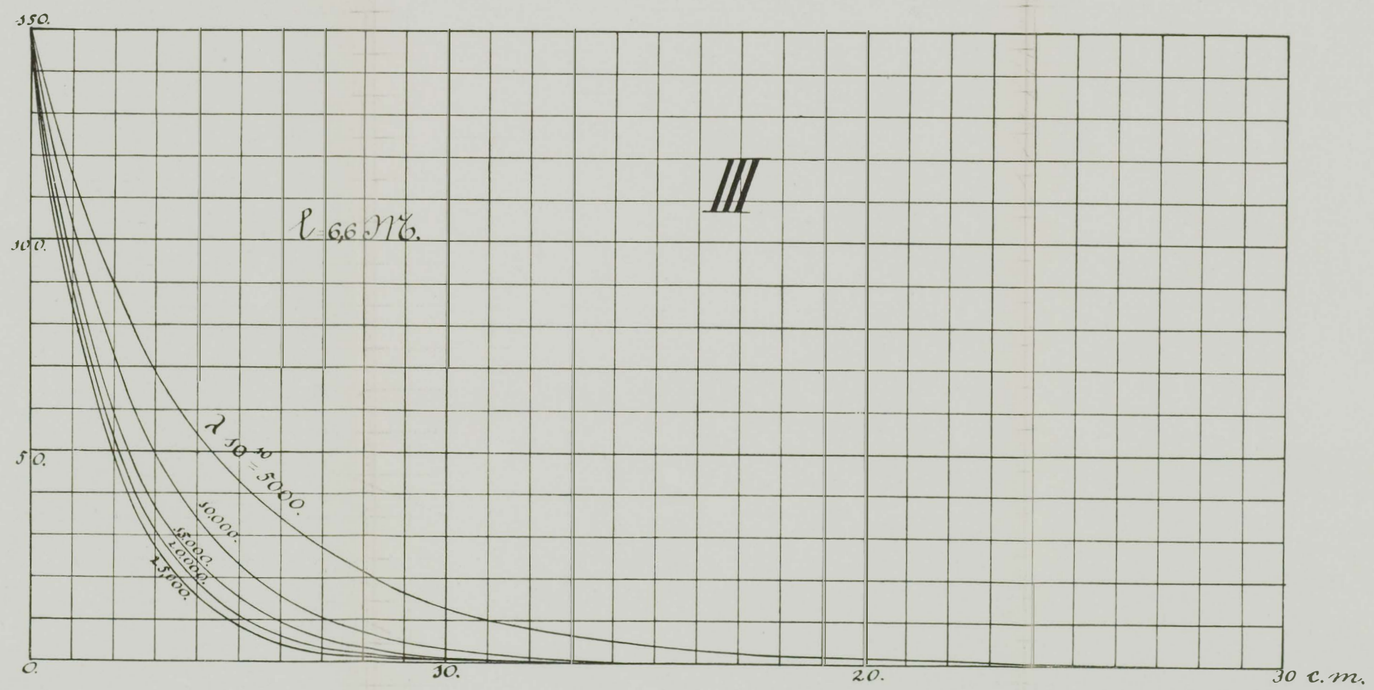
controlling experiments, I will explain in a few words.

The little Leyden jars with the bolometer are, we may say, an indifferent instrument. Independently of the frequency, *all* electrical waves are registered by the bolometer. Now we can interchange however the Leyden jars for something else. If we bring in the fluid a resonator isolated thereof and tuned to the frequency of the waves, of which the absorption is to be measured of course, it is quite another thing. The resonator will only be sensible for vibrations of the same period as the free ones. The intensity of the excited motion can of course be measured again by means of the bolometer and measures the operating forces. From some preliminary experiments I infer that it will be possible to make measurements with the sketched device. Future inquiry however must decide whether the results obtained in this manner admit of a tolerably simple interpretation.

Dr. P. ZEEMAN. Measurements Concerning the Variation of the Absorption of Electrical Vibrations with the Wave length and with the Concentration of the Electrolyte.







Zeeman.

Communication N° 30.

COMMUNICATIONS

FROM THE

PHYSICAL LABORATORY

AT THE

UNIVERSITY OF LEIDEN

BY

PROF. DR. H. KAMERLINGH ONNES,

Director of the Laboratory.

No. 31.

(REPRINT).

Dr. L. H. SIERTSEMA. A determination of the magnetic rotatory constant of water.

(*Translated from: Verslagen van de Afdeeling Natuurkunde der Kon. Akademie van Wetenschappen te Amsterdam*, **28 September 1896**, p. 131).

Dr. L. H. SIERTSEMA. Measurements on the magnetic rotatory dispersion in gases.

(*Translated from: the same p.* **132**).

EDUARD IJDO — PRINTER — LEIDEN.

Dr. L. H. SIERTSEMA. *A determination of the magnetic rotatory constant of water.*

In order to control the constant factors, used at the reduction of the measurements of magnetic rotation in gases to absolute units [1]), a measurement is made with the same apparatus on the absolute rotatory constant of water with *Na*-light. The experimental tube is filled with distilled and boiled water, and the magnetic rotation of *Na*-light is measured in the same way as with the other gases. Only a smaller current-intensity is used (7 amp.) and in consequence the shunt of the galvanometer was to be changed.

The ray *D*, which could not be pointed in the solar spectrum on account of bad weather, is made visible by throwing common salt on the carbons of the arc-lamp.

Four sets of measurements at a temperature of 13°.4 give for this constant:

0′.01303
1302
1302
1300

mean 0′.01302 (13°.4)

[1]) Communications etc. N°. 24.

In reducing it to 0° with the coefficient determined by RODGER and WATSON [1]), and comparing the result with those of other investigators, we find with a sufficient agreement:

ARONS [2])	0'.01298 (0°)
QUINCKE [3])	0'.01418 (0°)
RODGER and WATSON [1])	0'.01311 (0°)
SIERTSEMA	0'.01303 (0°)

[1]) RODGER and WATSON, Zeitschr. phys. Ch. 19, p.323.
[2]) ARONS, Wied. Ann. 24, p. 161 (1885).
[3]) QUINCKE, Wied. Ann. 24, p. 606 (1885).

Dr. L. H. SIERTSEMA. *Measurements on the magnetic rotatory dispersion in gases.*

In the former communications [1]) the magnetic rotations, found for some gases, are expressed in minutes for unity of magnetic potential-difference, by means of a reducing factor, calculated from the dimensions of the apparatus, the number of windings of the coils, and the constant of the tangent galvanometer with which the galvanometer of D'ARSONVAL is calibrated. This latter constant was deduced from comparisons with a copper-voltameter.

The accuracy of this reducing factor is controlled by a determination of the rotatory constant of water, which, as may be seen in the preceding pages, has showed a sufficient agreement with the values found by others.

Meanwhile it appeared necessary to add another correction to the manometer-readings, in consequence of which the following formulae are obtained:

Air (100 KG., 13°.2) $\quad n.10^6 = \frac{190.6}{\lambda}\left(1 + \frac{0.242}{\lambda^2}\right).$

Oxygen (100 KG., 7°.0) $\quad n.10^6 = \frac{271.7}{\lambda}\left(1 + \frac{0.0704}{\lambda^2}\right).$

Nitrogen (100 KG., 14°.0) $\quad n.10^6 = \frac{169.9}{\lambda}\left(1 + \frac{0.311}{\lambda^2}\right).$

[1]) Communications etc. N°. 24.

Carbonic acid (1 atm. 6°.5) $n.\,10^8 = \frac{269.5}{\lambda}\left(1 + \frac{0.307}{\lambda^2}\right)$.

Nitrogen monoxide (30.5 atm., 10°.9)

$$n\ 10^6 = \frac{75.50}{\lambda}\left(1 + \frac{0.306}{\lambda^2}\right).$$

Hydrogen (85.0 KG., 9°.5) $n.\,10^6 = \frac{138.6}{\lambda}\left(1 + \frac{0.325}{\lambda^2}\right)$.

In calculating the rotation of air at 13°.2 from that of *O* and *N*, we find:

$$n.\,10^6 = \frac{190.0}{\lambda}\left(1 + \frac{0.241}{\lambda^2}\right),$$

in close agreement with the formula which is derived from the direct measurements.

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No. 32.

(REPRINT.)

Dr. J. VERSCHAFFELT. On capillary ascension between two cylindrical tubes.

(*Translated from: Verslagen van de Afdeeling Natuurkunde der Kon. Akad. van Wetenschappen te Amsterdam*, **31 October 1896**, p. 175—181).

EDUARD IJDO — PRINTER — LEIDEN.

