Inflation

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Lessons learned in Leiden:

Leiden is beautiful

1) One cannot predict our personal future on the scale of one day

2) Predictions on the scale >> 100 years are quite reliable

3) It is even easier to predict the future of the universe☺

Two major cosmological discoveries:

- The new-born universe experienced rapid acceleration (inflation)

- A new (slow) stage of acceleration started 5 billion years ago (dark energy)

How did it start, and how it is going to end?

Our universe

Age: 13.8 billion years

The size of its observable part is proportional to 13.8 billion light years, about 10^{28} cm (up to a factor O(1))

Average energy density: 10^{-29} g/cм^3

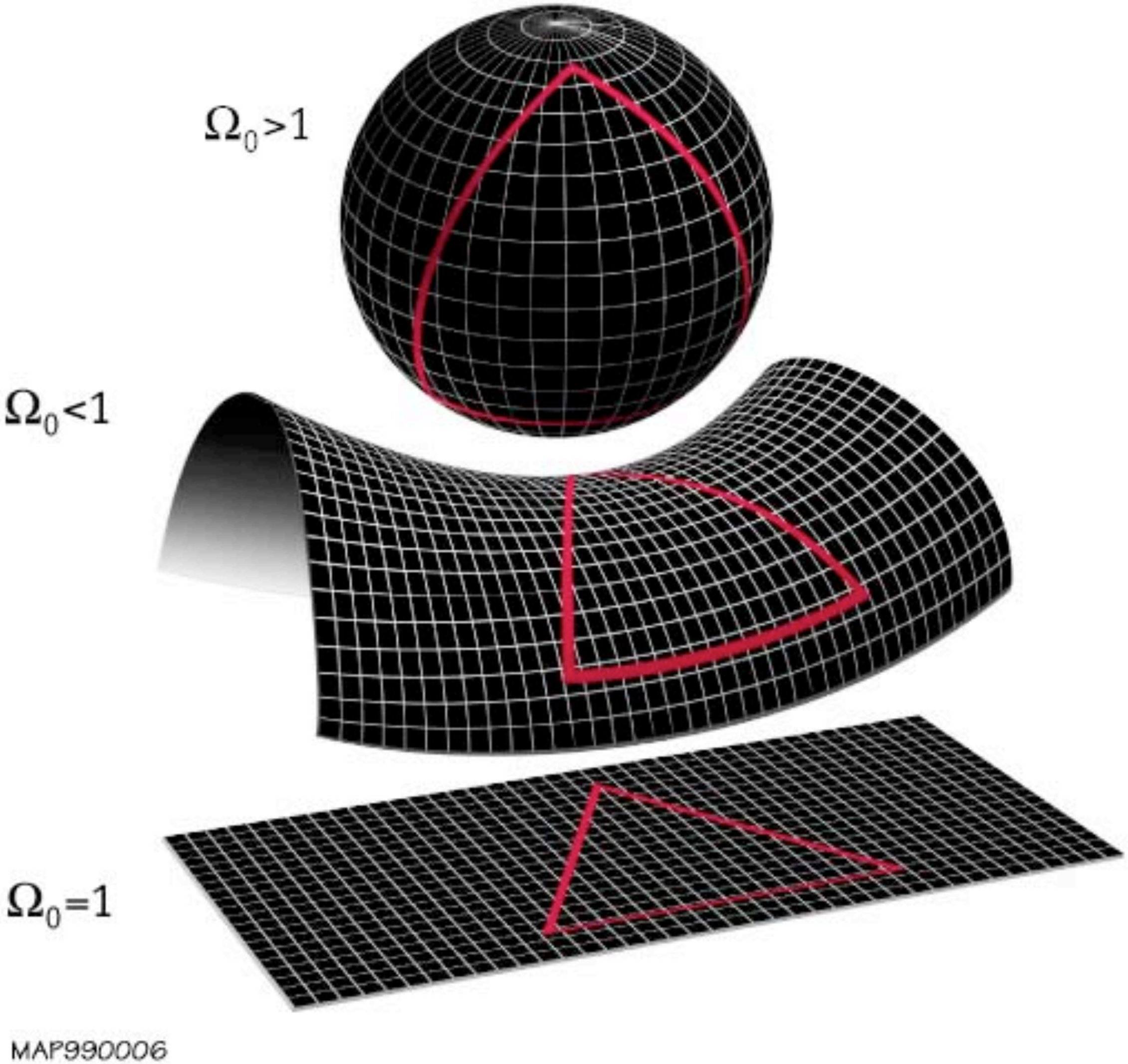
Weight: > 10^{50} tons

Weight at birth:

According to the Big Bang theory, the weight was either infinitely large, or at least **greater than about 10^{80} tons**

According to inflationary theory, the initial weight could be **smaller than a milligram.**

