Dr. E. VAN EVERDINGEN Jr. *On the* HALL-*effect and the magnetic increase of resistance in bismuth.*

1. At the end of my last communication [1]) is mentioned, „we shall, for an arbitrary position of the plane for which we wish to know the HALL-coefficient, find that coefficient with the aid of the ellipsoid of revolution, construed with the extreme values". In the accompanying figure 1 is further indicated how this should be understood. The plane of the plate is indicated by Pl, the principal crystallographic axis by OA. The intersection of the magnetisation-ellipsoid with the plane through OA and OH, the perpendicular to Pl, at the same time direction of magnetic force, is represented by the ellipse AB. Moreover the intersection BD is indicated of a second ellipsoid of revolution with this plane, whilst $\frac{OD}{OB} = \sqrt{\frac{OA}{OB}}$. The direction of the magnetisation is given by the point where the tangent plane perpendicular to OH touches the second ellipsoid; the magnitude of the magnetisation by the length OM of the radius-vector through this point to where it meets the first ellipsoid. Now the HALL-coefficient is given

[1]) Communications N⁰. 37, p. 19.

by OR, the length of that same radius-vector to where it meets the ellipsoid of revolution, the section of which is indicated by EF.

By a single line is indicated already in this figure the intersection with the plane AOH of the plane perpendicular to OM, in which the greatest increase of resistance is found. In the § mentioned we remarked, that the resistance ellipsoid will generally get three different axes. This is illustrated by fig. 2. Pl, OH and OM have the same signification as before; W is the plane perpendicular to OM. Round OS, the principal crystallographic axis as axis we imagine the resistance-ellipsoid, of which are drawn the intersection ACD with plane SOH and the intersection BD with plane W. The length OP of the radius-vector OM in the ellipsoid of revolution, the intersection of which with SOH is indicated by GK, determines the value of the increase of resistance, which is equal for all directions in the plane W. Thence the ellipse BD is enlarged proportionally to $B'D'$. The new ellipsoid of resistances passes through this ellipse and touches in M the old one. The principal directions (axes of symmetry) of the plate are still OA and OB; but the proportion between OA and OA' is another than that between OB and OB'. OC is not sensibly increased.

2. In my preceding communications on the above mentioned subjects [1]) some questions were proposed, which we will now try to answer, as far we are able to do so by this time.

[1]) Communications N°. 26 and 37.

In Communications N°. 26 we find on p. 6 a discussion of the question, how far we are allowed to write for the difference of potential at the secondary electrodes

$$e = \left\{ H + \tfrac{1}{2} \sin 2\alpha \, (K_{11} - K_{22}) \right\} \frac{I}{d}.$$

The correctness of this formula was then called in question first because of the small mean HALL-effect, found in all the round plates used. This argument no longer exists; later round plates gave sometimes very high values for the HALL-coefficient, and it is almost certain, that the small effect in the first plates was caused by a particular position of the principal crystallographic axis. (See Comm. No. 37 § 8)

A second reason for doubt was given by the observation of differences in mean HALL-effect after turning the plate about the lines of magnetic force through angles of 45°, 90° etc. In connection with this it was thought desirable to construct two differently situated quadratic plates out of *one* round plate and to investigate them.

This investigation has been performed indeed, but appeared not to possess sufficient demonstrative power, since the two halves may show large differences even in *corresponding* positions. Hence it was necessary to proceed as follows:

In a round plate of bismuth the axes of symmetry are determined and the amount of dissymmetry is measured.

Then the plate is sawed into two thinner halves (a and b); each of the halves is tested in the 8 principal positions, after a repeated determination of the axes.

Thereupon out of one a quadratic plate is cut with axes of symmetry as diagonals, out of the other one a quadratic plate with sides parallel to the axes of symmetry. These quadratic plates are tested each in 4 positions.

The following proportions of the HALL-effect in plates a to that in plates b resulted from such an investigation:

mean of the 4 positions of symmetry
and the 4 positions of dissymmetry in round plates 1,016

mean of the 4 pos. of diss. in b
to mean of the 4 pos. of symm. in a „ „ „ 1,035

mean of the 4 pos. of diss. in b
to mean of the 4 pos. of symm. in a „ quadratic „ 1.136.