Dr. J. VERSCHAFFELT. *On capillary ascension between two cylindrical tubes.*

In a former paper I communicated measurements on capillary ascension of liquefied carbonic acid. The experiments were taken in this way: the capillary tube was put in the axis of a wider one, and I measured the vertical distance between the lowest point of the meniscus in the capillary, and the horizontal plane tangent to the meniscus in the annular space.

Suppose the wider tube to have been left open at its inferior end and put upright amidst an infinite liquid surface; in order to find the *real* ascension H in the capillary tube, to the *apparent* one h we must add the elevation h^1 in the annular space.

r_1 and r_2 being the interior and exterior radii of the capillary, and r_3 the interior radius of the wider tube, we usually put [1])

$$2\,\pi\,r_1\,\alpha = \pi\,r_1^2\,h$$

and

$$2\,\pi\,(r_3 + r_2)\,\alpha = \pi\,(r_3^2 - r_2^2)\,h^1$$

hence

$$\frac{h^1}{h} = \frac{r_1}{r_3 - r_2}.$$

We find these relations by supposing that the quantity of liquid that is raised against the walls, above the

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[1]) Vide a. o. WINKELMANN, Handb. d. Physik, 1891, Erster Bd. p. 460.

horizontal plane tangent to the meniscus, is much smaller than the whole raised mass; we come to the same result by admitting the meridian section of the meniscus to be circular. In a narrow cylindrical tube both suppositions are not far from truth; moreover we can take into account a correction that in most cases is quite sufficient.

In a rather wide annular space, on the contrary, none of these hypotheses may be regarded even as an approximation; for not only the ascension is small, accordingly the error, made by neglecting the liquid raised against the walls, relatively very great; but when gravitation became infinitely small, the limit shape of the meridian section of the surface would not even be a circle, but a portion of the well known curve called *nodoïde* by PLATEAU.

Missing all exact theoretical basis for calculating h^1, in a former communication I used a hypothesis already made by HAGEN [1]) in the case of cylindrical tubes: i. e. the meridian section of the surface would be nearly an ellipse, of which r would be the half of the great axis, and the arrow d of the meniscus the half of the small one. According to HAGEN such a supposition applied to cylindrical tubes gives results agreeing very well with experiments.

The same hypothesis applied to an annular meniscus gives

$$\text{half of great axis} = \frac{r_3 - r_2}{2}$$

$$\text{» small »} = d$$

[1]) POGG. Ann., 67, 1846, p. 126.

d being now the height to which the liquid is raised against the wider tube, above the horizontal tangent plane.

The general equation of the surface

$$\sigma \left(\frac{1}{R} + \frac{1}{R^1}\right) = g\,(s_l - s_v)\,y$$

becomes in the capillary tube

$$\frac{2\sigma}{r_1} = g\,(s_l - s_v)\,H = g\,(s_l - s_v)\,(h + h^1).$$

In the horizontal plane tangent to the annular meniscus the radial radius of curvature is the radius at the top of the ellipse: $\frac{1}{R} = \frac{d}{\left(\frac{r_3 - r_2}{2}\right)^2}$; the second radius, according to the theorem of MEUSNIER is infinite: $\frac{1}{R^1} = 0$. In consequence

$$\frac{\sigma\, d}{\left(\frac{r_3 - r_2}{2}\right)^2} = g\,(s_l - s_v)\,h^1$$

hence

$$\frac{h^1}{h} = \frac{2d}{\frac{(r_3 - r_2)^2}{r_1} - 2d}.$$

It seemed necessary to try this somewhat bold hypothesis by a comparison between calculation and experiment, in order to decide whether the correction found in this way may be trusted or not. This comparison could not be made directly, as the value of h^1 itself cannot be observed; the calculated correction however could be tested by experiments.

By putting the same capillary in tubes of different diameter, we must observe that the apparent ascension is higher in wider tubes, and the difference between the apparent ascensions in two tubes is equal to the difference between the corresponding corrections.

The *calculated* differences must therefore agree well with the *real* observed ones, when the applied correction should be trustworthy. The correction itself, of course, still remains unknown, for the correction corresponding to the widest tube cannot be determined; yet, when the calculated differences are almost equal to the observed ones, we may conclude that the calculated corrections also are not far from truth.

The experiments were taken with liquid methylchloride. I chose a liquefied gas instead of an ordinary liquid, because last remnants can easier be removed.

In order to eliminate errors coming from an imperfect cylindrical shape of the capillary tube, I carefully calibrated it, by measuring capillary ascensions at several heights, from 10 mM. to 10 mM. nearly. For, besides the correction made necessary by the annular meniscus — and this correction we may consider as constant over the whole length of the capillary, the arrow of the meniscus remaining quite the same at every height, — this ascension only depends from the radius at the place at which the meniscus in the capillary has been observed.

As I have been longer in doing that calibration than a day, the temperature of the water taken from the supply did not always remain constant; the variations however did not exceed some tenths of a degree; as

preliminary experiments gave me an approximate value of the coefficient of temperature, all ascensions could be reduced to the same mean temperature 18° C.

We always suppose the section of the capillary to be circular; this is not quite so; as it is however necessary that this section should be circular as nearly as possible, I chose a capillary with very small excentricity.

Two series of experiments were made, each ranging along a half capillary; the first series finished, the capillary was turned upside down. In the middle of it some experiments were made in both series, and the agreement was perfect. After each displacement of the meniscus I waited till it remained absolutely quiet; sometimes observations were taken after liquid had been distilled in, another time after liquid had been boiled out, and in both cases the ascension was the same when equilibrium was obtained.

Now the capillary tube was brought in tubes of different diameters and ascensions measured at several spots near the middle of the capillary. It appeared indeed that the apparent ascension decreased with r_3, while the difference of ascension between two tubes was the same at all levels.

All observations were reduced to the middle of the capillary and 18° C. I found

tube n°	r_3 =	h =	d =
I	10.4 mM.	47.46 mM.	1.98 mM.
II	7.45	47.45	1.90
III	5.05	46.99	1.78
IV	3.25	46.36	1.40
V	2.95	46.20	1.24

The calibration was made in tube n° III.

it, and cutting out the so obtained figure, the weight of which l compared to that of 1 dM² of the same paper upon which the sketch was made. In order to make errors as small as possible I determined the surface $\int_0^l (h-47)\, dl$, because these integral was nearly $= 0$; the positive and negative portions were placed on different sides of the balance. In this way I found $h_m = 46.99$ mM., what is by chance exactly the ascension in the middle of the capillary; r_m is therefore also the radius in the middle of the capillary.

Now we are able to calculate the hypothetical correction h^1, and find

$h^1_I = 0.152$ mM.			(observed)
$h^1_{II} = 0.289$	$h^1_{II} - h^1_I = 0.137$ mM.		0.11 mM.
$h^1_{III} = 0.625$	$h^1_{III} - h^1_I = 0.473$		0.47
$h^1_{IV} = 1.295$	$h^1_{IV} - h^1_I = 1.143$		1.10
$h^1_V = 1.478$	$h^1_V - h^1_I = 1.326$		1.26

We see that the calculated differences agree prettily well with the observed ones, and therefore the calculated corrections may be considered as a good approximation.

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BY

PROF. DR. H. KAMERLINGH ONNES,

Director of the Laboratory.

No. 33.

(REPRINT).

Dr. P. ZEEMAN. On the influence of magnetism on the nature of the light emitted by a substance. (Part I.)

(*Translated from: Verslagen van de Afdeeling Natuurkunde der Kon. Akademie van Wetenschappen te Amsterdam*, **31 October 1896**, p. 181).

Dr. P. ZEEMAN. On the influence of magnetism on the nature of the light emitted by a substance. (Part II.)

Translated from: the same **28 November 1896,** p. 242.)

EDUARD IJDO — PRINTER — LEIDEN.

Dr. P. ZEEMAN. *On the influence of magnetism on the nature of the light emitted by a substance.* (*Part I.*)

1. Several years ago, in the course of my measurements concerning the KERR-phenomenon, it occurred to me whether the light of a flame if submitted to the action of magnetism would perhaps undergo any change. The train of reasoning by which I attempted to illustrate to myself the possibility of this is of minor importance at present, at any rate I was induced thereby to try the experiment. With an extemporized apparatus the spectrum of a flame, coloured with sodium, placed between the poles of a RUHMKORFF electromagnet, was looked at. The result was negative. Probably I should not have tried this experiment again soon had not my attention been drawn some two years ago to the following quotation from MAXWELL'S sketch of FARADAY'S life.