Closed, open and flat universes

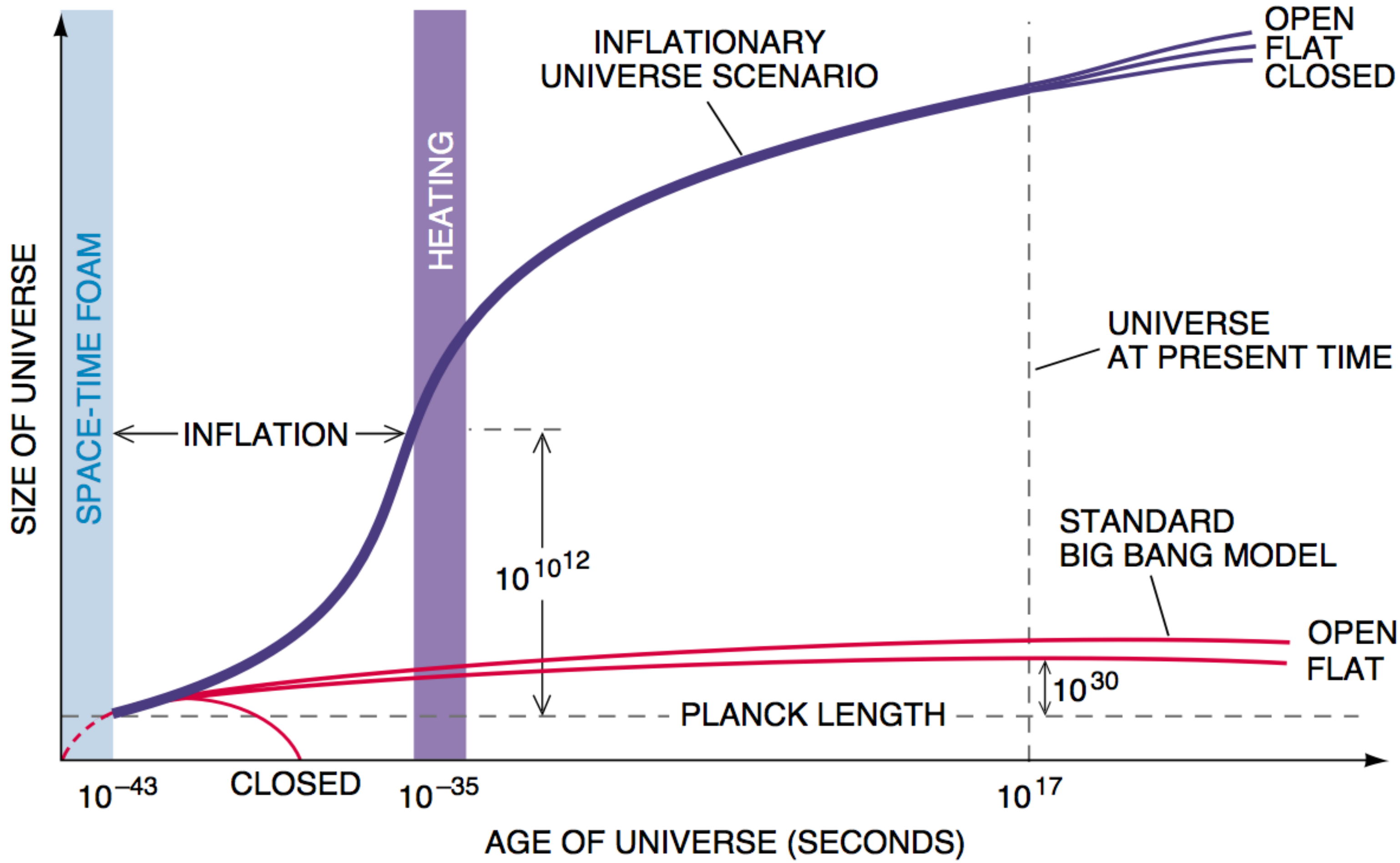


Closed universe

Open universe

Flat universe

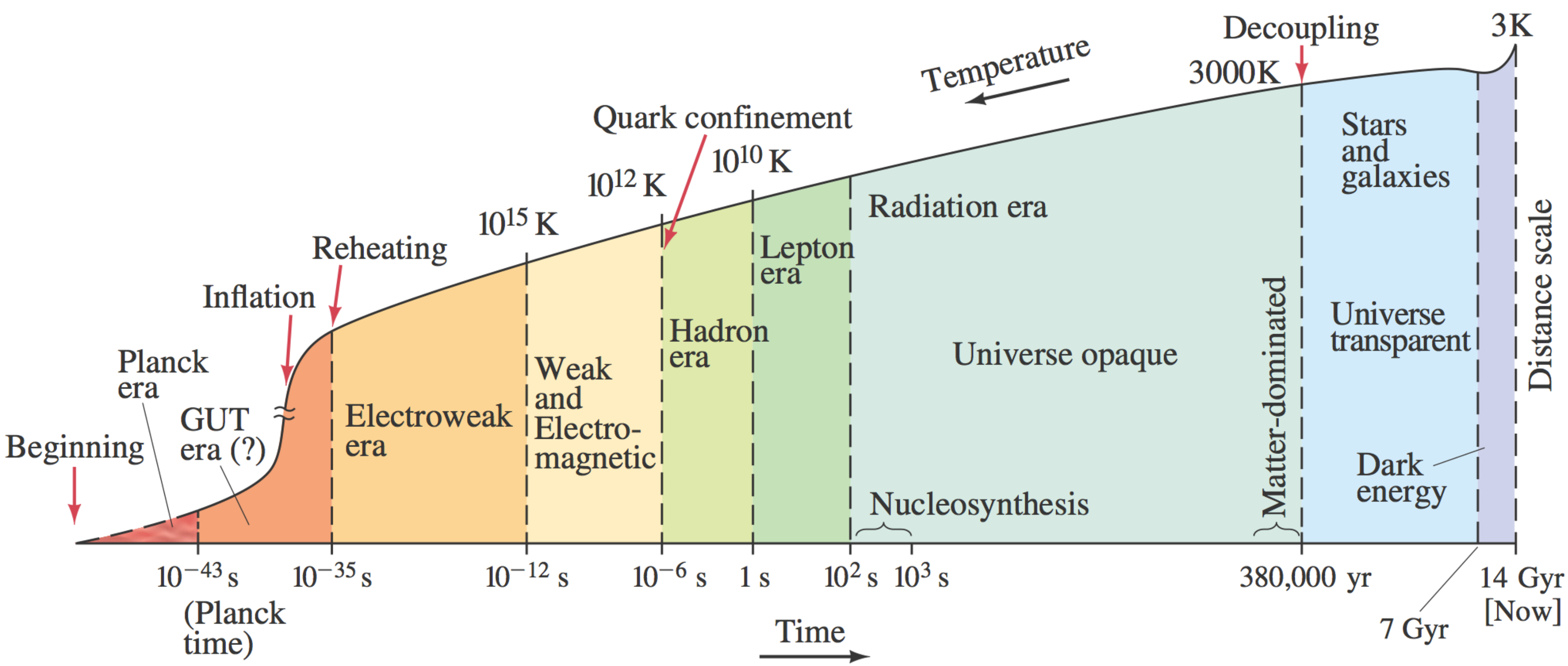
Inflationary Universe



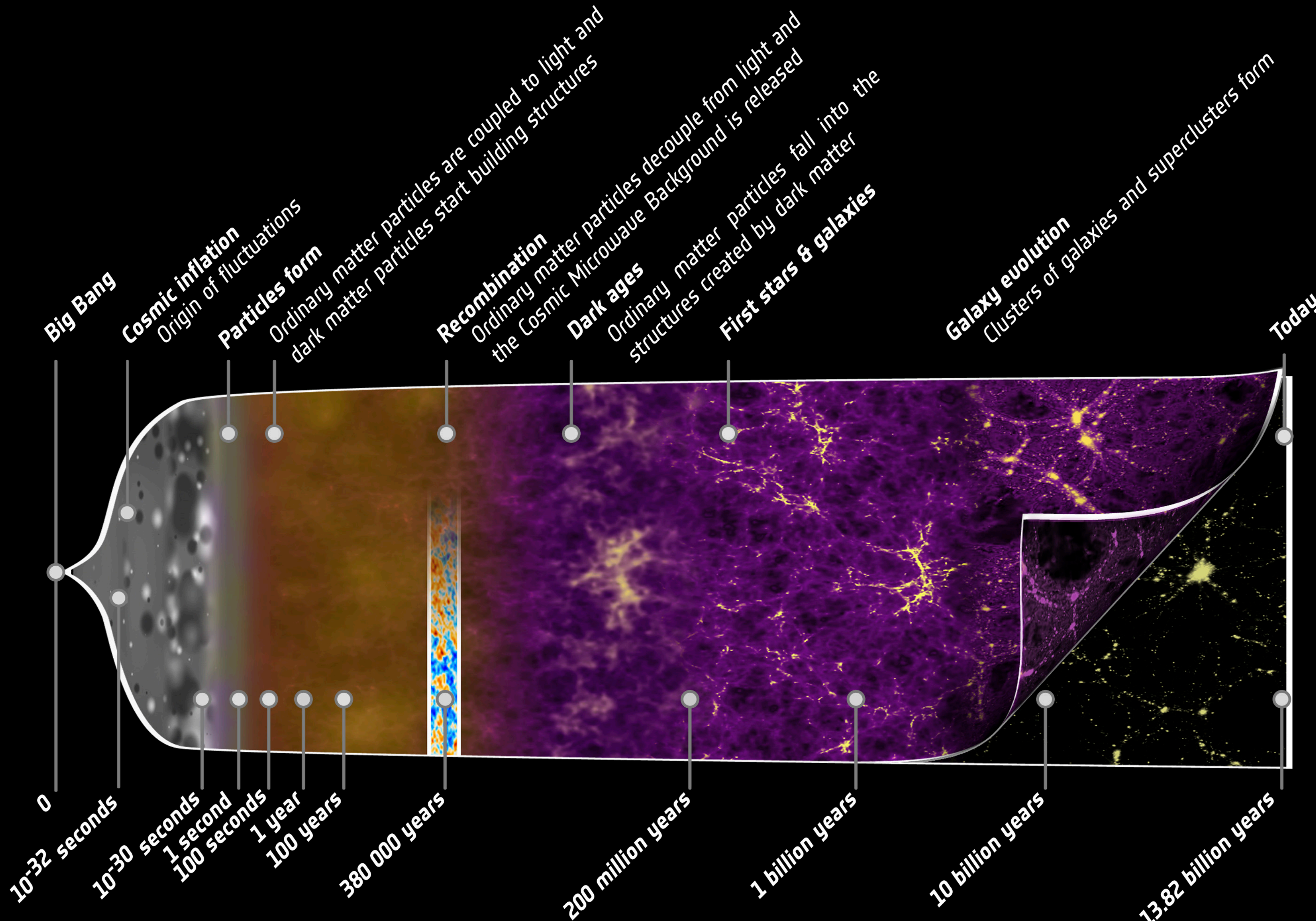
Inflation is an extremely rapid acceleration in the universe soon after its creation.

The Standard Big Bang Theory plus Inflation

(textbook version, see corrections later)



This lecture will be devoted to the first 10^{-35} seconds after the birth of the universe, and also to its very distant future.



Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
0
10^-32 seconds
10^-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years

Problems of the non-inflationary Big Bang theory:

- What was before the Big Bang?
- Why is our universe so **homogeneous** (better than 1 part in 10000)?
- Why is it **isotropic** (the same in all directions)?
- Why all of its parts started expanding simultaneously?
- Why is it **flat**? Why parallel lines do not intersect? Why is the universe so large? Why does it contain so many particles?

In the standard hot Big Bang theory it is assumed that the universe emerged from the singularity in a state of infinite density. Its description in terms of classical space and time is possible only after the Planck time $t_p \sim 10^{-43}$ seconds, when the size of the causally connected part of the universe was $l_p \sim c\, t_p \sim 10^{-33}$ cm, and the temperature of the universe was $T_p \sim 10^{32}$ K.

The part of the universe that we can see now has size 10^{28} cm. How large was this part at the Planck time?

Remember that now the temperature is $T_0 \sim 2.7$ K, and temperature was inversely proportional to the size of the universe: $T\, a = const$. Therefore the size of the universe at $T_p \sim 10^{32}$ K was approximately 10^{-32} of its present size, i.e. it was $10^{28} \times 10^{-32} \sim 10^{-4}$ cm.

Initial size 10^{-4} cm could seem tiny, but it is almost 10^{30} times greater than the initial size of the causally connected part of the universe $l_p \sim 10^{-33}$ cm

This means that **our part of the universe originally consisted of almost 10^{90} different regions which did not know about each other**.

Then why is our universe so uniform? Why did all of its parts started expanding simultaneously? Who 'polished' the universe? Who gave it the command to expand?

What would happen if the universe were much smaller?

The total lifetime (before collapse) of a **cold** closed universe of mass M is

$$t_{\max} \sim \frac{M}{M_p^2} \sim \frac{M}{M_p}\ 10^{-43}\mathrm{s}$$

Here M_p is the Planck mass, approximately 10^{-5} g

The total lifetime of a **hot** closed universe with a total entropy S (which is approximately equal to the total number of particles) is

$$t_{\max} \sim S^{2/3} \times 10^{-43}\mathrm{s}$$

Thus our universe must be **superheavy** and contain **exponentially many particles** when it was born.

Where did the energy come from?

Some basic facts:

1) Energy of matter in the universe **IS NOT CONSERVED:**

$$dE = -p\, dV$$

Volume V of an expanding universe grows, so its energy decreases if pressure p is positive.

2) **Total** energy of matter and of gravity (related to the shape and the volume of the universe) is conserved, but this conservation is somewhat unusual:

The sum of the energy of matter and of the gravitational energy is equal to zero

Energy of photons in the Big Bang theory

The total energy of radiation in the universe now is greater than **10^{53} g**. According to the Big Bang theory, the total number of photons in the universe practically did not change during its evolution, but the energy of each photon decreased as the temperature of the universe T.

The standard classical description of the universe becomes possible at the Planck time, when the temperature of the universe was 10^{32} times greater than now. At that time, the energy of radiation was greater than $10^{53} \times 10^{32}$ = **10^{85} g**

So before the Big Bang there was NOTHING, and then suddenly we got A HUGE AMOUNT OF ENERGY

Where did it come from?

Extending this investigation back to the cosmological singularity, where T was infinite, one finds that in order to create the universe in the Big Bang singularity one should have

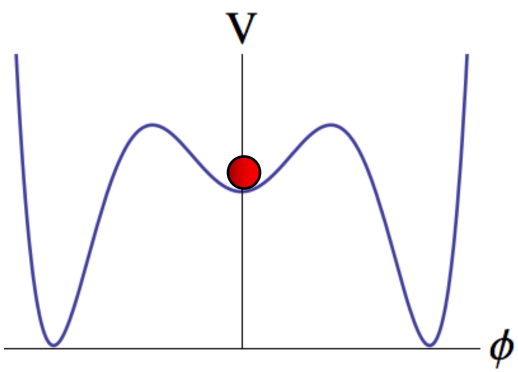
INFINITE AMOUNT OF ENERGY

Who could do it ?

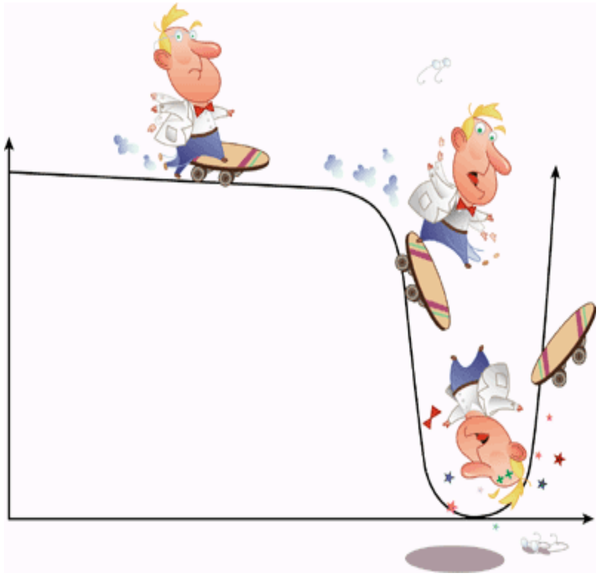
All these problems can be addressed in inflationary cosmology. In particular, inflation makes it possible to create our universe (and many others) from less than one milligram of matter.

Inflation

Starobinsky, 1980 – modified gravity, R + R^2. Original motivation was opposite to inflation: Instead of explaining uniformity of the universe, assumed that the universe was homogeneous from the very beginning. Observational predictions (Mukhanov and Chibisov) are great.

