Dr. P. ZEEMAN. *Measurements concerning the Variation of the Absorption of Electrical Vibrations with the Wave length and with the Concentration of the Electrolyte* (with 3 diagrams).

1. On a previous occasion [1]) was communicated to the Academy a determination, the first ever made, of the absorption-coefficient of electrical vibrations travelling through an electrolyte. The wave length of the vibrations used was 6.6 M. [2]) in air, and the conductivity of the solution 3340.10^{-10} that of mercury. In continuing this inquiry, I wished to investigate what function the absorption is of the concentration of the solution and of the wave length of the incident vibrations. I have now measured the absorption coefficient for two different vibrations travelling through solutions the conductivity of which lies between the limits 3500.10^{-10} and 40000.10^{-10}. I take the liberty to offer in this paper the results of these measurements.

2. *Apparatus.* Except some small changes of inferior

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[1]) ZEEMAN, Communications n⁰. 22.

[2]) I have given in my former communications 6.5 M. However 6.6 is more accurate.

importance, the apparatus used were the same as used with the former experiments and were then described.

3. *The double-wire circuit.* With the waves of 6.6 M. also the circuit, transferring the vibrations of the oscillator to the basin was the same as the one formerly used. That circuit having run through a corridor returned with a great curve into the room where the apparatus are placed. Using this circuit with waves greater than 8 M. perturbations appeared of the kind examined by v. GEITLER[1]), manifesting themselves in my experiments in irregularities of BJERKNES' curve of interference. I was therefore obliged to construct a new circuit. A first endeavour for that purpose failed. The circuit was made with much care in the garden of the laboratory, returning with a great arch parallel to the original direction in the room and continuing in a corridor with a second arch horizontally in a direction perpendicular to the original one. The horizontal part serves for the displacement of the bridge in order to get the interference-curve. One end of the horizontal part was relatively near to the vibrator. The two variations of the direction of the circuit and the last named circumstance were probably the reason that also on this occasion no satisfactory results were obtained. I obtained these only after disposing the oscillator in a separate building, the circuit now running in *one* straight line. The 2 parallel wires are continued straight forward in a horizontal plane to the extreme place intended for the bridge in the measurement of the curve of interference. The then following part of the double circuit, only destined for amortizing the waves passing the bridge in the mentioned measurement, naturally need not be arranged with so much care.

4. The observations concerning the absorption were made with the basin used on former occasions and also the method of measurement is the same. The diminution of the energy of the vibrations in the electrolyte was always determined by moving along the two parallel wires in the interior of the fluid, the two little Leyden jars, transferring the energy to the bolometer. The distance over which the jars are moved is entered in cM. in the following tables.

The final result of one series generally depends upon 3×4, sometimes upon 4×4 separate sets of measurements.

I have called coefficient of absorption the value of p in the expression Ae^{-2pz}, A being the incident energy, z the length of the traversed layer.

5. *Measurements with the wave of 6.6 M.* (logarithmic decrement $\gamma = 0.34$); diameter of the wire 0.70 mM. The diminution of the energy in the fluid is given by »observed deflection." Under »calculated deflection" have been entered the values following from the exponential function, by means of the value of p representing as well as possible the observations. That value of p is entered at the bottom of the table. In the fourth column the differences between the observed and calculated values are given.

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[1]) v. GEITLER. Wied. Ann. Bd. 49. p. 184. 1893.

	$\lambda = 3480.10^{-10}$			$\lambda = 8100.10^{-10}$		
Traversed layer	Observed Deflection.	Calculated Deflection.	Difference.	Observed Deflection.	Calculated Deflection.	Difference.
0	46.3	46.3	0.0	35.0	35.0	0.0
2.5	28.5	29.3	− 0.8	14.7	15.3	− 0.6
5	17.7	18.5	− 0.8	6.8	6.7	+ 0.1
7.5	11.7	11.7	0.0	2.3	3.0	− 0.7
10	8.5	7.4	+ 1.1	0.7	1.3	− 0.4
15	4.5	3.0	+ 1.5			
20	1.6	1.1	+ 0.5	0.3	0	+ 0.3
47	0	0	0.0	0	0	0
$p =$	0.091			=	0.165	