Here (MAXWELL. Collected Works II. p. 760) we read: „Before we describe this result we may mention that in 1862 he made the relation between magnetism and light the subject of his very last experimental work. He endeavoured, but in vain, to detect any change in the lines of the spectrum of a flame when the flame was acted on by a powerful magnet." If a FARADAY thought of the possibility of the above mentioned relation,

perhaps it might yet be worth while to try the experiment again with the excellent auxiliaries of the spectroscopy of the present time, as I am not aware that it has been done by others. I will take the liberty stating briefly the results I have obtained up till now.

2. The electromagnet used was one made by RUHMKORFF and of middle type. The magnetizing current furnished by accumulators was in most of the cases 27 ampères and could be raised to 35 ampères. The light of the source of light used was analysed by a ROWLAND grating, with a radius of 10 ft. and with 14438 lines per inch. The first spectrum was used and observed with a micrometer eye-piece with a vertical cross wire. An accurately adjustable slit is placed near the source of light under the influence of magnetism.

3. Between the paraboloidical poles of an electromagnet, the middle part of the flame from a BUNSEN burner was placed. A piece of asbestos impregnated with common salt, was put in the flame in such a manner that the two D-lines were seen as narrow and sharply defined lines on the dark ground. The distance between the poles was about 7 m.m. If the current was put on, the two D-lines were distinctly widened. If the current was put off, they returned in their original condition. The appearing and disappearing of the widening was simultaneous with the putting on and off of the current. The experiment could be repeated an indifferent number of times.

4. The flame of the BUNSEN burner was interchanged with a flame of lightgas fed with oxygen. In the same manner as in 3 asbestos impregnated with common salt was introduced in the flame. It ascended

vertically between the poles. If the current was put on again the D-lines were widened, becoming perhaps 3 or 4 times their former width.

5. With the red line of lithium, used as carbonate, wholly analogous phenomena were observed.

6. Possibly one will regard the observed phenomenon (3, 4, 5) as nothing particular.

One may reason in this manner: widening of the lines of the spectrum of an incandescent vapour is caused by increasing the density of the radiating substance and by increasing the temperature [1]). Now under the influence of the magnet, the outline of the flame is undoubtedly changed (as is easily seen) hence the temperature and possibly also the density of the vapour is changed. Hence one may be inclined to account in this manner for the phenomenon.

7. Not so easily is another experiment explained. A tube of porcelain, glazed inside and outside, is placed horizontally between the poles with its axis perpendicular to the line joining the poles. The inner diameter of the tube is 18 mm., the outer one 22 mm. The length of the tube is 15 cm. Caps are screwed on at each end of the tube [2]); these caps are closed by a plate of parallel glass at one end, and are surrounded by little waterjackets. In this manner, by means of a current of water the copper caps and the glass plates may be kept sufficiently cool, if the tube of porcelain is rendered incandescent.

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[1]) Cf. however also PRINGSHEIM (Wied. Ann. 45 p. 457. 1892).

[2]) PRINGSHEIM uses in his investigation concerning the radiation of gases analogous tubes l. c. p. 430.

In the neighbourhood of the glass plates, side-tubes provided with taps are fastened to the copper caps. With a large BUNSEN burner the tube could be made incandescent over a distance of 8 cm. The light of an electric lamp, placed sideways at about two metres from the electromagnet in order to avoid disturbing action on the arc, was made to pass the tube by means of a metallic mirror. The spectrum of the arc was made by means of the grating. With the eyeglass the D-lines are focussed. This may be done very accurately, as in the centre of the bright D-lines the narrow reversed lines are seen. Now a piece of sodium was introduced into the tube. The BUNSEN flame is ignited and the temperature begins to raise. A coloured vapour soon begins to fill the tube, being at first of a violet, then of a blue and green colour and at last quite invisible to the naked eye. The absorption soon diminishes as the temperature is increased. The absorption is especially great in the neighbourhood of the D-lines. At last the two dark D-lines are visible. At that moment the poles of the electromagnet are pushed close to the tube, their distance now being about 24 m.m. The absorption lines now are rather sharp the greatest part of their length. At the top they are thicker, where the spectrum of the lower denser vapours was observed. Immediately after the closing of the current the lines *widen* and are seemingly *blacker*, if the current is put off they immediately recover their initial scharpness. The experiment could be repeated several times, till all the sodium had disappeared. The disappearing of the sodium is chiefly to be attributed to the chemical action of it on the glazing of the tube. For further

experiments therefore unglazed tubes were also used.

8. One will perhaps try to account for the last experiment (7) in this direction. It is true that the tube used at the top and at the bottom was not of the same temperature, further it appears from the shape of the D lines (7) that the density of the vapour of sodium is obviously different at different height. Hence certainly convection currents caused by differences of temperature between the top and bottom were present. Under certain suppositions one may calculate that by the putting on of the electromagnet, differences of pressure are originated in the tube of the same order of magnitude as those caused by the difference of temperature.

Hence the magnetization will push e. g. the denser layer at the bottom in the direction of the axis of the tube. The lines become widened. For their width at a certain height is chiefly determinated by the number of incandescent particles at that height in the direction of the axis of the tube. Although this explanation yet gives rise to some difficulties, certainly something may be said for it.

9. The explanation of the widening of the lines initiated in (8) is no more applicable to the following variation of the experiment, in which a un-glazed tube is used. The inner diameter of the tube, about 1 m.m. thick, was 10 m.m. The poles of the electromagnet could be moved till the distance was 14 m.m. The tube now was heated by means of the blow pipe instead of with the BUNSEN burner and became in the middle part red hot. The blow pipe and the smaller diameter of the tube make it easier to bring the upper and lower parts to the same temperature. This was now higher than

before (7) and the sodium lines remained visible continuously [1]). One now can wait till the density of the sodium vapour is the same at various heights.

By rotating the tube continuously round its axis, I have still further advanced this. The absorption lines now are equally broad from the top to the bottom. If the electromagnet was put on, the absorption lines immediately widened along their whole length. Now the explanation in the manner of (8) fails.

10. I should like to have studied the influence of magnetism on the spectrum of a solid. Oxide of erbium has, as was found by BUNSEN and BAHR the remarkable property of giving by incandescence a spectrum with bright lines. With the dispersion used however the edges of these lines were too indistinct to serve my purpose.

11. The different experiments from 3 to 9, make it more and more probable, that the absorption- and hence also the emission-lines of an incandescent vapour, are widened by the action of magnetism. Hence, if this is really the case, then by the action of magnetism in addition to the free vibrations of the atoms, which are the cause of the line spectrum, other vibrations of changed period appear.

I hope to decide by future investigation whether it is really inevitable to admit this specific action of magnetism.

1) PRINGSHEIM l. c. p. 456.

Dr. P. ZEEMAN. *On the influence of magnetism on the nature of the light emitted by a substance.* (*Part II.*)

12. From the representation I had formed to myself of the nature of the forces, acting in the magnetic field on the atoms, it seemed me to follow that with a band-spectrum and with external magnetic forces the phenomenon I had found with a line-spectrum would not occur.

It is however very probable that the difference between a band- and a line-spectrum is not of a quantitative but of a qualitative kind [1]). In the case of a band-spectrum the molecules are complicated, in the case of a line-spectrum the widely dissociated molecules contain but a few atoms. Further investigation has shown that the representation I had formed of the cause of the widening in the case of a line-spectrum, in the main was really true.

13. A glass tube closed at both ends by glass plates with parallel faces, was placed between the poles of the RUHMKORFF electromagnet in the same manner as the tube of porcelain in § 7. A small flame under the tube vapourized the iodine, the violet vapour filling the tube.

[1]) KAYSER in WINKELMANN's Handbuch II. 1. p. 421.

By means of electric light the absorption spectrum could be examined. As the temperature is low this is the band-spectrum. With the high dispersion used, one sees in the bands a very great number of fine dark lines. If the current round the magnet is being closed, one observes, contrary to what the experiments with sodium vapour teach, that *no* change in the dark lines is observed.

The absence of the phenomenon in this case supports the explanation, that even in the first experiment, with sodium vapour (§ 7) the convection currents have had no influence. For in the case now considered, the convection currents originated by magnetism, which I believed to be possible in that case, apparently are insufficient to cause a change of the spectrum, and though I could not see it in the appearance of the absorption lines (cf. § 7) also the band-spectrum is like the line-spectrum very sensible to changes of density and of temperature.

14. Although the means at my disposal did not enable me to execute more than a preliminary approximate measurement, I yet thought it of importance to determine approximately the value of the magnetic change of the period.

The widening of the sodium lines to both sides amounted to about $^1/_{40}$ of the distance of the said lines, the intensity of the magnetic field being about 10^4. Hence follows a positive and negative magnetic change of $^1/_{40000}$ of the period.