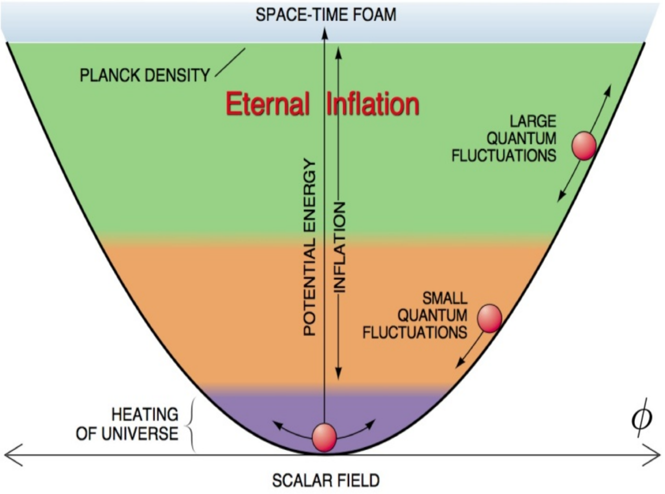


Guth, 1981 - old inflation. Beautiful idea, first outline of the new paradigm, but did not quite work, and did not predict inflationary perturbations



A.L., 1982 - new inflation (also Albrecht, Steinhardt)

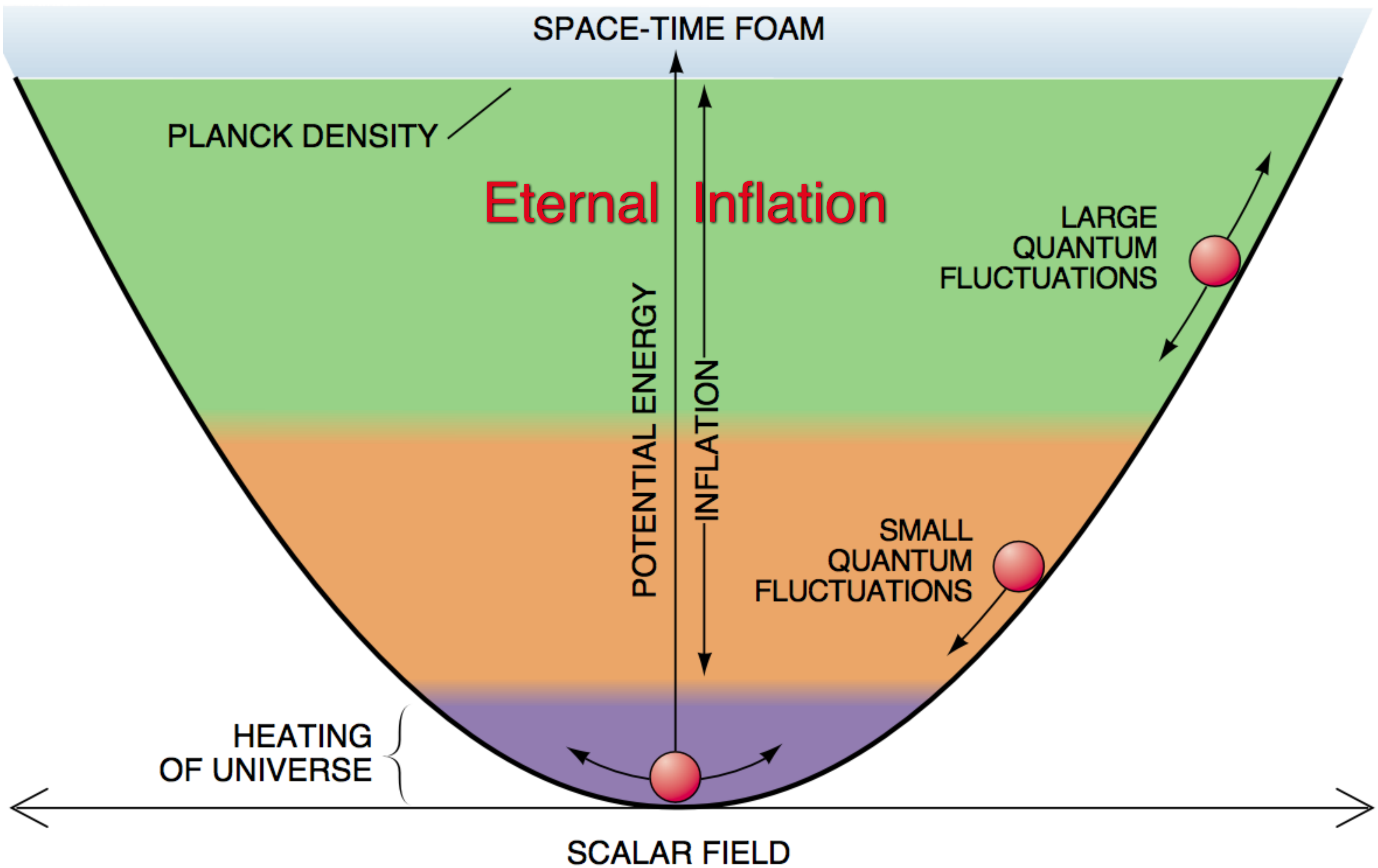
$$V(\phi) = \frac{m^2}{2}\phi^2$$



1983 - chaotic inflation

Inflation as a theory of a harmonic oscillator

$$V(\phi) = \frac{m^2}{2}\phi^2$$



Equations of motion:

- **Einstein equation:**

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{m^2}{6}\phi^2$$

- **Klein-Gordon equation:**

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi$$

Compare with equation for the harmonic oscillator with friction:

$$\ddot{x} + \alpha\dot{x} = -kx$$

Logic of Inflation:

Large φ → large H → large friction

→ field φ moves very slowly, so that its potential energy for a long time remains nearly constant

$$H = \frac{\dot{a}}{a} = \frac{m\phi}{\sqrt{6}} \approx \text{const}$$

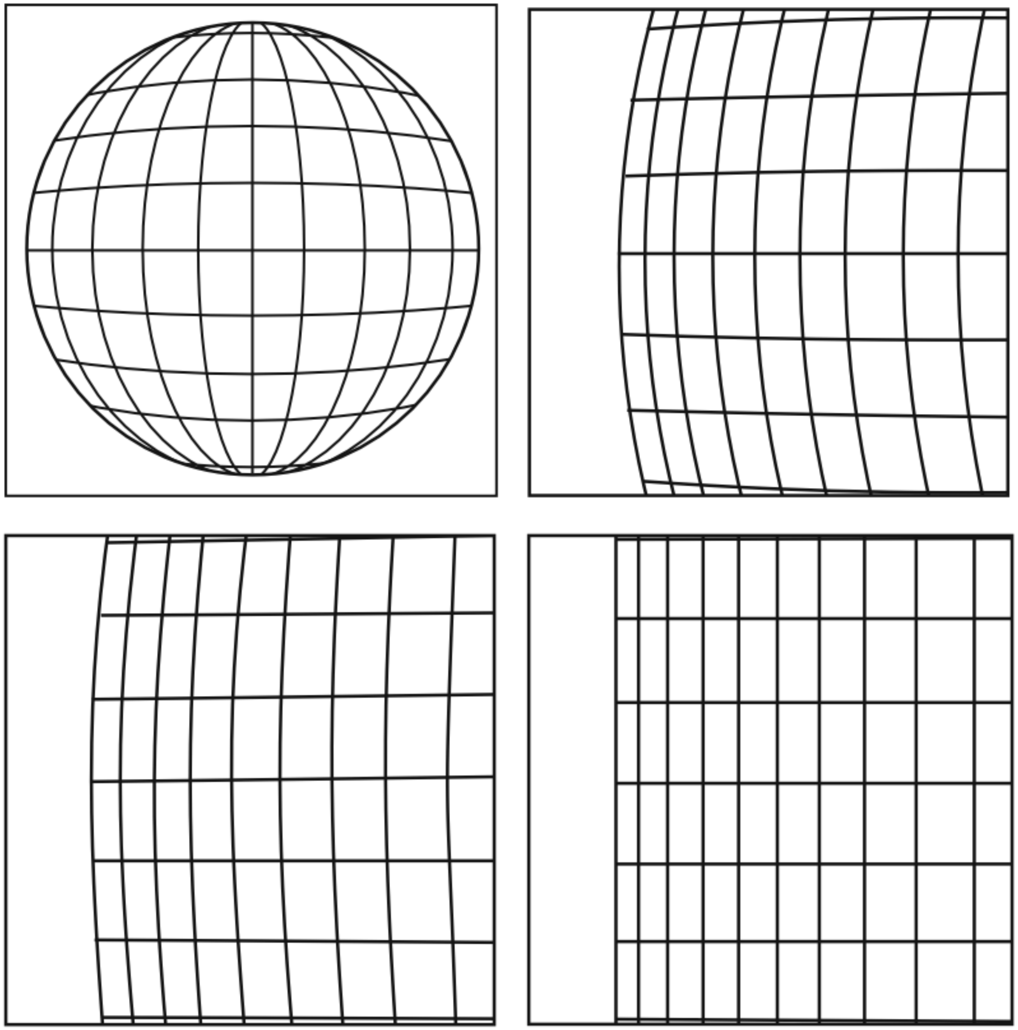
⇓

$$a \sim e^{Ht}$$

This is the stage of inflation

Inflation makes the universe flat, homogeneous and isotropic

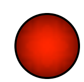
In this simple model the universe typically grows $10^{1000000000000}$ times during inflation.



Now we can see just a tiny part of the universe of size ct = 10^{10} light yrs. That is why the universe looks homogeneous, isotropic, and flat.

A newborn universe could be as small as 10^{-33} cm and as light as 10^{-5} g

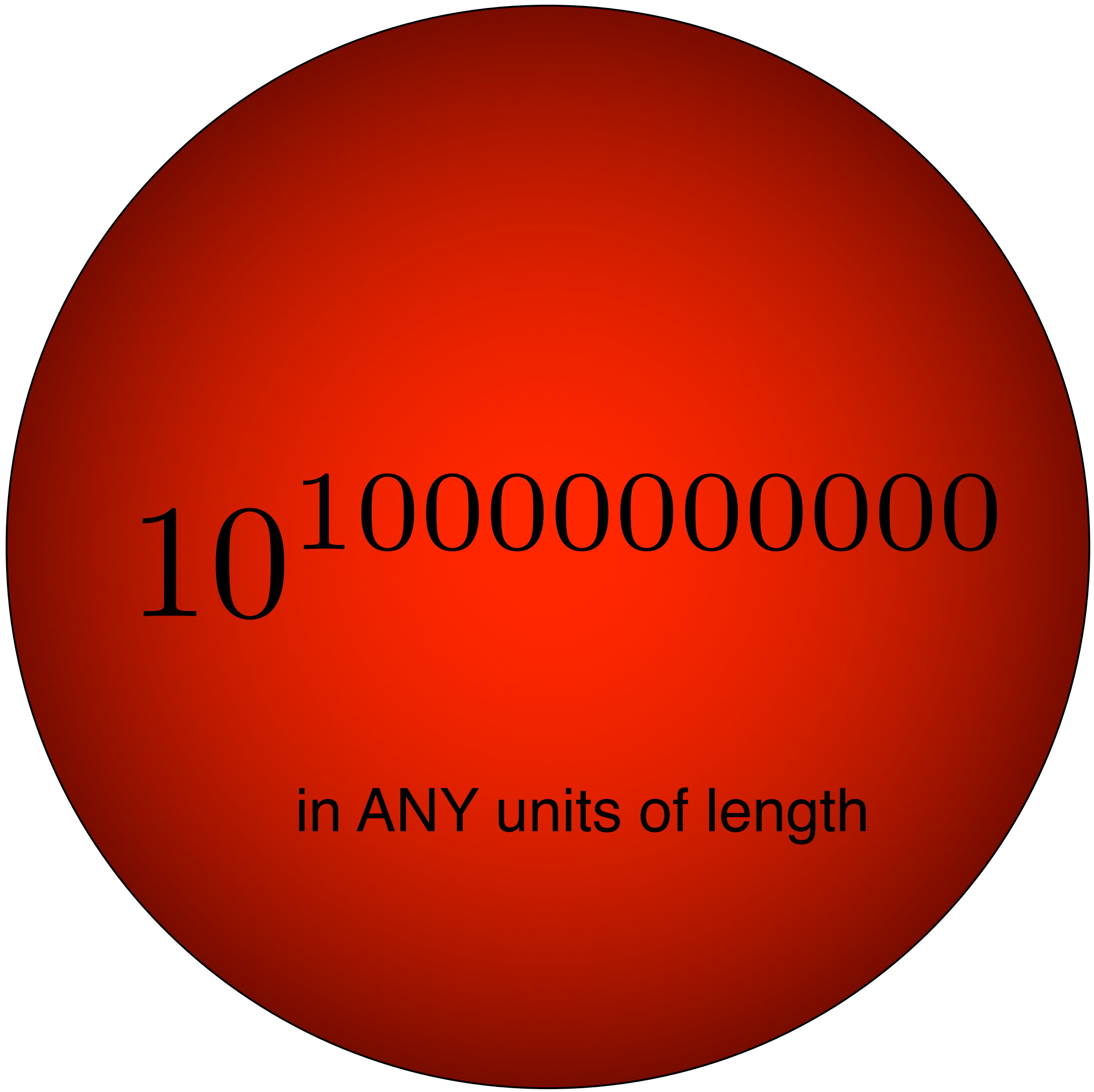
(it could be born from nothing at all...)