	$\lambda = 14600.10^{-10}$			$\lambda = 28000.10^{-10}$		
Traversed layer	Observed Deflection.	Calculated Deflection.	Difference.	Observed Deflection.	Calculated Deflection.	Difference.
0	22.6	22.6	0.0	13.9	13.9	0.0
1	14.3	14.3	0.0	7.8	7.6	+ 0.2
2	8.1	8.9	− 0.8	4.2	4.2	0.0
3	4.8	5.6	− 0.8	1.9	2.3	− 0.4
4	3.6	3.5	+ 0.1			
6	0.9	1.4	− 0.5			
10	0.1	0.2	− 0.1	0.5	0.6	− 0.1
30	0	0	0	0	0.0	0
$p =$	0.231			=	0.300	

6. *Measurements with the wave of 11.8 M.* [1] ($\gamma = 0.38$) diameter of wire 0.83 mM.

	$\lambda = 11400.10^{-10}$			$\lambda = 16000.10^{-10}$		
Traversed layer	Observed Deflection.	Calculated Deflection.	Difference.	Observed Deflection.	Calculated Deflection.	Difference.
0	43.0	43.0	0.0	33.0	33.0	0 0
1	34.5	32.8	+ 1.7	22.5	23.6	− 1.1
2	25.5	25.1	+ 0.4	19.0	16.8	+ 2.2
3	19.7	19.1	+ 0.6	11.5	12.0	− 0.5
6	7.2	8.5	− 1.3	4.5	4.4	+ 0.1
9	3.0	3.8	− 0.8	3.5	1.6	+ 1.9
19	0.3	0 3	0.0	0	0.1	− 0.1
$p =$	0.135			=	0.170	

	$\lambda = 20600.10^{-10}$			$\lambda = 29800.10^{-10}$		
Traversed layer	Observed Deflection	Calculated Deflection	Difference.	Observed Deflection.	Calculated Deflection.	Difference.
0	51.0	51.0	0.0	47.7	46.5	+ 1.1
1	33.7	34.0	− 0.3	28.0	28.7	− 0.7
2	22.7	22.7	0.0	18.0	17.7	+ 0.3
3	19.3	15.1	+ 4.2	14.3	10.9	+ 3.4
6	7.3	4.5	+ 2.8	5.7	2.6	+ 3.1
9	3.3	1.3	+ 2.0	1.4	0.6	+ 0.8
19	0	0	0	0	0	0
$p =$	0.200			=	0.240	

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[1] Determined according to BJERKNES' method. This value of the wavelength was still verified by calculating the frequency from the dimensions of the oscillator by means of the formula $2\ \pi \sqrt{L.C}$. The value of the inductance L I determined by means of a formula given by MASCART. The capacity C I measured directly.

$\lambda = 40000.10^{-10}$

Traversed layer	Observed Deflection.	Calculated Deflection.	Difference.
0	27.1	27.1	0.0
1	15.9	15.3	+ 0.6
2	7.9	8.7	— 0.8
3	4.9	4.9	0.0
6	1.5	0.9	+ 0.6
9	0	0.2	— 0.2
19	0	0	0

$p =$ 0.285

7. *Influence of galvanic resistance of the wires.* From the theory of the propagation of electrical waves it follows, that in some cases, e. g. with great resistance of the parallel wires, the electrical lines of force are no longer perpendicular to the wire surface. Herewith is intimately connected that in these cases part of the propagated energy is converted into heat and hence that then the measured coefficient of absorption no longer is that of the electrolyte. However in our case one may convince oneself by actual calculation that this source of error lies entirely between the limits of the errors of observation. Hence it is to be expected that in my experiments a change in the diameter of the wire does not interfere with the measured coefficient of absorption. In one case I have verified experimentally that the diameter of the wire has no perceptible

influence on the measured coefficient of absorption [1]. For that purpose the wires of diameter 0.70 mM in the basin and about 60 cM. at the head of the basin were substituted by thicker ones. The wave length was 6.6M. as given above and the conductivity $\lambda = 3800.10^{-10}$. The following was found:

Measurements with thick wires.