15. The train of reasoning, mentioned in (1) and by which I was induced to search after an influence of magnetism was at first the following. If the consideration is true that in a magnetic field a rotatory motion of the ether is going on, the axis of rotation being

the direction of the magnetic forces (KELVIN and MAXWELL) and if the radiation of light may be represented as caused by the motion of the atoms, relative to the centre of mass of the molecule, revolving in all kinds of orbits, suppose for simplicity circles, then the period or what comes to the same, the time of describing the circumference of these circles will be determined by the forces acting between the atoms and then deviations of the period to both sides will occur by the influence of the perturbating forces between ether and atoms. The sign of the deviation of course will be determined by the direction of motion, as seen from out the lines of force. The deviation will be the greater the more the plane of the circle approximates a position perpendicular to the lines of force.

16. Somewhat later I elucidated the subject by representing to myself the influence exercised on the period of a vibrating system if this is linked together with another in rapid rotatory motion. Lord KELVIN, now 40 years ago [1]), gave the solution of the following problem. Let the two ends of a cord of any length be attached to two points at the ends of a horizontal arm made to rotate round a vertical axis through its middle point at a constant angular velocity, and let a second cord bearing a material point be attached to the middle of the first cord. The motion now is investigated in the case, the point is infinitely little disturbed from its position of equilibrium. With great angular velocity the solution becomes rather simple. Circular vibrations of the point in contrary directions have slightly different

[1]) Proc. R. S. 1856.

periods. If for the double pendulum we substitute a lumineferous atom and for the rotating arm the rotational motion about the magnetic lines of force, the relation of the mechanical problem to our case will be clear.

It needs not to be proved that the above mentioned considerations are at most of any value as indications of somewhat analogous cases. I however communicate them because they were the first motive of my experiments.

17. A real explanation of the magnetic change of the period seemed me to follow from Prof. LORENTZ's theory [1]).

In this theory it is accepted that in all bodies, small electrically charged ponderable particles are present, that all electric phenomena are dependent upon the configuration and motion of these „ions" and that the light vibrations are vibrations of these ions. Then the charge, configuration and motion of the ions completely determine the state of the ether. The said ion, moving in a magnetic field, experiences mechanical forces of the kind as above mentioned and which must explain the variation of the period. Prof. LORENTZ to whom I communicated these considerations, at once kindly informed me of the manner, in which according to his theory the motion of an ion in a magnetic field is to be calculated, and pointed out to me that, if the explanation following from his theory was true, the edges of the lines of the spectrum ought to be circularly polarized. The amount of widening might then be used

[1]) LORENTZ. „La Théorie électromagnétique de MAXWELL". Leyde 1892 and „Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpern." Leiden, 1895.

to determine the ratio of charge and mass to be attributed in this theory to a particle giving out the vibrations of light.

The above mentioned extremely remarkable conclusion of Prof. LORENTZ relating to the state of polarization in the magnetically widened line, I have found to be fully confirmed by experiment (§ 20).

18. We shall now proceed to establish the equations of motion of a vibrating ion, when it is moving in the plane of (x, y) in a uniform magnetic field in which the magnetic force is everywhere parallel to the axis of z and equal to H. The axes are chosen so that if x is drawn to the east, y to the north, z is upwards. Let e be the charge (in electromagnetic measure) of the positively charged ion, m its mass. The equation of motions then are:

$$\left.\begin{aligned} m\frac{d^2x}{dt^2} &= -k^2x + eH\frac{dy}{dt} \\ m\frac{d^2y}{dt^2} &= -k^2y - eH\frac{dx}{dt} \end{aligned}\right\} \quad \ldots (1)\ [1])$$

The first term of the second member expresses the elastic force, drawing back the ion to its position of equilibrium, the second term gives the mechanical force due to the magnetic field.

This is satisfied by:

$$\left.\begin{aligned} x &= \alpha e^{st} \\ y &= \beta e^{st} \end{aligned}\right\} \quad \ldots (2)$$

provided

$$\left.\begin{aligned} ms^2\alpha &= -k^2\alpha + eHs\beta \\ ms^2\beta &= -k^2\beta - eHs\alpha \end{aligned}\right\} \quad \ldots (3)$$

[1]) The equations of relative motion.

Now m, k, e, are to be regarded as known quantities.

For us the period T is particularly interesting. If $H = o$, it follows from (3).

$$s = i\,\frac{k}{\sqrt{m}} = i\,\frac{2\pi}{T}$$

$$\text{or} \quad T = \frac{2\pi\sqrt{m}}{k} \quad . \quad . \quad . \quad (4)$$

If H is not o, it follows from (3) approximately

$$s = i\,\frac{k}{\sqrt{m}}\left(1 \pm \frac{eH}{2k\sqrt{m}}\right)$$

Putting T' for the period in this case, we have:

$$T' = \frac{k}{2\pi\sqrt{m}}\left(1 \pm \frac{eH}{2k\sqrt{m}}\right) \quad . \quad . \quad . \quad (5)$$

Hence the ratio of the change of the period to the original period becomes:

$$\frac{eH}{2k\sqrt{m}} = \frac{e}{m}\,\frac{HT}{4\pi} \quad . \quad . \quad . \quad (6)$$

A particular solution of (1) is that representing the motion of the ions in circles. If revolving in the positive direction (viz. in the direction of the hands of a watch for an observer standing at the side towards which the lines of force are running) the period is somewhat less then if revolving in the negative direction. The period in the first case is determined by the value of (5) with the minus sign, in the second with the plus.

The general solution of (1) proves that by the ions are described besides circles, also elliptical slowly rotating orbits. In the general case, the original motion of the ion having an arbitrary position in space, it is perfectly clear that the projection of the motion in the plane of (x, y) has the same character. The motion decomposed in the direction of the axis of z is a simple

harmonic motion, independent of and not disturbing the one in the plane of (x,y) and hence not influenced by the magnetic forces. Of course the consideration of the motion of an ion now given, is only to be regarded as the very first sketch of a theory of the luminiferous motions.

19. Imagine an observer looking at a flame placed in a magnetic field in a direction such, that the lines of force run towards or from him.

Let us suppose that the said observer could see the very ions of § 16, as they are revolving, then the following will be remarked. There are some ions moving in circles and hence emitting circularly polarized light, if the motion is round in the positive direction the period will for instance be longer than with no magnetic field, if in the negative direction shorter. There will also be ions seemingly stationary and really moving parallel to the lines of force with unaltered period. In the third place there are ions which seem to move in rotating elliptical orbits.

If one desires to know the state of the ether originated by the moving ions, one may use the following rule, deduced by Prof. LORENTZ from the general theory. Let us suppose that in a molecule an ion P—of which the position of equilibrium be P_0— has two or more motions *at the same time*, viz. let the vector P_0P always be obtained by adding the vectors P_0P which should occur in each of the composing motions at that moment, then the state in the ether, at a very great distance, in comparison with P_0P, will be obtained by superposing the states, which would occur in the mentioned distinct cases.

Hence it follows in the first place that a circular motion of an ion, gives circularly polarized light in points on the axis of the circle.

Further one may choose instead of the above considered elliptical orbits a resolution more fit to our purpose. One may resolve the motion of the ion, existing before the putting on of the magnetic force, in a rectilinear harmonic motion parallel to the axis of z and two circular (right-hand and left-handed) motions in the plane of (x,y).

The first remains unchanged under the influence of the magnetic force, the periods of the last are changed.

By the action of the grating the vibrations originated by the motion of the ions are sorted according to the period and hence the complete motion is broken up into three groups. The line will be a triplet. At any rate one may expect that the line of the spectrum will be wider than in the absence of the magnetic field and that the edges will give out circularly-polarized light[1]).

20. A confirmation of the last conclusion may be certainly taken as a confirmation of the guiding idea of Prof. LORENTZ's theory. To decide this point by experiment, the electromagnet of § 2, but now with pierced poles was placed so that the axes of the holes are in the same straight line with the centrum of the grating. The sodium lines were observed with an eye-

[1]) I saw afterwards that STONEY, Trans. Dublin, IV, endeavours to explain the existence of doublets and triplets in a spectrum by the rotation of the elliptical orbits of the „electrons" under the influence of perturbating forces.

piece with a vertical cross wire. Between the grating and the eye-piece were placed, the quarter undulation plate and Nicol, which I formerly used in my investigation of the light normally reflected from a polarly magnetized iron mirror [1]).