$$l \sim 10^{-33}\ \text{cm}$$

$$m \sim 10^{-5}\ \text{g}$$

Inflationary universe 10^{-35} seconds old

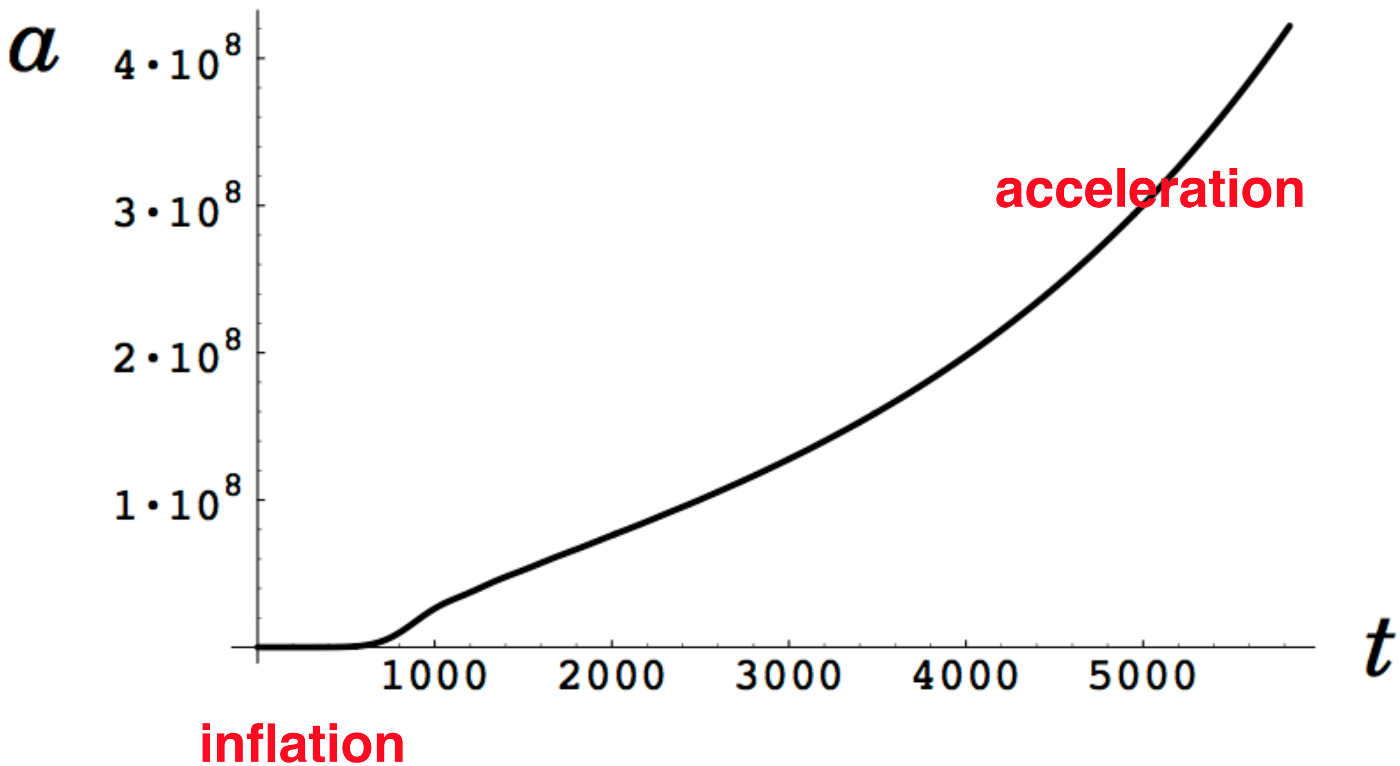


Add a constant to the inflationary potential

- get inflation and dark energy

$$V = \frac{m^2}{2}\phi^2 + \Lambda$$

The simplest model of inflation AND dark energy



Note that the energy density of the scalar field during inflation remains nearly constant, because at that stage the field practically does not change.

Meanwhile, the total volume of the universe during inflation grows exponentially, as $a^3(t) \sim e^{3Ht}$.

Therefore the total energy of the scalar field also grows exponentially, as $E \sim e^{3Ht}$.

After inflation, scalar field decays, and all of its energy is transformed into the exponentially large energy/mass of particles populating our universe.

We can start with a tiny domain of the smallest possible size (Planck length $l_P = M_P^{-1} \sim 10^{-33}$ cm) at the largest possible density (Planck density $M_P^4 \sim 10^{94}$ g/cm^3). The total energy of matter inside such a domain is $l_P^3 M_P^4 \sim M_P \sim 10^{-5}$ g. Then inflation makes this domain much larger than the part of the universe we see now.

What is the source of this energy?

Energy density and pressure for the scalar field:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

If the scalar field moves slowly, its pressure is negative,

$$p = w\rho, \qquad w \approx -1$$

Therefore energy of matter grows, $dE = -p\, dV > 0$

Existence of matter with $\mathrm{p} < 0$ allows the total energy of matter to grow at the expense of the gravitational energy, which becomes equally large but negative.

$$E_{matter} \sim + e^{3Ht}$$

$$E = 0$$

Total energy of the universe

Exponential instability

Simultaneous creation of space and matter

$$E_{space} \sim - e^{3Ht}$$

If such instability is possible, it appears over and over again. This leads to <u>eternal inflation,</u> which we will discuss later.

Inflation may start in the universe of the Planck mass (energy) $E \sim M_P \sim 10^{-5}$ g, at the Planck time $t_P \sim M_P^{-1} \sim 10^{-43}$ s.

But where did these initial 10^{-5} g of matter came from?

Uncertainty relation (in units $c = \hbar = 1$):

$$\Delta E \cdot \Delta t = M_P \cdot M_p^{-1} = 1$$

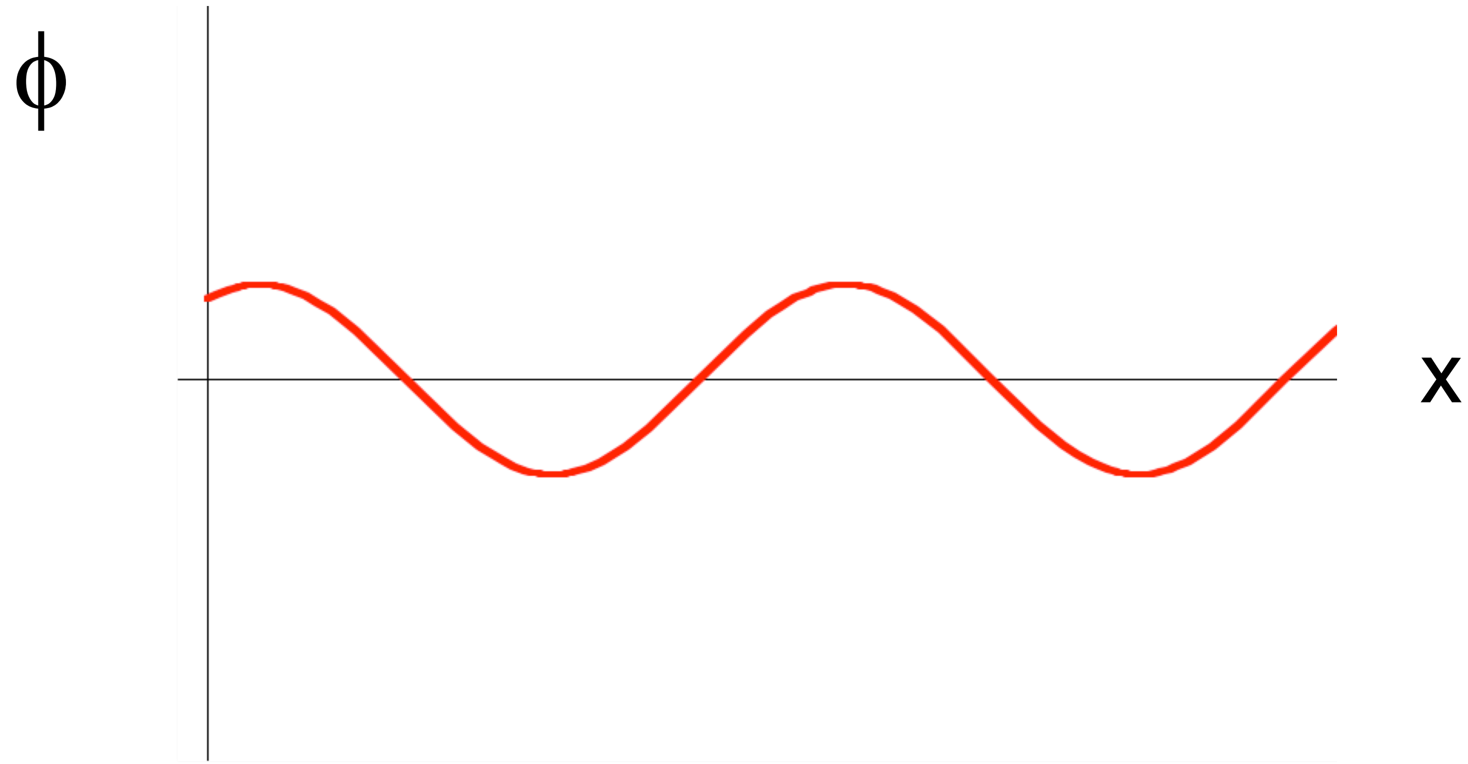
Thus the emergence of the initial 10^{-5} g of matter is compatible with the quantum mechanical uncertainty principle. And once we have 10^{-5} g of matter in the form of a scalar field, inflation begins, and energy becomes exponentially large.

If one can create the whole universe from one milligram of matter, or from nothing at all, what other miracles are possible?