So we find the difference between positions of symmetry and dissymmetry also in quadratic plates, even in a higher degree than in the round ones.

3. These experiments rendered it rather probable, that not the round form of the plates caused the observed differences. Certainty with regard to this matter was obtained by calculating the difference of potential between the secundary electrodes of a round plate of bismuth, placed in the magnetic field, to within terms of the third order. This calculation, in which H and $(K_{11} - K_{22})$ where regarded as very small compared with $(K_{11} + K_{22})$, and which consisted in an approximation [1]) successively of terms without H or $(K_{11} - K_{22})$, terms with the first power of those quantities, etc., is too long to be given here; therefore I will communicate merely the results.

[1]) For this idea I am indebted to Mr. J. WEEDER, phil. doct. at Leiden.

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1°. For round plates, in whatever position they stand with regard to the primary electrodes, the HALL-effect is always equal to

$$H \frac{I}{d},$$

and therefore always the full HALL-effect is found.

2°. Whether we adjust the secundary electrodes so as to annul the difference of potential in the zero magnetic field, or measure this difference of potential and allow for the increase of resistance [1]), always the observed dissymmetry is equal to

$$\frac{4}{\pi} \sin 2\alpha \frac{I}{d} \left\{ (K_{11} + K_{22}) \triangle \left(\frac{K_{11} - K_{22}}{K_{11} + K_{22}} \right) \right\}$$

where $\triangle$ means the increase in the magnetic field of the quantity in brackets.

The result from this calculation as to the total resistance of the plate between the primary electrodes was given in the communication of 20 April 1897 (§ 2) [2]).

4. Also after the investigation mentioned in § 2 the difference between the values of the mean HALL-effect in positions of symmetry and of dissymmetry remains unexplained. As no theoretical meaning could be given as yet to this difference, it seemed that it had to be attributed to some new phenomenon; but doubt arose again because with some round plates tested afterwards the sign of the difference was reversed, viz. the greatest HALL-effect was found in the positions of dissymmetry. For the present we might still assume, in spite of the

[1]) See Communications N°. 26, p. 5.
[2]) Communications N°. 37, p. 5.

apparent regularity of the phenomenon [1]), that the difference is due to the influence of irregular crystallisation (see communication 2 of 21 April 1897, § 7 [2])) especially as the electrodes in the positions of dissymmetry stand in places quite different from those of the positions of symmetry.

5. Many observations have been made also of the differences between the mean HALL-effect in the 4 possible positions of symmetry or the 4 possible positions of dissymmetry, to which as little theoretical meaning could be given as yet.

Here were taken into account the possible influences of:

Instability of the magnetising current (escaping observation by inaccuracy of the ampère-meter).

Non-homogeneity of the field.

Differences of temperature.

Bad contacts at the secondary electrodes.

The influence of these sources of errors was annulled or diminished by the use of more accurate instruments, by measuring the temperature and the contact-resistance, etc. but in spite of all this in the observed differences generally no traces of regularity could be discovered, and even in one position at different periods rather different values for the mean HALL-current were found *These* differences therefore may very likely be ascribed to irregularity in cristallisation.

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[1]) This came out strongly especially in plate R 3 where in a later series for all positions of symmetry $\pm$ 9,10 , for all positions of dissymmetry 7,80 was found.

[2]) Communications N^0. 37, p. 17.

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6. With a view to the results of the calculation communicated in § 3, the calculation of the resistances in the magnetic field in two directions at right angles, contained in § 1 and 2 of the second communication of 30 May 1896 [1]), should be revised. We will confine ourselves here to the calculation for plate R 1. As nothing is known about the resistances in the zero magnetic field in the directions of the axes of symmetry, we must assume those resistances to have equal values. The formula for the dissymmetry becomes then simply

$$\frac{4}{\pi}(K_{11} - K_{22})\frac{I}{d}$$

This therefore represents $e_A - e_B$.