The plate and the Nicol were placed relatively in such a manner, that right-handed circularly-polarized light was quenched. Now according to the preceding the widened line must at one edge be right-handed circularly-polarized, at the other edge left-handed. By a rotation of the analyser over 90° the light that was first extinguished must be admitted and vice versâ. Or, if first the right edge of the line is visible in the apparatus, a reversal of the direction of the current makes the left edge visible. The cross wire of the eye-piece was set in the bright line. At the reversal of the current the visible line removed! This experiment could be repeated any number of times.

21. A small variation of the preceding experiment is the following. With unchanged position of the $\frac{\lambda}{4}$ plate the analyser is turned round. The widened line is then by one revolution twice wide and twice fine.

22. The electromagnet was turned 90° in a horizontal plane from the position of § 20, the lines of force now being perpendicular to the line joining the slit with the grating. The edges of the widened line now appeared to be plane polarized, at least in so far as the present apparatus permitted to see, the plane of polar-

[1]) ZEEMAN. Communications Nr. 15.

isation being perpendicular to the line of the spectrum. This phenomenon is at once evident from the consideration of § 19. The circular orbits of the ions being perpendicular to the lines of force are now seen on their edges.

23. The experiments 20 to 22 one may consider as a proof that the light vibrations are caused by the motion of ions, as introduced by Prof. LORENTZ in his theory of electricity. From the measured widening (§ 14) by means of relation (6), now the ratio $\frac{e}{m}$ may be deduced. It then appears that $\frac{e}{m}$ is of the order of magnitude 10^7. Of course this result from theory is only to be considered as a first approximation.

24. It may be deduced from the experiment of § 20 whether the positive or the negative ion revolves.

If the lines of force were running towards the grating, the right-handed circularly-polarized rays appeared to have a greater [1]) period. Hence in connection with § 17 it follows that the negative [1]) ions revolve or at least describe the greater orbit.

25. Especially now the magnetisation of the lines of a spectrum can be interpreted in the theory of Prof. LORENTZ, the further inquiry of it becomes very attractive. A series of further questions already present themselves. It seems very promising to investigate the

[1]) In this reprint we have corrected the erroneous statement in the first edition of this paper. This correction has been previously mentioned *Phil. Mag.* July '97 p. 59 *and Archives Néerl. d. Sc. exactes et nat.* Sér. II, T. I, p. 3[illegible], 1898.

motion of the ions for various substances, under varying circumstances of temperature and pressure, with varying intensities of the magnetisation. Further inquiry has also to decide in how far the strong magnetic forces, existing according to some, at the surface of the sun, may change its spectrum.

COMMUNICATIONS

FROM THE

PHYSICAL LABORATORY

AT THE

UNIVERSITY OF LEIDEN

BY

PROF. DR. H. KAMERLINGH ONNES,

Director of the Laboratory.

No. 34.

(REPRINT).

Dr. L. H. SIERTSEMA. On thermal coefficients of aneroids of Naudet.

(*Translated from: Verslagen van de Afdeeling Natuurkunde der Kon. Akad. van Wetenschappen te Amsterdam*, **28 November 1896**, p. 233—241).

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Dr. L. H. SIERTSEMA. *On thermal coefficients of aneroids of Naudet.*

§ 1. On the occasion of the determination of the thermal coefficients and index-errors of some aneroids, which I undertook for the sake of observations of altitudes by Prof. MARTIN in the Moluccas, and by Prof. MOLENGRAAFF and others in Borneo, I consulted the extensive literature on this subject. Little more than indications are found there of the causes to which the usually rather great thermal coefficients can be attributed. Three causes are enumerated:

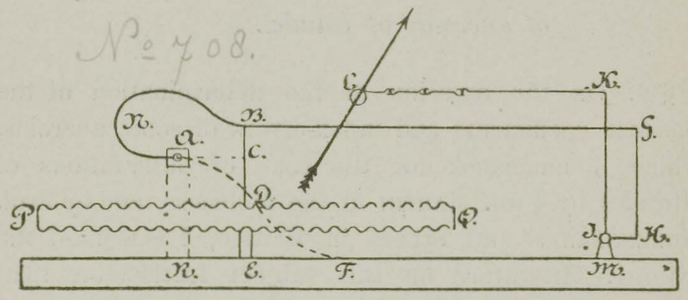
1°. Rise of temperature will cause expansion of the different parts of the instrument. Among others the surface of the vacuum-box will increase, and thus the atmospheric pressure will act more strongly.

2°. The coefficients of elasticity of the metals, of which the box and the spring are made, will decrease with rising temperature. The deflection therefore increases, and will cause a higher reading.

3°. The air, which still remains in the box, will expand by heating, and more counterbalance the exterior air-pressure by which an apparent decrease of the barometric pressure is caused.

A closer examination of the consequences of 1° will show us, that this cause explains but a small part of the thermal coefficient. For this purpose we will exa-

mine more in detail the construction of the instrument, in doing which we shall make use of the subjoined sketch of the aneroid. In this sketch PQ represents the vacuum-box, NB the spring, $BGHIK$ the system



of levers for the transmission of the deflexion of the spring to the index, KL the chain and L the axis of the index. We will now investigate the influence, which the expansion of the different parts of the instrument alone will have on the readings. The changes caused by 2^0 and 3^0 will not be taken into account here.

Let us begin with the box. When the rise of temperature is t, the surface of the box will increase in the proportion $1 + 2\,a\,t$, where a is the coefficient of expansion for the box (argentan). The total pressure on the box therefore increases in the same proportion. The force which acts in B in consequence of the air-pressure, will also increase in the proportion $1 + 2\,a\,t$. This force will be balanced by the elastic forces of the spring and the box. If for the sake of simplicity we neglect those of the box, which probably are small compared with those of the spring, and when we besides suppose the

spring wholly unchanged, the deflection s will increase in the proportion $1 + 2\,a\,t$. But the spring does expand, and in order to find the exact deflection, we should have to know the exact relation between the deflection, the elastic force and the dimensions of the spring. By analogy with other phenomena of deflection we may assume, that when all dimensions increase in the same proportion, the deflection will decrease in this proportion. If now b is the coëfficient of expansion for the spring (steel), the total deflection after the rise of temperature will be

$$s\,\frac{1 + 2\,a\,t}{1 + b\,t} = s\,\{1 + (2\,a - b)\,t\}$$

and the increase of the deflection

$$s\,(2\,a - b)\,t.$$

With this amount then is increased the deflection of the point B from the position which it would have if no force were applied. But this last position (zero) will change by the expansion in a way which can be found in the following manner. The brass piece AR will by the expansion rise $A\,R\,c\,t = C\,E\,c\,t$, where c is the coefficient of expansion for brass. For the same reason the point B will rise $C\,E\,c\,t + B\,C\,b\,t$, and we find for the total rise of B

$$-\,s\,(2\,a - b)\,t + C\,E\,c\,t + B\,C\,b\,t.$$

When we take N as the fixed point of the lever NBG, which point rises $C\,E\,c\,t + B\,C\,b\,t$, we find for the rise of G

$$-\,\frac{N\,G}{N\,B}\,s\,(2\,a - b)\,t + (C\,E\,c + B\,C\,b)\,t$$

Yet there is another cause by which G will move. Fixed to the spring, which can turn in A, we find a curved steel rod, connected with the bottom-plate in F by a screw. Now AF being made of steel, AR of brass, which metals expand in a different degree, the spring will turn in A. The sinking of G by this reason is easily calculated to be $CE\,(c-b)\,t\,\frac{AG}{RF}$. Adding this quantity to the rise found above, we obtain

$$-\frac{NG}{NB}s\,(2a-b)\,t-CE\,(c-b)\,\frac{AG}{RF}t+(CEc+BCb)\,t.$$

The expansion of BG has no influence on the reading, because the levers in G and H are connected with turning joints. Of course we leave out of consideration the case of the lever BG bending on expanding, which arrangement is used for the compensation of the thermal coefficient. GH is made of steel, and therefore the rise of H will be

$$-\frac{NG}{NB}s\,(2a-b)\,t-CE\,(c-b)\,\frac{AG}{RF}t+(CEc+BCb)\,t-GHbt.$$

The axis I, connected with the bottomplate by brass supports, will rise $MIct$, so that the angular motion at I amounts to

$$\frac{1}{HI}\Big(-\frac{NG}{NB}s\,(2a-b)-CE\,(c-b)\,\frac{AG}{RF}+$$
$$+CEc+BCb-GHb-MIc\Big)\,t$$

and the displacement of K to the left becomes

$$\frac{KI}{HI}\Big(-\frac{NG}{NB}s\,(2a-b)-CE\,(c-b)\,\frac{AG}{RF}+$$
$$+(CE-MI)\,c+(BC-GH)\,b\Big)t.$$

