Dr. J. VERSCHAFFELT. *On capillary ascension between two cylindrical tubes.*

In a former paper I communicated measurements on capillary ascension of liquefied carbonic acid. The experiments were taken in this way: the capillary tube was put in the axis of a wider one, and I measured the vertical distance between the lowest point of the meniscus in the capillary, and the horizontal plane tangent to the meniscus in the annular space.

Suppose the wider tube to have been left open at its inferior end and put upright amidst an infinite liquid surface; in order to find the *real* ascension H in the capillary tube, to the *apparent* one h we must add the elevation h^1 in the annular space.

r_1 and r_2 being the interior and exterior radii of the capillary, and r_3 the interior radius of the wider tube, we usually put [1])

$$2 \pi r_1 \alpha = \pi r_1^2 h$$

and $$2 \pi (r_3 + r_2) \alpha = \pi (r_3^2 - r_2^2) h^1$$

hence

$$\frac{h^1}{h} = \frac{r_1}{r_3 - r_2}.$$

We find these relations by supposing that the quantity of liquid that is raised against the walls, above the

[1]) Vide a. o. WINKELMANN, Handb. d. Physik, 1891, Erster Bd. p. 460.

horizontal plane tangent to the meniscus, is much smaller than the whole raised mass; we come to the same result by admitting the meridian section of the meniscus to be circular. In a narrow cylindrical tube both suppositions are not far from truth; moreover we can take into account a correction that in most cases is quite sufficient.

In a rather wide annular space, on the contrary, none of these hypotheses may be regarded even as an approximation; for not only the ascension is small, accordingly the error, made by neglecting the liquid raised against the walls, relatively very great; but when gravitation became infinitely small, the limit shape of the meridian section of the surface would not even be a circle, but a portion of the well known curve called *nodoïde* by PLATEAU.

Missing all exact theoretical basis for calculating h^1, in a former communication I used a hypothesis already made by HAGEN [1]) in the case of cylindrical tubes: i. e. the meridian section of the surface would be nearly an ellipse, of which r would be the half of the great axis, and the arrow d of the meniscus the half of the small one. According to HAGEN such a supposition applied to cylindrical tubes gives results agreeing very well with experiments.

The same hypothesis applied to an annular meniscus gives

$$\text{half of great axis} = \frac{r_3 - r_2}{2}$$

$$\text{» small »} = d$$

[1]) POGG. Ann., 67, 1846, p. 126.

d being now the height to which the liquid is raised against the wider tube, above the horizontal tangent plane.

The general equation of the surface

$$\sigma\left(\frac{1}{R} + \frac{1}{R^1}\right) = g\,(s_l - s_v)\,y$$

becomes in the capillary tube

$$\frac{2\,\sigma}{r_1} = g\,(s_l - s_v)\,H = g\,(s_l - s_v)\,(h + h^1).$$

In the horizontal plane tangent to the annular meniscus the radial radius of curvature is the radius at the top of the ellipse: $\frac{1}{R} = \frac{d}{\left(\frac{r_3 - r_2}{2}\right)^2}$; the second radius, according to the theorem of MEUSNIER is infinite: $\frac{1}{R^1} = 0$. In consequence

$$\frac{\sigma\,d}{\left(\frac{r_3 - r_2}{2}\right)^2} = g\,(s_l - s_v)\,h^1$$

hence

$$\frac{h^1}{h} = \frac{2\,d}{\frac{(r_3 - r_2)^2}{r_1} - 2\,d}.$$

It seemed necessary to try this somewhat bold hypothesis by a comparison between calculation and experiment, in order to decide whether the correction found in this way may be trusted or not. This comparison could not be made directly, as the value of h^1 itself cannot be observed; the calculated correction however could be tested by experiments.

By putting the same capillary in tubes of different diameter, we must observe that the apparent ascension is higher in wider tubes, and the difference between the apparent ascensions in two tubes is equal to the difference between the corresponding corrections.

The *calculated* differences must therefore agree well with the *real* observed ones, when the applied correction should be trustworthy. The correction itself, of course, still remains unknown, for the correction corresponding to the widest tube cannot be determined; yet, when the calculated differences are almost equal to the observed ones, we may conclude that the calculated corrections also are not far from truth.

The experiments were taken with liquid methylchloride. I chose a liquefied gas instead of an ordinary liquid, because last remnants can easier be removed.

In order to eliminate errors coming from an imperfect cylindrical shape of the capillary tube, I carefully calibrated it, by measuring capillary ascensions at several heights, from 10 mM. to 10 mM. nearly. For, besides the correction made necessary by the annular meniscus — and this correction we may consider as constant over the whole length of the capillary, the arrow of the meniscus remaining quite the same at every height, — this ascension only depends from the radius at the place at which the meniscus in the capillary has been observed.

As I have been longer in doing that calibration than a day, the temperature of the water taken from the supply did not always remain constant; the variations however did not exceed some tenths of a degree; as

preliminary experiments gave me an approximate value of the coefficient of temperature, all ascensions could be reduced to the same mean temperature 18° C.

We always suppose the section of the capillary to be circular; this is not quite so; as it is however necessary that this section should be circular as nearly as possible, I chose a capillary with very small excentricity.

Two series of experiments were made, each ranging along a half capillary; the first series finished, the capillary was turned upside down. In the middle of it some experiments were made in both series, and the agreement was perfect. After each displacement of the meniscus I waited till it remained absolutely quiet; sometimes observations were taken after liquid had been distilled in, another time after liquid had been boiled out, and in both cases the ascension was the same when equilibrium was obtained.

Now the capillary tube was brought in tubes of different diameters and ascensions measured at several spots near the middle of the capillary. It appeared indeed that the apparent ascension decreased with r_3, while the difference of ascension between two tubes was the same at all levels.

All observations were reduced to the middle of the capillary and 18° C. I found

tube n⁰ I	$r_3 =$ 10.4 mM.	$h =$ 47.46 mM.	$d =$ 1.98 mM.
II	7.45	47.45	1.90
III	5.05	46.99	1.78
IV	3.25	46.36	1.40
V	2.95	46.20	1.24

The calibration was made in tube n⁰ III.

it, and cutting out the so obtained figure, the weight of which I compared to that of 1 dM^2 of the same paper upon which the sketch was made. In order to make errors as small as possible I determined the surface $\int_0^l (h-47)\, dl$, because these integral was nearly $= 0$; the positive and negative portions were placed on different sides of the balance. In this way I found $h_m = 46.99$ mM., what is by chance exactly the ascension in the middle of the capillary; r_m is therefore also the radius in the middle of the capillary.

Now we are able to calculate the hypothetical correction h^1, and find

		(observed)
$h^1_I = 0.152$ mM.		
$h^1_{II} = 0.289$	$h^1_{II} - h^1_I = 0.137$ mM.	0.11 mM.
$h^1_{III} = 0.625$	$h^1_{III} - h^1_I = 0.473$	0.47
$h^1_{IV} = 1.295$	$h^1_{IV} - h^1_I = 1.143$	1.10
$h^1_V = 1.478$	$h^1_V - h^1_I = 1.326$	1.26

We see that the calculated differences agree prettily well with the observed ones, and therefore the calculated corrections may be considered as a good approximation.

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Dr. P. ZEEMAN. On the influence of magnetism on the nature of the light emitted by a substance. (Part II.)

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