If we mean by:

w_A and w_B the compensative resistances for the two directions of magnetisation;

w_r the resistance of the rheotan-wires of the compensative current;

w_s " " in the secundary circuit;

then the difference of the secundary currents is:

$$I.\left(\frac{w_r}{w_A} - \frac{w_r}{w_B}\right)$$

So

$$e_A - e_B = I.\,w_r.\,w_s.\left(\frac{1}{w_A} - \frac{1}{w_B}\right).$$

The quantity D, always used as measure for the dissymmetry, is [2])

$$1000\left(\frac{1}{w_A} - \frac{1}{w_B}\right)$$

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[1]) Communications N^0. 26, p. 13 and 16.

[2]) " N^0. 26, p. 11.

whence we find

$$K_{11} - K_{22} = \frac{\pi}{4} d \frac{w_r \cdot w_s \cdot D}{1000} \cdot 10^9 \text{ c. g. s.}$$

By means of this formula we found for plate R 1 in a field of 8600 c. g. s.

$$K_{11} - K_{22} = \pm 4200.$$

7. The general formula for the dissymmetry and the conception of the influence of a magnetic field with the resistance of bismuth, worked out in § 1, enable us to inquire, what we may expect the variation of the dissymmetry and the increase of resistance to be in varying magnetic field and temperature. A very simple calculation shows for instance, that if we call p the increase of resistance in $\%$ for the direction OB (fig. 2) and neglect higher powers of p, the increase of resistance in the direction OA is given by $p\cos^2\alpha$, where α represents the value of the angle MOH. So long as α remains constant, also the ratio between the two increments of resistance will remain the same, and therefore the dissymmetry will be proportional to the mean increase of resistance. If however α varies, this proportionality cannot exist any longer. Now the value of α is determined wholly by the ratio of the permeability in the directions OA and OB (fig. 1). Hence from the observations with plate R 2 [1]) would follow, that in this plate the ratio spoken of varies considerably, when we alter the magnetising force.

8. In Communications N°. 26, § 4, p. 20 attention was directed to a similarity in the variation with temperature of conductivity and HALL-effect in bismuth. Inter alia we deduced from the observations of HENDERSON, that probably in a field of 6000 c. g. s. units a maximum conductivity would be found for bismuth at a temperature below 0°; in connection with the observation of such a maximum at low temperature for the HALL-effect with bismuth II by LEBRET and with bismuth I by myself this was regarded as a confirmation of the similarity mentioned. Since the publication of that communication the maximum conductivity for electrolytic bismuth has really been observed by FLEMING and DEWAR [1]) below 0°; but my further experiments at low temperatures (from 0° to — 70°) with bismuth from MERCK and bismuth from OBERSCHLEMA show for neither of these specimens a maximum HALL-effect, not even an approach towards a maximum. Hence the similarity between conductivity and HALL-effect is not confirmed by these later experiments.

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[1]) Communications No. 26, p. 15.

[1]) Proc. Roy. Soc. 60, p. 425.

Fig. I.

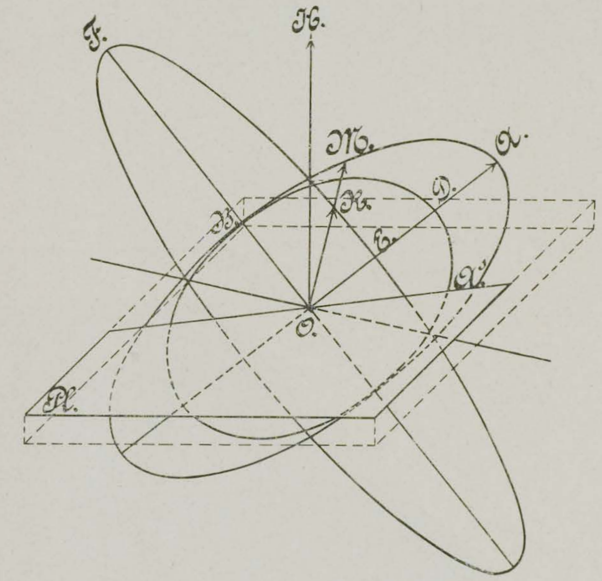

Fig. II.