$\lambda = 3800.10^{-10}$

Traversed layer	Observed Deflection.	Calculated Deflection.	Difference.
0	56.8	57.2	— 0.4
2.5	37.2	36.0	+ 1.2
5	22.4	22.7	— 0.3
7.5	14.0	14.3	— 0.3
10	7.3	9.0	— 1.7
15	2.5	3.6	— 1.1
31	0.0	0.2	- 0.2
40	0.0	0.0	0

$p =$ 0.093

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[1] These measurements were already made when DRUDE published measurements (Berichte der Sächs. Gesellsch. d. Wiss. p. 318, 320 1896) whence it follows that the refractive index of electrical vibrations remains the same if the diameter of the wires is changed from 1 mM. to $^1/_2$ mM. (distance of the wires 18 mM.) Direct experiments concerning the influence of the diameter on the absorption however, as far as I know, have not been made by DRUDE.

From the graphical representation of the measurements with the same wave and *thin* wires (5) it follows that to a conductivity 3800.10^{-10} corresponds 0,096. Within the limits of the errors of measurement this is the same value as found above with *thick* wires. Hence the results arrived at (5,6) cannot contain a considerable systematical error, dependent on the diameter of the wire.

8. *Results*. Respecting a few of the results that may be derived from the observations, I have something more to say.

a. We ma yask: how does, the wave length being given, the coefficient of absorption change with the conductivity? It seems to me, with regard to theory, of some importance to see how far p is proportional to $\sqrt{\lambda}$. In the following tables the quotients $\frac{p}{10^3 \sqrt{\lambda}}$ have been entered, as derived from the results of 5 en 6.

Wave length 6,6 M.

p	$\lambda.\ 10^{10}$	$\frac{p}{10^3.\sqrt{\lambda}}$
0.091	3480	0.154
0.165	8100	0.183
0.231	14600	0.191
0.300	28000	0.179

Wave length 11,8 M.

p	$\lambda.\ 10^{10}$	$\frac{p}{10^3.\sqrt{\lambda}}$
0.135	11400	0.126
0.170	16000	0.134
0.200	20600	0.139
0.240	29800	0.139
0.285	40000	0.142

The numbers of the last column in each of the tables, appear to be nearly constant with greater concentration and hence within the explored region *with given wave length, the coefficient of absorption is approximately proportional to the square root of the conductivity*.

The graphical representation of the observations is given in fig. I.

b. Another question is this: suppose that for a given wave length and given conductivity, the coefficient of absorption is known, how is with increasing wave length the conductivity to be changed, if the absorption is to remain unchanged? The answer to this question is to be found in the following table. From the diagram I for 6 different values of p the corresponding λ's have been taken and the quotients $\frac{\lambda 10^{10}}{l}$ evaluated for the two waves.

	Wave length 6,6 M.		Wave length 11,8 M.	
p	$\lambda.\ 10^{10}$	$\frac{\lambda}{l}.\ 10^{10}$	$\lambda.\ 10^{10}$	$\frac{\lambda}{l}.\ 10^{10}$
0.120	5000	758	10000	847
0.150	7000	1060	13000	1100
0.180	9300	1410	17400	1470
0.210	12200	1850	23200	1970
0.240	16200	2450	29800	2520
0.270	21600	3270	37400	3170

Taking into account the possible sources of error we have with greater concentration the law: *If the wave length is increased and in the same ratio the conductivity* *of the solution, the absorption remains unchanged.* In a diagram with the coordinates l and λ the points belonging to the same p are nearly in a straight line through the origin, cf. fig. II.

c. The variation of the absorption of the energy with the conductivity appears from the graphical representation III. For $\lambda.10^{10} = 5000$ till $= 25000$ and $l = 6.6$ M. lines representing the intensity at different depth in the fluid are given.

d. From *a* and *b* together it follows within the same limits, that with different wave length the intensity sinks at $^1/_e$ of the original value at distances proportional to the square root of the wave length.