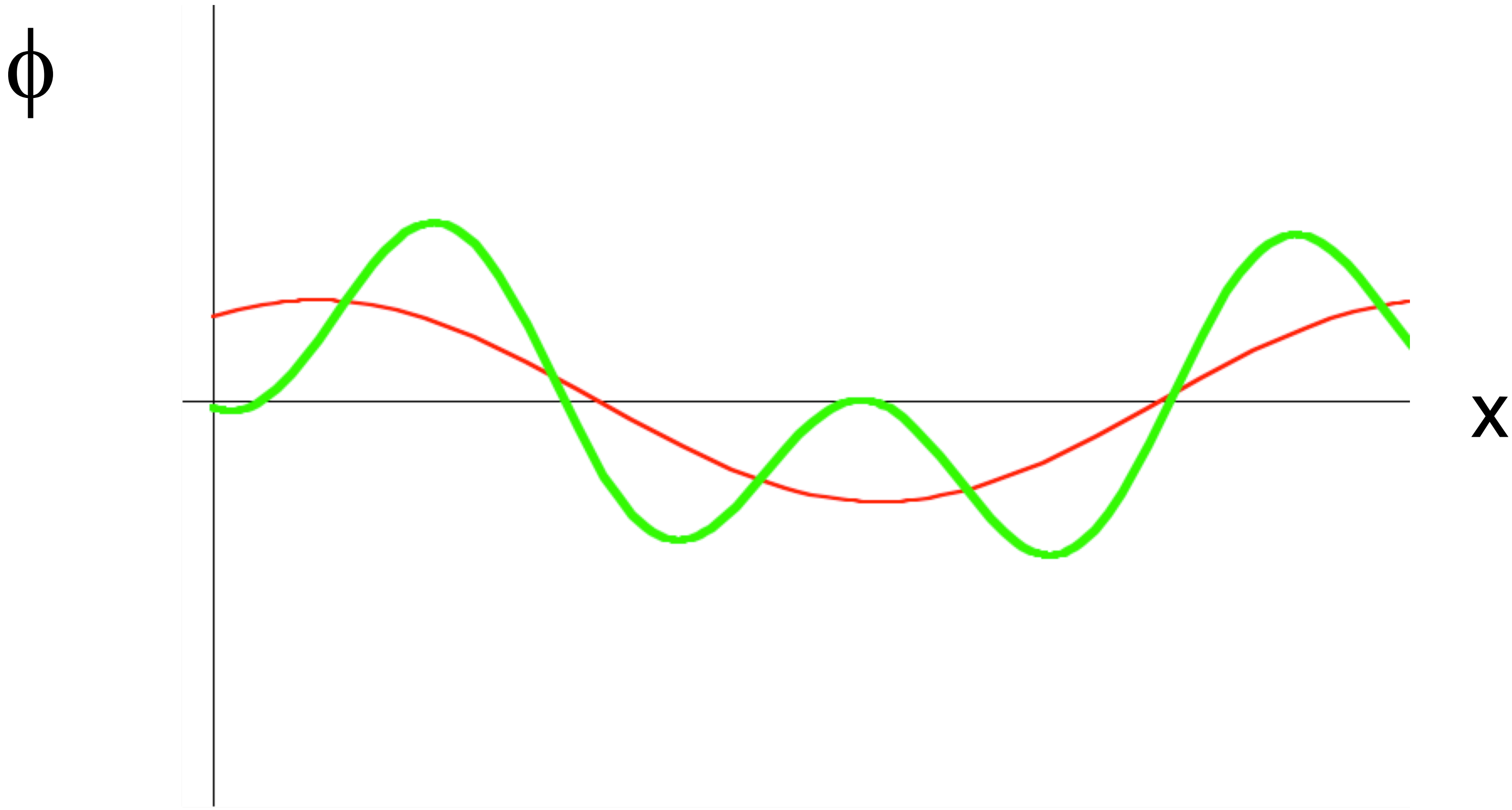
1) Inflation can create galaxies from quantum fluctuations.

2) Inflationary fluctuations can create new exponentially large parts of the universe (eternal inflation).

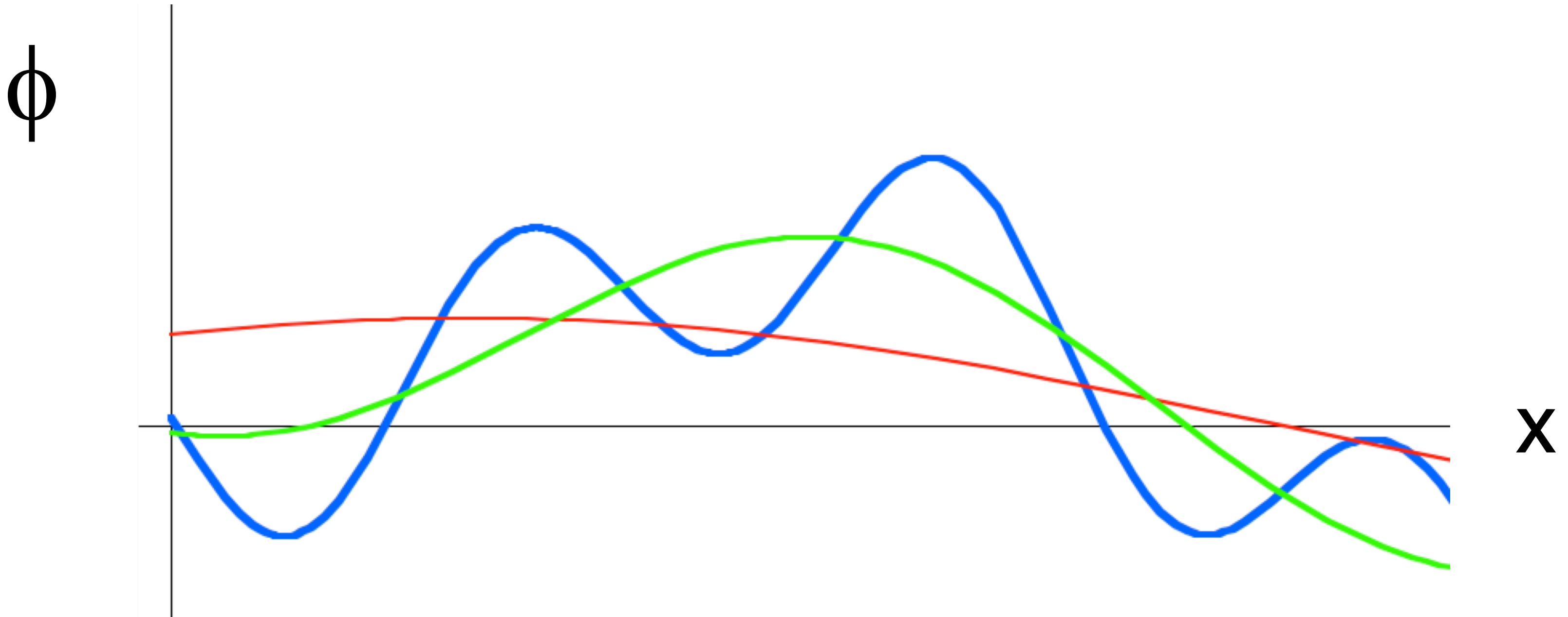
Quantum fluctuations produced during inflation



Small quantum fluctuations of all physical fields exist everywhere. They are similar to waves in the vacuum, which appear and then rapidly oscillate, move and disappear. Inflation stretched them, together with stretching the universe. When the wavelength of the fluctuations became sufficiently large, they stop moving and oscillating, and do not disappear. They look like frozen waves.



When expansion of the universe continues, new quantum fluctuations become stretched, stop oscillation and freeze on top of the previously frozen fluctuations.



This process continues, and eventually the universe becomes populated by inhomogeneous scalar field. Its energy takes different values in different parts of the universe. These inhomogeneities are responsible for the formation of galaxies.

Sometimes these fluctuations are so large that they substantially increase the value of the scalar field in some parts of the universe. Then inflation in these parts of the universe occurs again and again. In other words, the process of inflation becomes eternal.

We will illustrate it now by computer simulation of this process.

Inflationary perturbations and Brownian motion

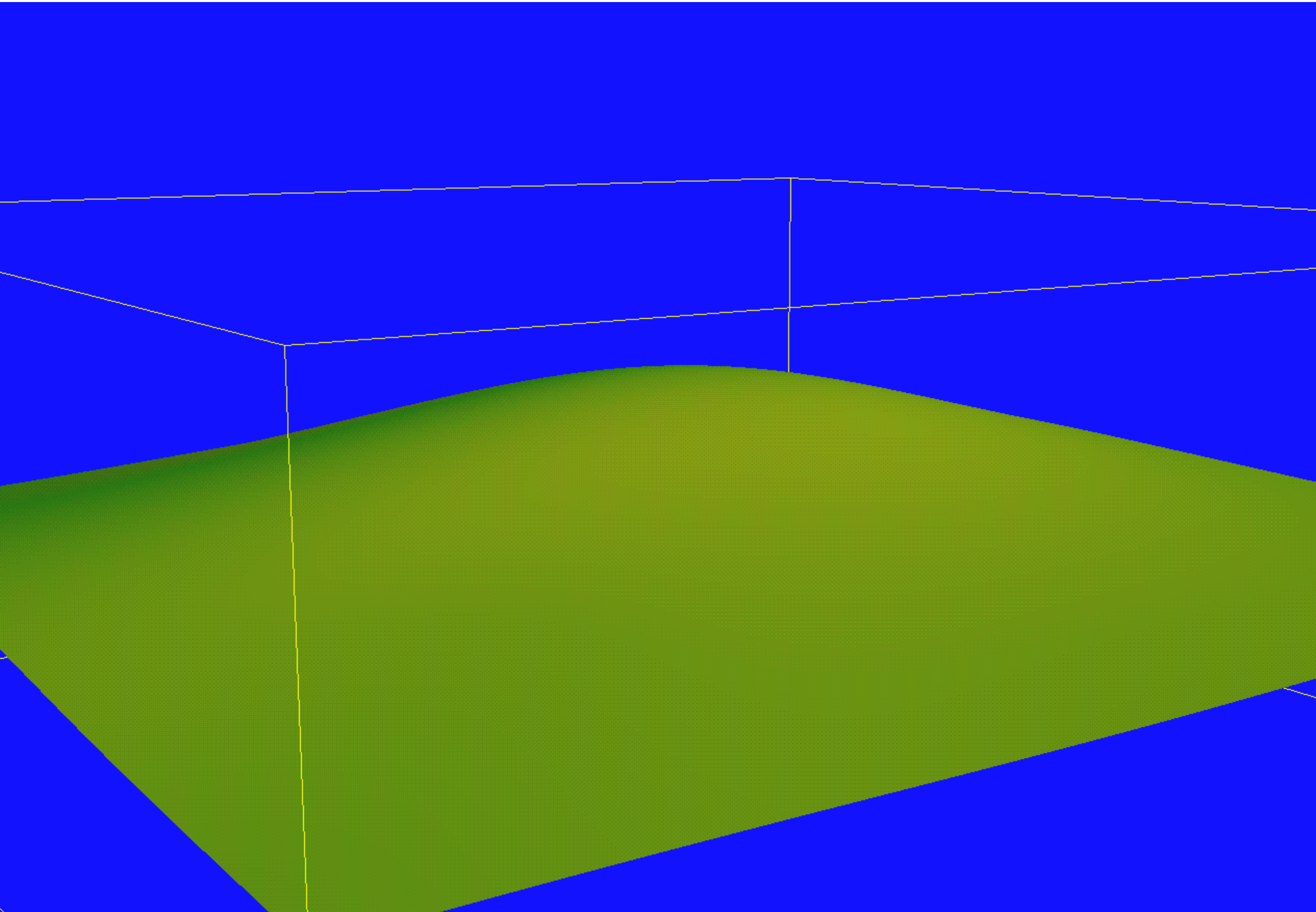
Perturbations of the massless scalar field are frozen each time when their wavelength becomes greater than the size of the horizon, or, equivalently, when their momentum k becomes smaller than H.