The expansion of the steel chain, and the brass arm, by which the taps of the axis L are connected with I, will cause another alteration in the displacement, which for a point of the chain L reaches the value

$$\frac{KI}{HI}\Big(-\frac{NG}{NB}s\,(2a-b)-CE\,(c-b)\,\frac{AG}{RF}+$$
$$+(CE-MI)\,c+(BC-GH)\,b\Big)\,t-KL\,(c-b)\,t.$$

In order to find from this the variation of the reading, we will determine the consequences of a variation ∂A of the barometric pressure A. The deflection s will increase with $\frac{s}{A}\,\partial A$, which consequently will be the displacement of B. For that of KL we find $\frac{s}{A}\cdot\frac{NG}{NB}\cdot\frac{KI}{HI}$ ∂A. From this we deduce, that a rise of temperature t corresponds to an apparent fall of barometric pressure

$$\frac{\frac{KI}{HI}\Big(-\frac{NG}{NB}s(2a-b)-CE(c-b)\,\frac{AG}{FR}+(CE-MI)c+(BC-GH)b\Big)-LK(c-b)}{\frac{s}{A}\cdot\frac{NG}{NB}\cdot\frac{KI}{HI}}\,t$$

or

$$\Big(-A\,(2a-b)+\frac{A}{s}\frac{NB}{NG}\Big\{-CE\frac{AG}{RF}\,(c-b)+$$
$$+(CE-MI)\,c+(BC-GH)\,b-KL.\frac{HI}{KI}\,(c-b)\Big\}\Big)t.$$

The coefficient of t represents the thermal coefficient.

In order to obtain an estimate of the value of this coefficient, a rough measuring is performed on an aneroid. When we assume in connection with this measuring

$$a=0.000018,\ b=0.000011,\ c=0.000019,\ A=760,$$

$\frac{A}{s} = 200$ [1]), $CE = 16$. $\frac{AG}{RF} = \frac{60}{28} = 2.14$, $BC = 6$, $GH = 19$,

$MI = 6.5$ $\frac{NB}{NG} = \frac{23}{83} = 0.28$, $KL = 43$, $\frac{HI}{KI} = \frac{6}{23} = 0.26$,

we find for the thermal coefficient -0.037.

Ordinarily the thermal coefficients are much larger in absolute value. This result therefore confirms the assertion that the expansion of the instrument is not one of the principal causes of the thermal coefficient.

§ 2. The two other above named causes for variations by heating are better adapted to explain the thermal coefficient. Often has attention been directed to both of them; the last has been used to compensate the thermal coefficient.

From these causes we can easily deduce an approximate value for the thermal coefficient. In doing this we shall, for the sake of simplicity, and on account of the preceding results, neglect the expansion of the different metallic parts.

Let A be the barometric pressure,

p the pressure of the air, which still remains in the box, measured at 0°,

α the coefficient of expansion for air.

The effect of the atmospheric pressure on the box at $t°$ will be that of a force, applied in the centre, amounting to

$$\{A - p(1 + \alpha t)\} c$$

[1]) See REINHERTZ, Zeitschr. f. Instrumentenk. VII, p. 157 (1887), where for the deflection of the box is given in the mean 0.005 mM. for a variation of barometric pressure of 1 mM.

where c depends on the shape and the dimensions of the box.

With this force balance the elastic forces, which are excited by the deflection f at the centre of the box, and by an equal deflection of the spring.

E_1 being the coefficient of elasticity for the metal of the box at 0°, we can represent the force excited by a deflection f at 0° by $f E_1 k_1$, where k_1 again is a quantity depending on the shape and dimensions of the box. When now we suppose that at $t°$ E_1 is changed into $E_1(1 - \eta_1 t)$ we find for our force at $t°$

$$f E_1 (1 - \eta_1 t) k_1.$$

If we call E_2, η_2, k_2 the corresponding quantities for the spring, the force produced by the deflection of the spring will be

$$f E_2 (1 - \eta_2 t) k_2$$

and the equation of equilibrium becomes

$$f[k_1 E_1 (1 - \eta_1 t) + k_2 E_2 (1 - \eta_2 t)] = c[A - p(1 + \alpha t)].$$

If at a first approximation we neglect the expansion of the metal (see above), we can consider c, k_1 and k_2 as constants, and unite them with E_1 and E_2 to a new constant

$$k = \frac{c}{k_1 E_1 + k_2 E_2}$$

so that we find

$$f\left(1 - \frac{k_1 E_1 \eta_1 + k_2 E_2 \eta_2}{k_1 E_1 + k_2 E_2} t\right) = k\left(A - p(1 + \alpha t)\right).$$

Now taking

$$\eta = \frac{k_1 E_1 \eta_1 + k_2 E_2 \eta_2}{k_1 E_1 + k_2 E_2}$$

a quantity that lies between η_1 and η_2, this equation becomes

$$f(1-\eta t) = k[A - p(1+\alpha t)],$$

or, neglecting terms with t^2

$$f = k[A - p - \{p\alpha - (A-p)\eta\}t].$$

Instead of the barometric reading A we shall therefore find (neglecting index-errors)

$$A - p - \{p\alpha - (A-p)\eta\}t.$$

The correction becomes

$$p + \{p\alpha - (A-p)\eta\}t,$$

and the thermal coefficient

$$\lambda = p\alpha - (A-p)\eta = p(\alpha+\eta) - A\eta.$$

We find here that λ depends on the pressure A, agreeing to what has often been observed [1]).

§ 3. For a numeric comparison of the deduced expression of λ with experiments we should have to know the values of η_1 and η_2 and the proportion $\frac{E_1 k_1}{E_2 k_2}$, which is equal to that of the elastic forces caused at 0° in the box and in the spring, and moreover the air-pressure p, which would require a special experimental research. In default of such complete data, we can try a rough comparison by means of what is known about these quantities.

If $p = 0$, that is to say, if the box is completely exhausted, λ reaches its greatest negative value $-A\eta$. Regarding η we may suppose that the spring produces a much greater elastic force than the box for the same deflection. We may therefore thake $\eta = \eta_2$, and using the results of MAYER [2]) for steel, assume for η a value between 0.000224 and 0.000309. The greatest possible negative value of λ would

[1]) See e. g. WIEBE, Zeitschr. f. Instrumentenk. X, p. 429 p. (1890).

[2]) MAYER, Americ. J. of Sc. (4) I, p. 81 (1896).

then be $-A\eta = -760 \times 0.000309 = -0.235$. This agrees pretty well with the results of JELINEK [1]), who finds among 108 aneroids only 3 which have $-\lambda$ greater than 0.235. Also HARTL [2]), testing 81 instruments, finds as highest limit -0.24.

Ordinarily a much smaller value is found, as is apparent from the above mentioned investigation of JELINEK. He finds λ [3])

for	9	instruments	between	$+0.23$	and 0.00
»	9	»	»	0.00	» -0.07
»	82	»	»	-0.07	» -0.17
»	8	»	»	-0.17	» -0.37

The positive values are for the most part found with instruments of smaller size (pocket-size) with which a less accurate workmanship, and greater air-pressure are not impossible.

To λ between -0.07 and -0.17, when $A = 760$, $\eta = 0.0003$, $\alpha + \eta = 0.004$, would correspond a pressure in the box of 39 to 14 mM. When $\eta = 0.000224$ these limits would be 25 and 0 mM. Such values of p are quite possible.

The relation of λ to the pressure has been investigated a. o. by WIEBE. He expresses the thermal coefficient in the form

$$\lambda = a + b(760 - A)$$

and finds for b values, which mostly (8 out of 9) lie between 0.0002 and 0.0003. When we compare this form with our expression

[1]) JELINEK, Carl's Repert. XIII, p. 72 (1877).

[2]) HARTL, Zeitschr. f. Vermessungsw. 1882, p. 458.

[3]) See JORDAN, Handbuch der Vermessungsk. II, p. 499, 3 ed.

$$\lambda = p\,(\alpha + \eta) - A\,\eta$$

we see that b must be equal to η, which is sufficiently confirmed by the observed values.

HARTL [1]) has determined thermal coefficients at different pressures of aneroids after removing the box, and charging the spring with weights. The thermal coefficient in this case is expressed by $\lambda' = -A\,\eta$, and so is proportional to the pressure. HARTL finds with four aneroids:

Aneroid	pressure	therm. coeff. spring.	
	A	$-\lambda'$	$-\frac{\lambda'}{A}$
I	735	0.262	0.00036
II	759	0.365	0.00048
	682	0.350	51
	600	0.289	48
III	793	0.428	0.00054
	748	0.400	53
	644	0.397	62
IV	759	0.335	0.00044

The proportionality with the pressure is confirmed by II. By III only when we exclude the last number.