9. *Conclusion.* Though I have allowed myself to deduce some conclusions from my measurements, I think I must expressly declare that I believe it still necessary to confirm these conclusions by another method. For it must be conceded, that many perturbating circumstances can influence the measurements. Especially other electrical motions can have been superposed on the one defined by $l = 1.18, \gamma = 0.30$, and supposed by us to be quite alone. We may even go so far as to suppose that in the course of the experiments the superposed movements have changed. Perhaps even the differences between the last lines of the columns headed »calculated deflection" and »observed deflection" in 6 concerning $\lambda = 20000.10^{-10}$ and $\lambda = 29800.10^{-10}$ are to be ascribed to this cause. It is certainly remarkable that there »differences" attain a higher value than with higher and lower concentrations, though the differences *can* yet be attributed to accidental errors. The manner in which I intend to make

controlling experiments, I will explain in a few words.

The little Leyden jars with the bolometer are, we may say, an indifferent instrument. Independently of the frequency, *all* electrical waves are registered by the bolometer. Now we can interchange however the Leyden jars for something else. If we bring in the fluid a resonator isolated thereof and tuned to the frequency of the waves, of which the absorption is to be measured of course, it is quite another thing. The resonator will only be sensible for vibrations of the same period as the free ones. The intensity of the excited motion can of course be measured again by means of the bolometer and measures the operating forces. From some preliminary experiments I infer that it will be possible to make measurements with the sketched device. Future inquiry however must decide whether the results obtained in this manner admit of a tolerably simple interpretation.

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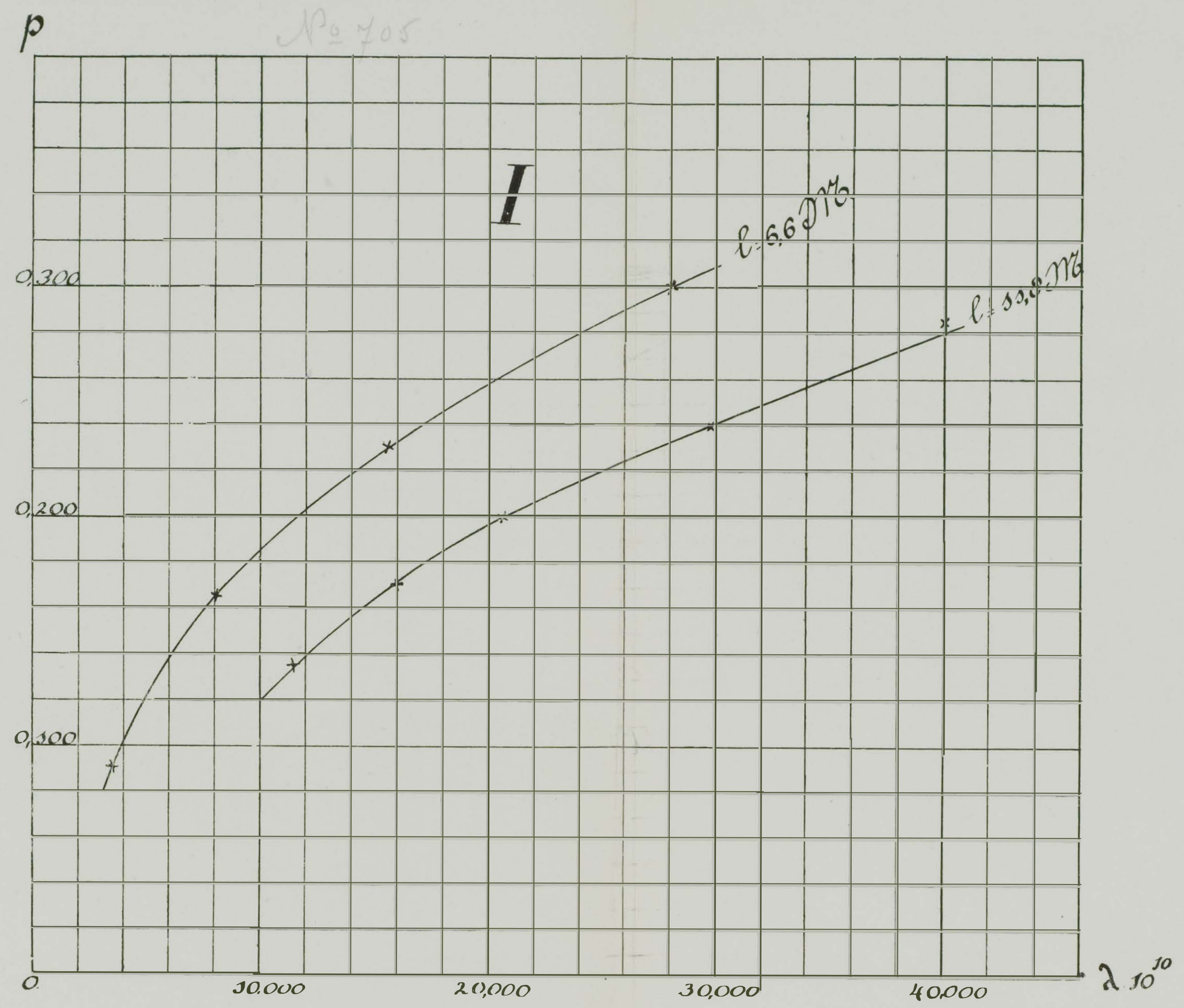