Each time $t = H^{-1}$ the perturbations with $H < k < e\, H$ become frozen. Since the only dimensional parameter describing this process is H, it is clear that the average amplitude $\delta\phi$ of the perturbations frozen during this time interval is proportional to H. A detailed calculation shows that

$$\delta\phi = \frac{H}{2\pi} = T_H$$

This is the Hawking temperature in dS space. This process repeats each time $t = H^{-1}$, but the sign of $\delta\phi$ each time can be different, like in the Brownian motion. This produces chaotically distributed classical scalar field with exponentially large wavelengths, due to inflation.

Generation of Quantum Fluctuations



Predictions of inflation:

1) The universe is flat, $\Omega = 1$. (In the mid-90's, the consensus was that $\Omega = 0.3$, until the discovery of dark energy.)

2) The observable part of the universe is uniform.

3) It is isotropic. In particular, it does not rotate. (Back in the 80's we did not know that it is uniform and isotropic at such an incredible level.)

4) Perturbations produced by inflation are adiabatic

5) Unlike perturbations produced by cosmic strings, inflationary perturbations lead to many peaks in the spectrum

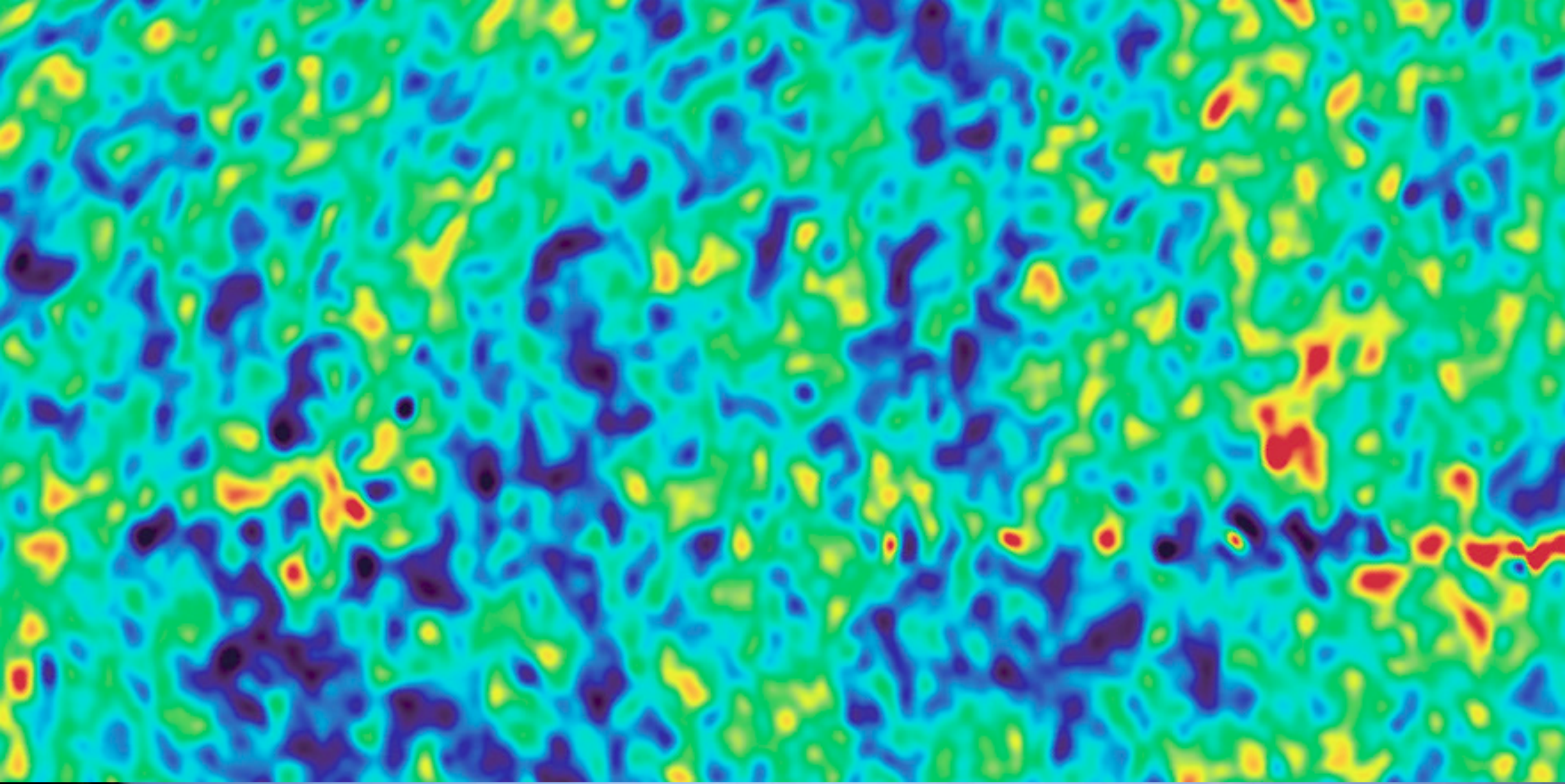
6) The large angle TE anti-correlation (WMAP, Planck) is a distinctive signature of superhorizon fluctuations (Spergel, Zaldarriaga 1997), ruling out many alternative possibilities

Predictions of inflation:

7) Inflationary perturbations should have a nearly flat (but not exactly flat) spectrum. A small deviation from flatness is one of the distinguishing features of inflation. It is as significant for inflationary theory as the asymptotic freedom for the theory of strong interactions.

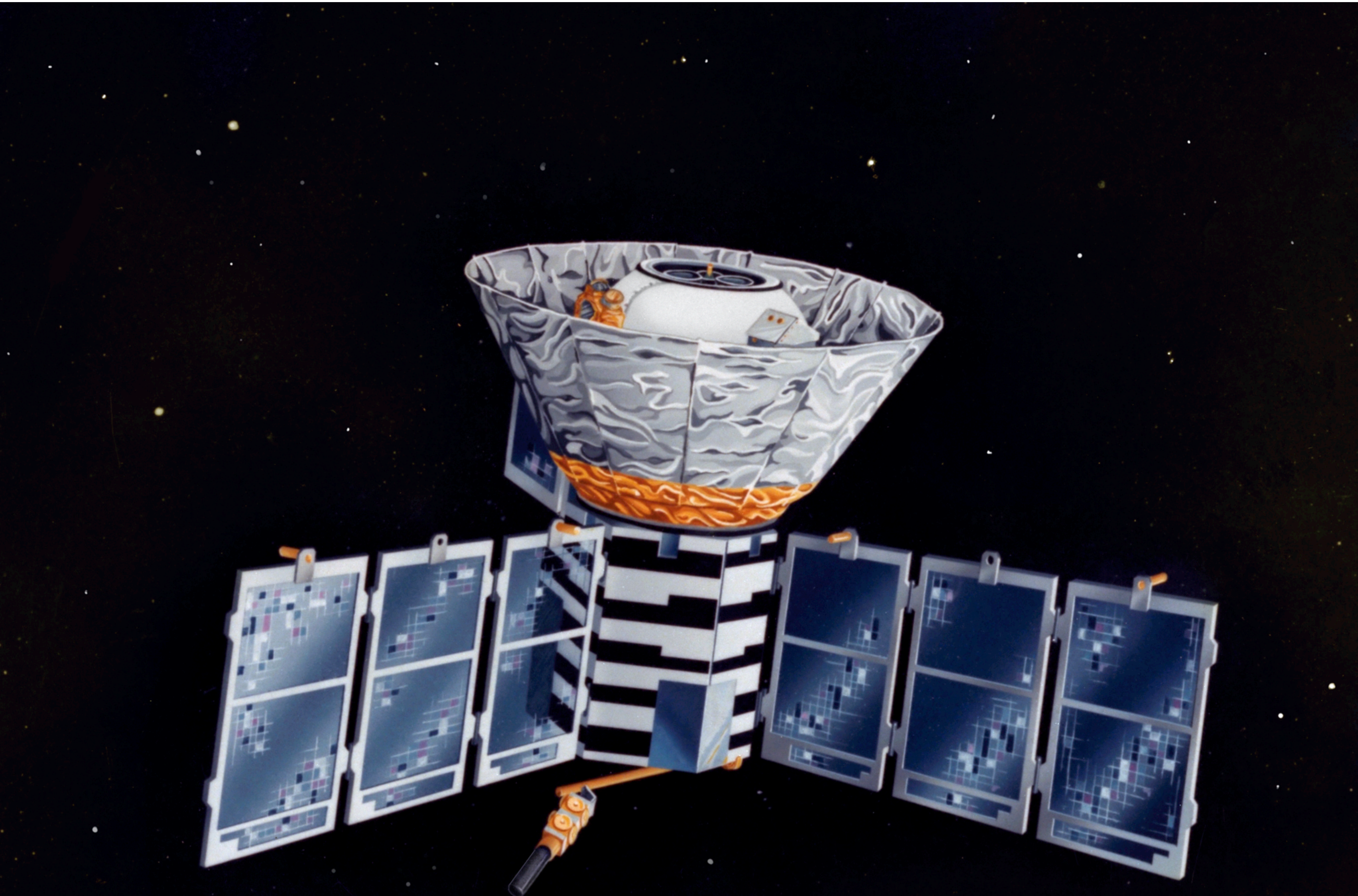
8) Inflation produces scalar perturbations, but it also produces tensor perturbations with nearly flat spectrum, and it does not produce vector perturbations. There are certain relations between the properties of scalar and tensor perturbations.

9) Scalar perturbations are Gaussian. In non-inflationary models, the parameter f_{NL}^{local} describing the level of local non-Gaussianity can be as large as 10^4, but it is predicted to be O(1) in all single-field inflationary models. Prior to the Planck2013 data release, there were rumors that f_{NL}^{local} >> O(1), which would rule out all single field inflationary models. The final WMAP results (2012) were $-3 < f_{NL}^{local} < 77$ at 95% CL.