The value found for $-\frac{\lambda'}{A}$, which should be equal to η, is however much greater than the values found by MAYER.

As far as is to be concluded from these data, the observed thermal coefficients are on the whole not in contradiction with the expression found above. For a more accurate treatment better data would be required. More researches such as promised by HARTL, are therefore much to be desired.

[1]) HARTL, Zeitschr. f. Instrumentenk. VI (1886).

COMMUNICATIONS

FROM THE

PHYSICAL LABORATORY

AT THE

UNIVERSITY OF LEIDEN

BY

PROF. DR. H. KAMERLINGH ONNES,

Director of the Laboratory.

No. 35.

(REPRINT.)

Dr. L. H. SIERTSEMA. On the effect of pressure on the natural rotation of the plane of polarisation in solutions of cane-sugar. (*Translated from: Zittingsverslagen van de Afdeeling Natuurkunde der Kon. Akad. van Wetenschappen te Amsterdam*, **2 Januari 1897**, p. 305—309).

EDUARD IJDO — PRINTER — LEIDEN.

Dr. L. H. SIERTSEMA. *On the effect of pressure on the natural rotation of the plane of polarisation in solutions of cane-sugar.*

The apparatus with which the magnetic rotation in gases has been determined, can also serve for the investigation of magnetic and natural rotations in other matter at high pressure. The determinations which will here be communicated, have been made in consequence of a conversation with Prof. TAMMANN, and relate to the natural rotation in solutions of sugar under various pressures. These determinations will afford materials for verifying the hypothesis of TAMMANN on the effect of the internal pressure of liquids on their molecular properties.

The apparatus needed only some unimportant additions. It appeared to be somewhat difficult to withdraw all the air out of the great nicol-holder, which caused some inconvenience when the pressure was applied and removed. Therefore between the experimental tube and the pressure-tube a reservoir was interposed, partly filled with the solution, so that the surface of the liquid can rise and fall in it when the pressure varies.

The most obvious way for determining the effect of pressure would be a direct measurement of the rotation.

For doing this we should have to fill the experimental tube, after adjusting the nicol so that the light is extinguished, with a solution of such a strength, that in the middle part of the spectrum a black band appears, indicating a rotation of 180°, and then to try to observe a displacement of this band when pressure is applied by means of a reservoir of high pressure, e. g. a filled oxygen-cylinder. An accurate examination of this method however shows, that it is not convenient for the observation of small changes of rotation. For although the rotation is considerable, a small change of the rotation will cause a very small displacement of the band or, in other terms, the mobility of the band is too small. The experiment also showed that in this case no displacement was to be seen.

Better results were obtained with the following compensation-method. Between the nicols a plate of quartz, rotating to the left, was interposed, which nearly compensated the rotation of the sugar. A small change of the rotation α_s of the sugar will now greatly affect the difference $\alpha_s - \alpha_k$ to the rotation of sugar and quartz, and the black band in the spectrum will soon move perceptibly. In this way very small changes of α_s will become visible.

In choosing the concentration of the solution and the thickness of the quartz, we must consider that, when we diminish the remaining dispersion, although the mobility of the band increases, it also widens, and the pointings therefore become less accurate. Experience must teach us to choose the best degree of compensation.

It is obvious that in this way also the difference between

the rotatory dispersion of sugar and of quartz can be accurately investigated. This difference is rather irregular, as appears from the irregular changes of the black band when we turn the smaller nicol. So it was once observed, in a case in which $\alpha_k > \alpha_s$, that the band, at first very narrow, showed no perceptible displacement at the rotation above mentioned, but widened, till at last it almost totally vanished. On turning in the other direction the same thing was observed, and moreover in another part of the spectrum a new band appeared, which on the rotation going on, also vanished.

In order to give a narrow black band the quartz must be made of very pure homogeneous material, and of accurate workmanship. Little impurities, which could not be detected with other methods, here became clearly visible, so that we have here a very effective method for investigating the purity of quartz.

The solutions were prepared by solving a weighed quantity of sugar-candy in distilled water, and filtrating the solution before it was used. The concentration was controlled several times by filling the experimental tube with the solution, the nicols having been adjusted so that they cut of the light, and then the wave-length of the black band was determined.

When we know the thickness of the plate and the concentration of the solution, we can calculate the angle of the principal planes of the nicol, which will afford a band in a selected part of the spectrum. Of course the nicols must be adjusted under this angle before closing the flanges of the nicol-holder.

The wave-lengths were determined by pointings on

the Na-lines, produced by putting common salt on the carbons of the arc-lamp.

The application of pressure now caused a considerable displacement of the band. The observations consisted in pointings of the band with and without pressure, four or five times in each case. After applying the ordinary corrections, means were taken of the readings of each set.

From these means was then calculated the number of degrees $\Delta\alpha$, with which the rotation would have changed at a pressure of 100 atm., assuming this change to be proportional to the pressure. Our being justified in making this supposition appears from the following observations

			change.	
c	λ	p	$\Delta\Phi$	$\Delta\Phi/p$.
9.50	562	48.0	0°.196	0.00408
		98.7	0°.402	0.00407
9.30	562	97.9	0°.455	0.00465
		81.4	0°.382	0.00469

in which c indicates te concentration (G. in 100 cc. of the solution), λ the wave-length, p the pressure in KG. per cM².

The angles $\Delta\alpha$, calculated in this way for 100 atm. would be equal to the change of the rotation of the solution, if that of quartz remained unchanged. Now we have $\Delta\alpha = \Delta\alpha_s - \Delta\alpha_k$, where $\Delta\alpha_s$ is the variation of α_s, $\Delta\alpha_k$ that of α_k. In default of direct data we shall deduce the variation $\Delta\alpha_k$ from the coefficients of compressibility, found by VOIGT.

The rotation α_k can change:

1° by the thickness of the plate undergoing some alteration,

2° by a change in the rotation per unit of length.

VOIGT finds in the case of hydrostatic pressure as coefficient of compressibility in the direction of the principal axis 6.73×10^{-8} (mM², G.). For a pressure of 100 atm. the rotation α_k would therefore vary by the first cause with $-1033 \times 6.73 \times 10^{-8}\,\alpha_k = -70 \times 10^{-6}\,\alpha_k$. Concerning the second cause, we will assume that this variation is determined by that of the density. For this variation we have, according to VOIGT, the coefficient $25{,}97 \times 10^{-8}$ (mM², G.), and the variation of the rotation by this cause amounts to $+1033 \times 25.97 \times 10^{-8}\,\alpha_k = +268 \times 10^{-6}\,\alpha_k$. Hence both causes together give a variation $\Delta\alpha_k = +198 \times 10^{-6}\,\alpha_k$. From this we can then calculate $\Delta\alpha_s = \Delta\alpha + \Delta\alpha_k$.

In this way the following results were obtained:

I. Thickness of the quartz 6.88 mM.

c	λ	$\Delta\alpha$	$\Delta\alpha_k$	$\Delta\alpha_s$	α_s	$\Delta\alpha_s/\alpha_s$	n
9.50	492	+0°.555	+0°.043	+0°.598	231°.4	+0.00258	10
»	528	0 .467	0 .037	0 .504	198 .4	254	6
»	559	0 .423	0 .033	0 .456	175 .0	260	8
»	560	0 .438	0 .033	0 .471	174 .6	270	6
»	561	0 .468	0 .033	0 .501	173 .9	288	9
»	»	0 .413	0 .033	0 .446	173 .9	256	12
»	562	0 .437	0 .033	0 .470	173 .2	271	21
»	589	0 .408	0 .030	0 .438	156 .5	280	22
»	»	0 .392	0 .030	0 .422	156 .5	270	24
9.30	537	0 .483	0 .036	0 .519	187 .0	277	21

concentration 9.47, mean value of $\frac{\Delta\alpha_s}{\alpha_s} = 0.00271 \pm 0.00003$.

II. Thickness of the quartz 13.835 mM.

c	λ	$\Delta\alpha$	$\Delta\alpha_k$	$\Delta\alpha_s$	α_s	$\frac{\Delta\alpha_s}{\alpha_s}$	n
18.45	529	+0°.937	+0°.075	+1°.012	397.1	0.00255	24
»	567	0 .840	0 .064	0 .904	330.7	273	16
18.86	530	0 .942	0 .075	1 .017	391.2	260	24
»	558	0 .777	0 .067	0 .844	349.5	241	24
»	579	0 .755	0 .062	0 .817	323.3	253	12

concentration 18.70, mean value of $\frac{\Delta\alpha_s}{\alpha_s} = 0.00255 \pm 0.00005$

Here n represents the weight of each determination, taken equal to the number of pointings from which it has been deduced.