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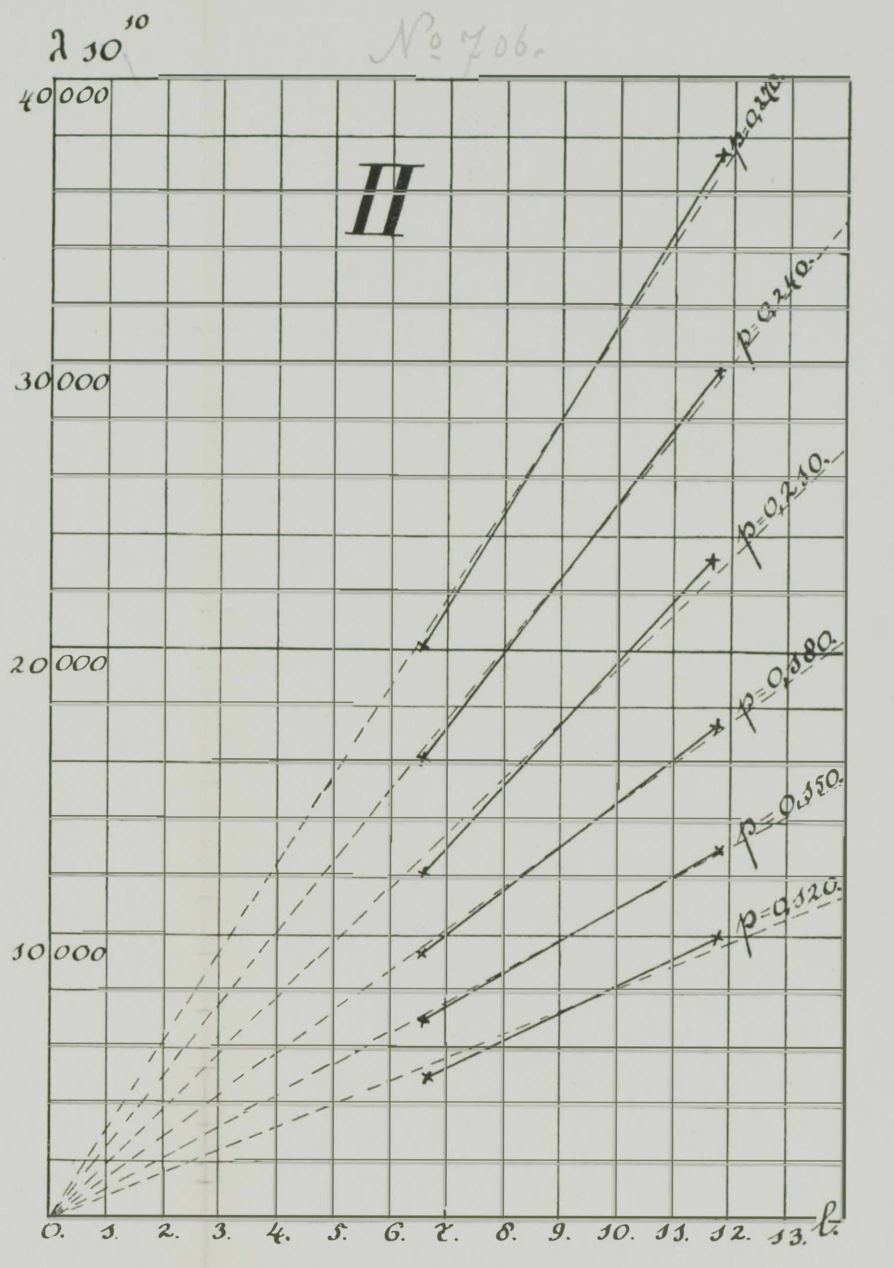