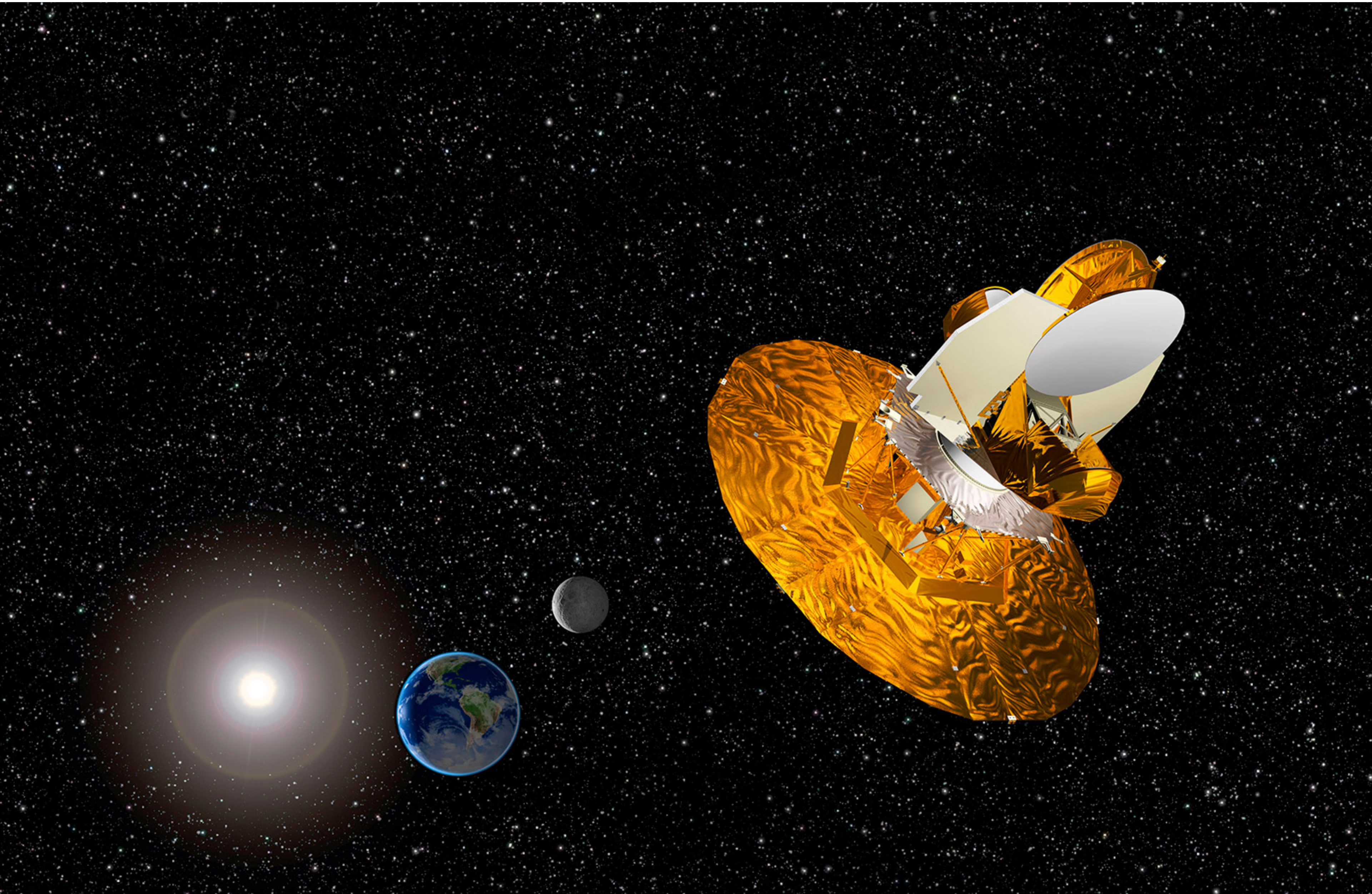


Observations

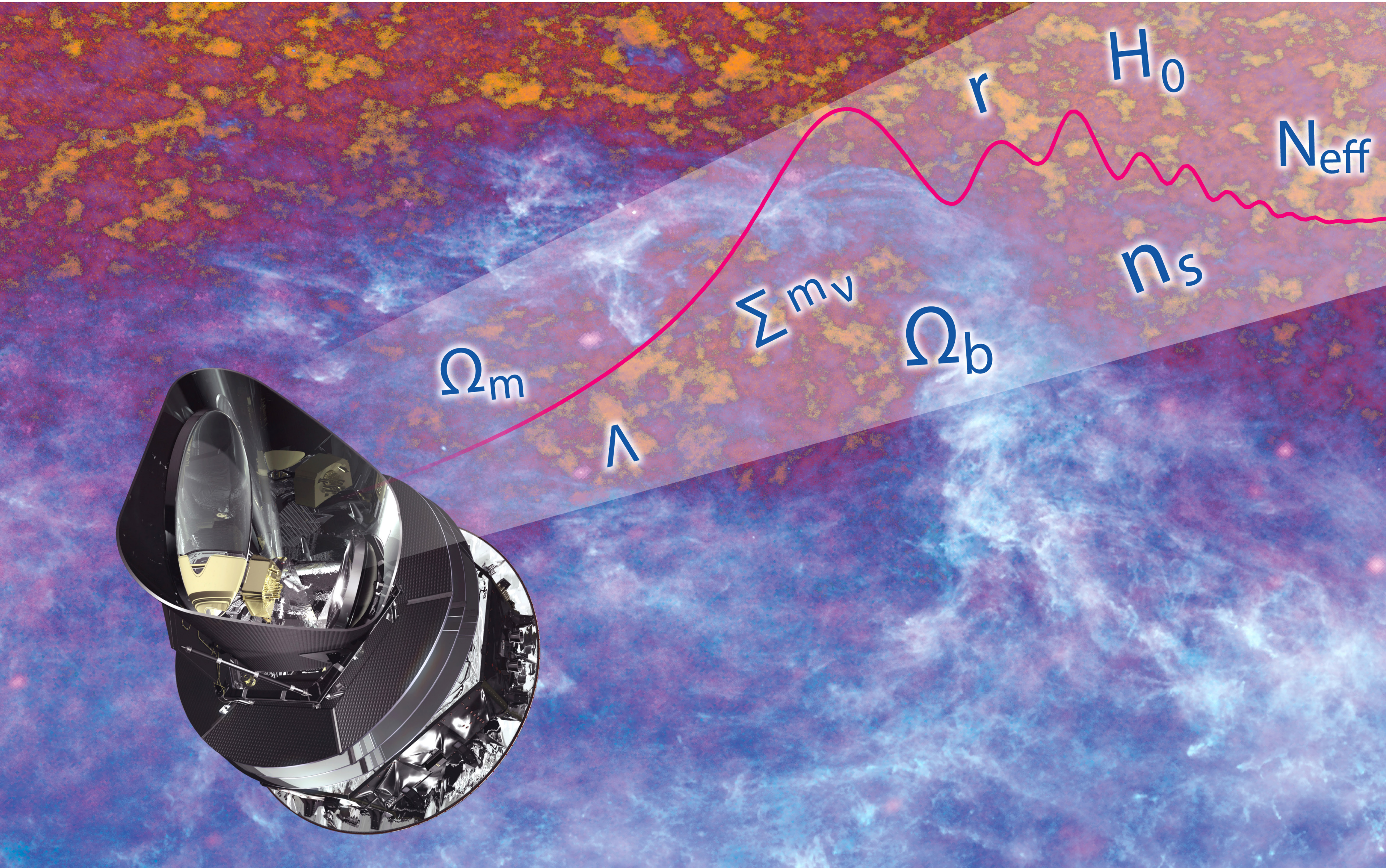
COBE



WMAP

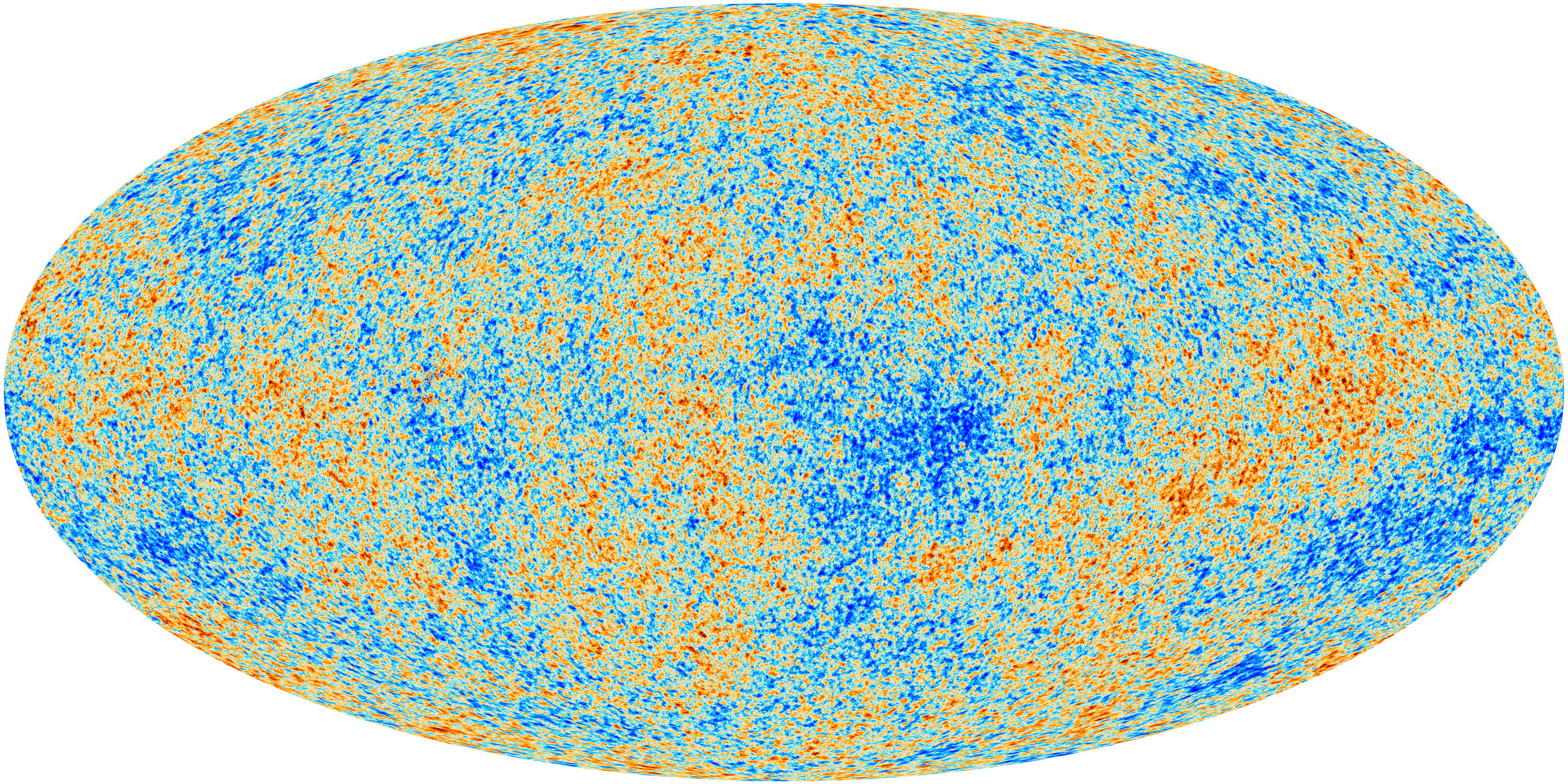


Planck satellite



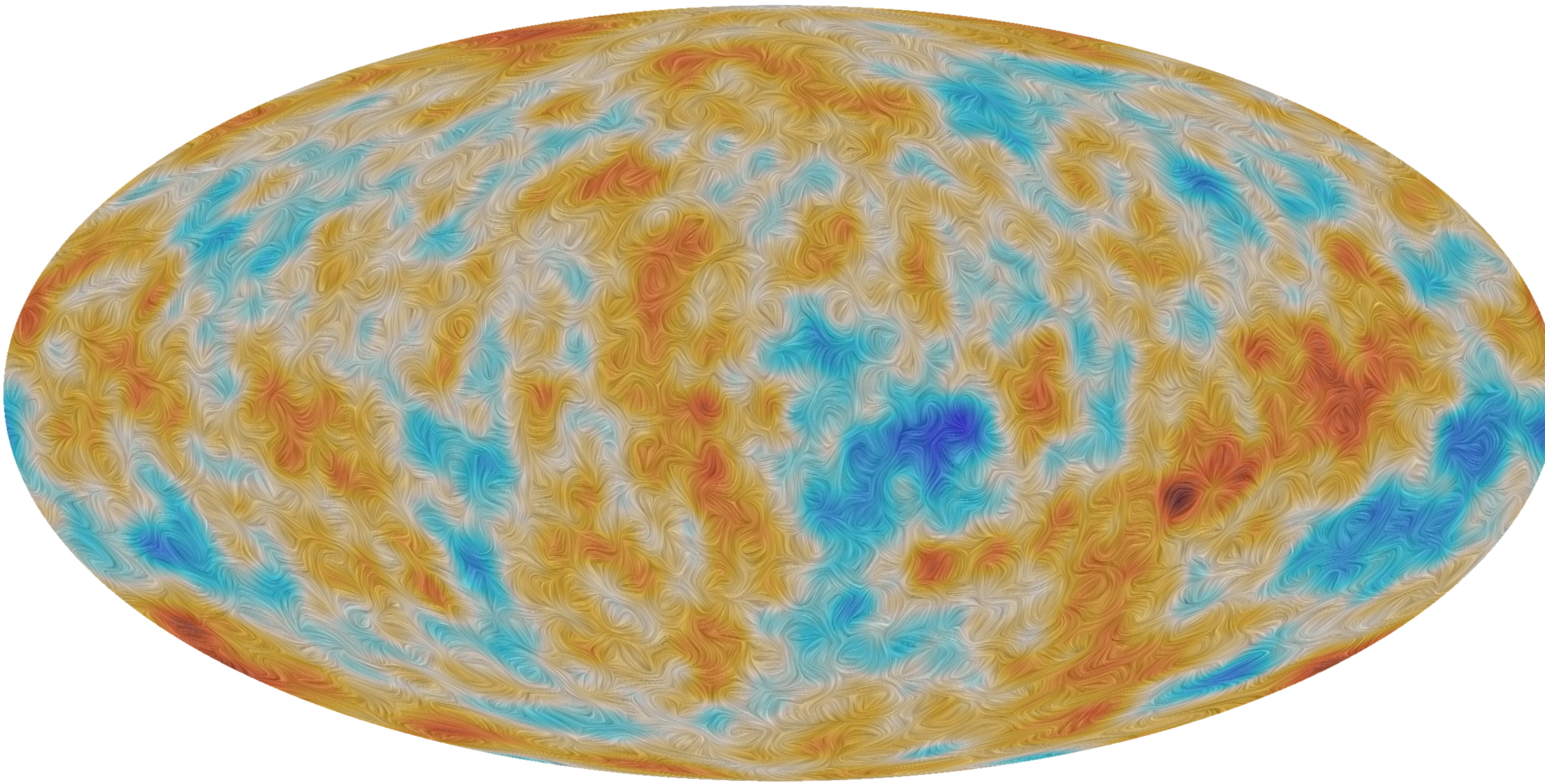
Planck 2013: Perturbations of temperature

This is an image of quantum fluctuations produced 10^{-35} seconds after the Big Bang. These tiny fluctuations were stretched by inflation to incredibly large size, and now we can observe them using all sky as a giant photographic plate!!!

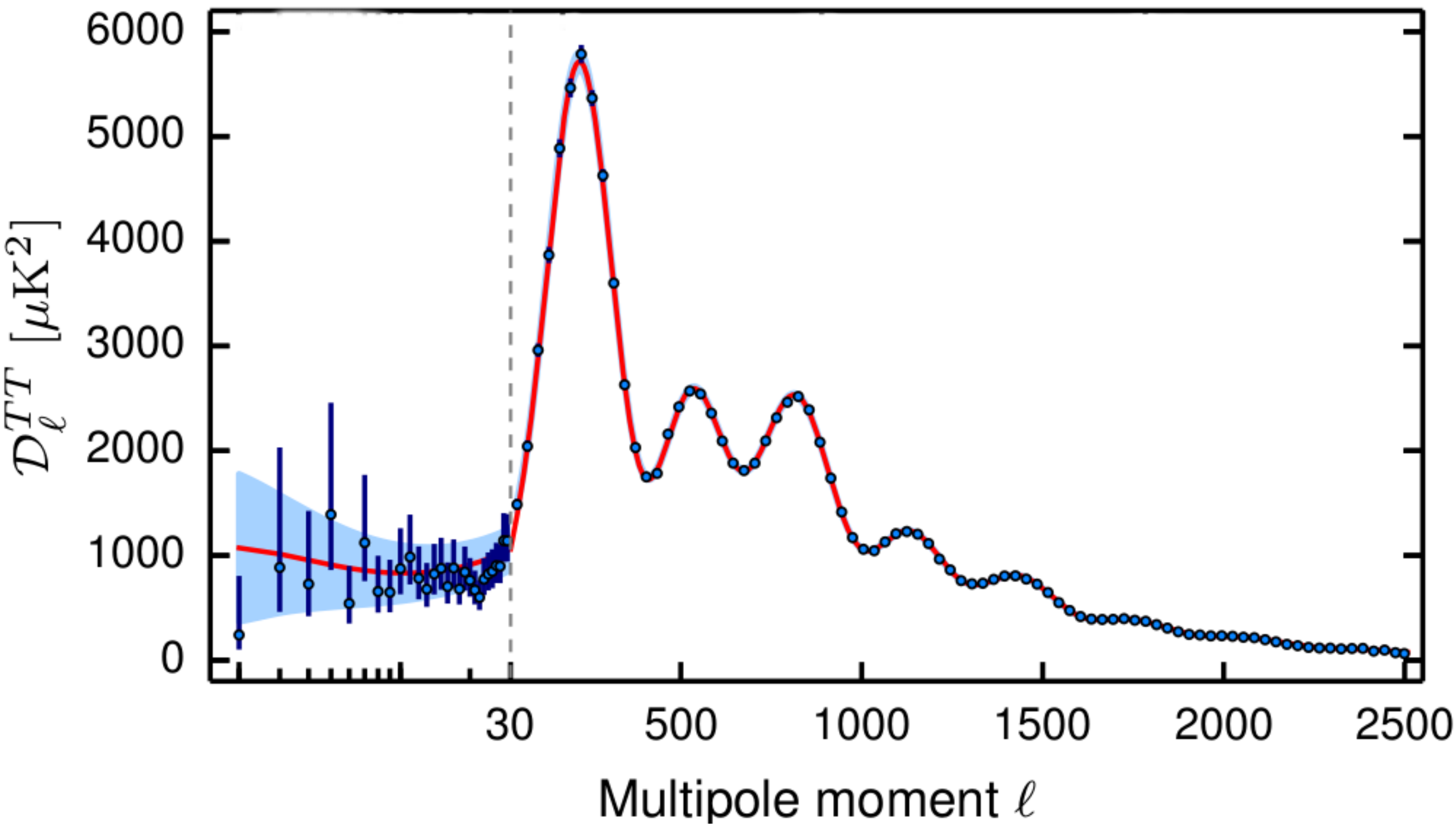