If β represents the rotation of the solution of sugar per unit of length, and l the length of the experimental tube, we have

$$\alpha_s = \beta l$$

from which follows:

$$\frac{\Delta\alpha_s}{\alpha_s} = \frac{\Delta\beta}{\beta} + \frac{\Delta l}{l}.$$

The expansion Δl of the experimental tube was measured with a couple of reading-microscopes, placed on a board entirely detached from the apparatus, and beside from it. These microscopes were pointed each on one of the nicol-holders, and so the displacement of these pieces was observed when the pressure was applied. We found $\Delta l = 0.06$ mM. for a pressure of 100 K.G., and $l = 248$ cM., so that $\frac{\Delta l}{l} = 0.00003$.

After applying of this correction we find for our two means:

c	$\Delta\beta/\beta$
9.47	0.00268
18.70	252

The question whether the proportion $\frac{\Delta\beta}{\beta}$ depends on the concentration, cannot yet be solved from these means. Determinations with greater concentrations, which have yet to be undertaken, will perhaps throw more light on the subject.

Considerations on the change of molecular rotatory power and of the concentration by pressure, in connection with the hypothesis of TAMMANN, will be deferred till the conclusion of these determinations.

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No. 36.

Dr. P. ZEEMAN. Appendix to No. 33.
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EDUARD IJDO — PRINTER — LEIDEN.

Dr. P. ZEEMAN. *On the influence of magnetism on the nature of light emitted by a substance. Appendix to No. 33.* (Phil. Mag. March 1897).

Since the publication of my original paper in the Proceedings of the Academy at Amsterdam, and while the present paper was in the press, I have become acquainted with two attempts, till now unknown to me, in the same direction, and also with the original account of FARADAY's experiment referred to in § 1 [1]). The last is to be found in FARADAY's 'Life' by Dr. BENCE JONES, vol. II. p. 449 (1870), and as it is extremely remarkable I will reprint it here:—

»1862 was the last year of experimental research. STEINHEIL's apparatus for producing the spectrum of different substances gave a new method by which the action of magnetic poles upon light could be tried. In January he made himself familiar with the apparatus, and then he tried the action of the great magnet on the spectrum of chloride of sodium, chloride of barium, chloride of strontium, and chloride of lithium."

On March 12 he writes:—"Apparatus as on last day (January 28), but only ten pairs of voltaic battery for the electromagnet.

»The colourless gas-flame ascended between the poles

[1]) Comm. N°. 33.

of the magnet, and the salts of sodium, lithium, &c. were used to give colour. A NICOL's polarizer was placed just before the intense magnetic field, and an analyser at the other extreme of the apparatus. Then the electromagnet was made, and unmade, but not the slightest trace of effect on or change in the lines in the spectrum was observed in any position of polarizer or analyser.

»Two other pierced poles were adjusted at the magnet, the coloured flame established between them, and only that ray taken up by the optic apparatus which came to it along the axis of the poles, *i. e.* in the magnetic axis, or line of magnetic force. Then the electromagnet was excited and rendered neutral, but not the slightest effect on the polarized or unpolarized ray was observed."

»This was the last experimental research that FARADAY made."

In 1875 we have a paper by Prof. TAIT, who has kindly sent me a copy. »On a Possible Influence of Magnetism on the Absorption of Light, and some correlated subjects" (Proc. Roy. Soc. of Edinburgh, Session 1875—76, p. 118). Prof. TAIT remarks that a paper by Professor FORBES, read at the Society, and some remarks upon it by MAXWELL, have recalled to him an experiment tried by him several times, but which hitherto has led to no result. Then the paper proceeds:—

»The idea is briefly this.—The explanation of FARADAY's rotation of the plane of polarization of light by a transparent diamagnetic requires, as shown by THOMSON, molecular rotation of the luminiferous medium. The

plane-polarized ray is broken up, while in the medium, into its circularly-polarized components, one of which rotates with the aether so as to have its period accelerated, the other against it in a retarded period. Now, suppose the medium to absorb one definite wavelength only, then—if the absorption is not interfered with by the magnetic action—the portion absorbed in one ray will be of a shorter, in the other of a longer, period than if there had been no magnetic force; and thus, what was originally a single dark absorption line might become a double line, the components being less dark than the single one."

Hence here the idea is perfectly clearly expressed of the experiment, tried in vain; an idea closely akin to that of § 15 above [1]), both being in fact founded on KELVIN's theory of the molecular rotation of the luminiferous medium, though not directly applicable to the experiment of § 9, in which case the lines of magnetic force are perpendicular to the axis of the tube.

In the second place I have to mention two papers by the late M. FIEVEZ, to which attention has been drawn by M. VAN AUBEL, in a letter to Prof. ONNES and intended for communication to the Academy of Sciences, Amsterdam. Prof. ONNES read the letter at the January meeting, and made at the same time some explanatory remarks of which in the following I make free and extensive use [2]). The papers referred to are:—

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[1]) Comm. N°. 33.

[2]) As the remarks of Mr. ZEEMAN resume the paper of mine, it seems superfluous to give a translation of it. H. KAMERLINGH ONNES.

M. FIEVEZ, »De l'Influence du Magnétisme sur les caractères des Raies spectrales" (*Bulletin de l'Acad. des Sciences de Belgique,* 3e série, tome IX. p. 381, 1885); and FIEVEZ, »Essai sur l'Origine des Raies de FRAUNHOFER, en rapport avec la Constitution du Soleil" (*l. c.* 3e série, tome XII. p. 30, 1886). Here experiments are described as in §§ 4 and 13 of the present paper. Nothing, however, is observed about the widening of the absorption-lines, nor about the polarization of the emitted light. The results obtained by M. FIEVEZ merit careful attention and consideration. He has observed with a flame in a magnetic field not only widening but reversal and double reversal of the lines of the spectrum, the lines at the same time becoming more brilliant. Unfortunately quantitative details are not given. The facts observed in some cases by FIEVEZ are qualitatively not in accordance with my observations or what is to be deduced from my results. Hence even in the cases where the results are qualitatively in accordance, the question remains whether FIEVEZ has observed *the same phenomenon.* The field used by FIEVEZ seems to have been more intense than the one I had at my disposal. Is it possible perhaps to account in this manner for the »double renversement (c'est-à-dire l'apparition d'une raie brillante au milieu de la raie noire élargie)"? I think the answer must be in the negative. For, arguing from § 19, a line must widen, or else, the field being very intense, become a triplet. We cannot but understand from FIEVEZ's description of the experiment that the light was emitted perpendicular to the lines of force. Now the double reversed line of FIEVEZ is not

the triplet to be expected from theory, for it is expressly stated by FIEVEZ that the line experimented upon is not the simple line of the spectrum, but one previously widened and reversed (by some agency independent of magnetism). By the action of magnetism a brilliant line in the centre of the black line appears. Hence perhaps one may interpret the case of double reversal as a direct action of magnetism, but then only as a doubling of the absorption-line and not as a division of the original line into three parts. As the application of LORENTZ's theory given in § 18 is confessedly only a very first sketch, further theoretical and experimental evidence is wanted before we are able to decide whether in the experiment of FIEVEZ a specific action of magnetism on light or perturbing circumstances have been prevalent. Indeed one may make the same objection to M. FIEVEZ's experiment as I myself have made to my own analogous experiment in § 6.

The whole of the phenomena observed by FIEVEZ can readily be attributed to a change of temperature by the well-known actions of the field upon the flame (change in its direction or outline, magnetic convection, &c.); and the last sentence of his paper states that »les phénomènes qui se manifestent sous l'action du magnétisme sont identiquement les mêmes que ceux produits par une élévation de température." The negative result obtained by FIEVEZ with absorption-spectra would without further consideration (as in § 12 [1])) point

[1]) Comm. N°. 33.

in the same direction. The inference to be drawn from FIEVEZ's experiments alone would rather be, I think, that the temperature of the flame is changed in his experiments than that a specific action of magnetism on the emission and absorption of light exists. By experiments already in progress I hope to settle the dubious points.

Summarizing we may say:—Had the experiments of FIEVEZ come to my knowledge they would have been a motive for me to further investigation, FIEVEZ not having prosecuted his inquiry up to a decisive result. At least at present it remains even doubtful whether the phenomenon observed by FIEVEZ with a magnetized flame is really to be attributed to *the specific action of the magnetic field on the period of the vibrations of light*, which I have found and undoubtedly proved by the experimental confirmation of LORENTZ's predictions.