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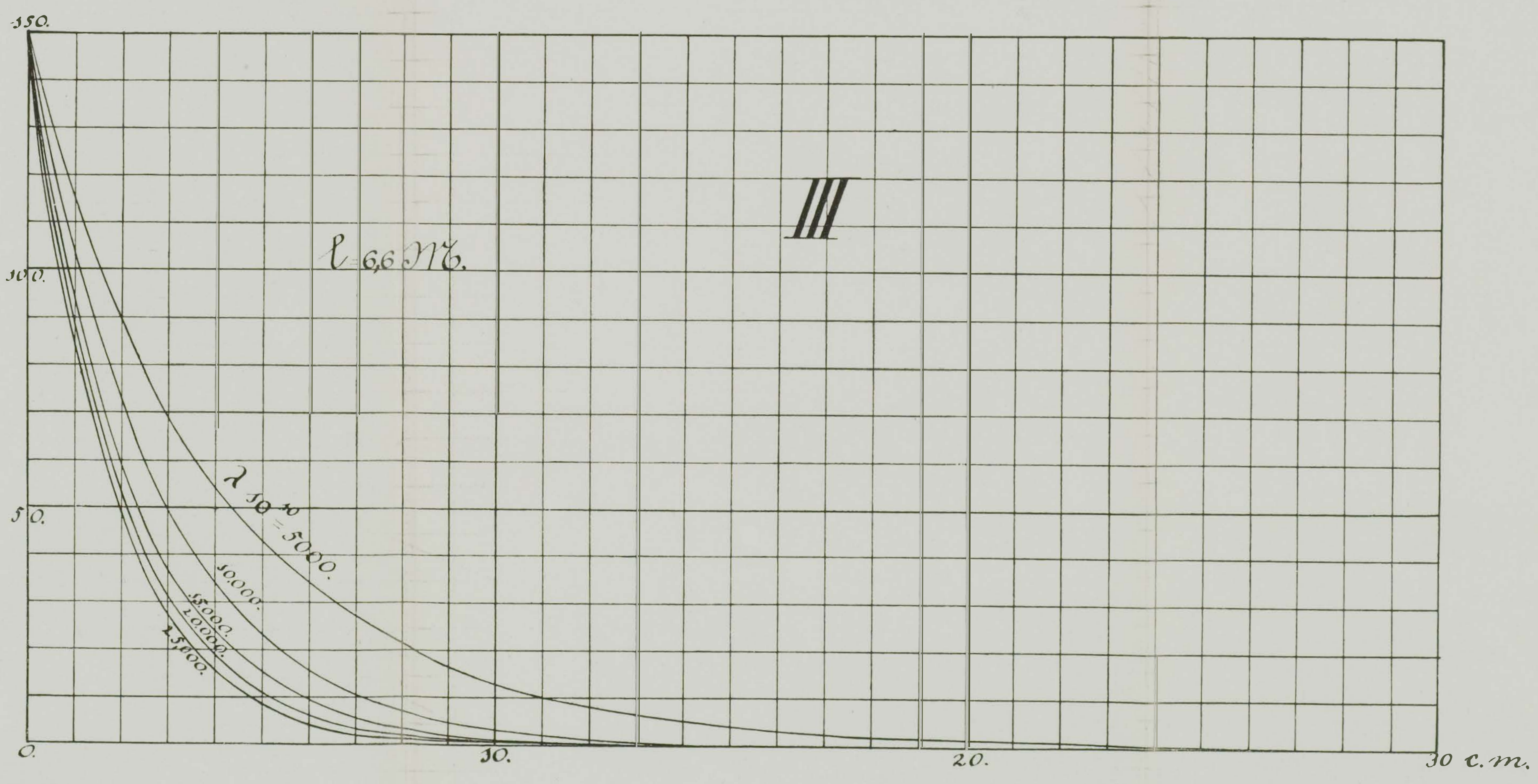

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