Planck 2015: Perturbations of polarization

E-modes



Planck 2015: Temperature perturbations (blue dots) and predictions of inflationary theory (red line)



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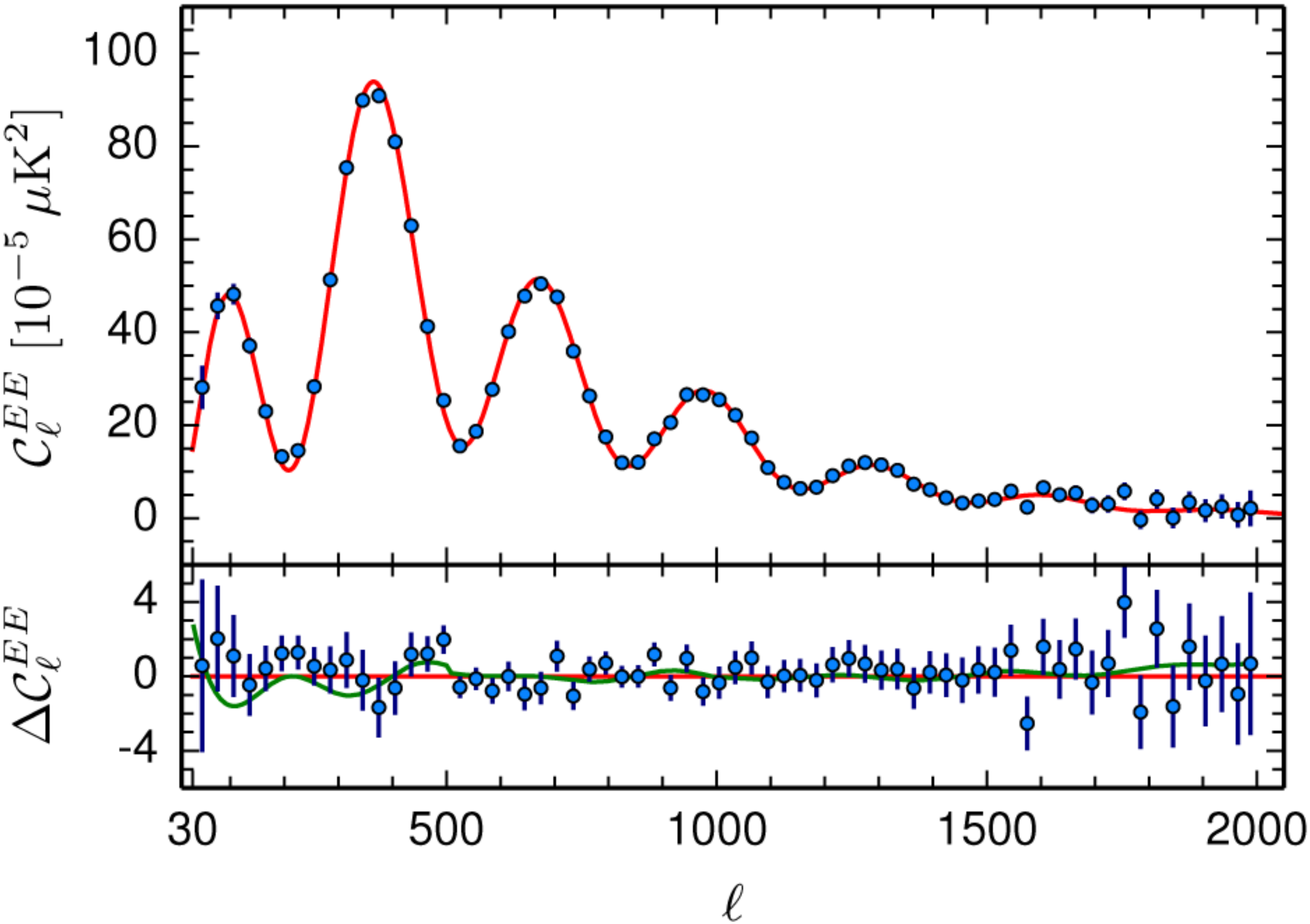

Inflation and Planck2015

$$\Omega = 1 \pm 0.005$$

Universe is flat with accuracy 10^{-2}

$$n_s = 0.968 \pm 0.006$$

Spectrum of perturbations is nearly flat

Non-inflationary HZ spectrum with $n_s = 1$ is ruled out at a better than 6σ level, just as predicted in 1981 by Mukhanov and Chibisov. (This is an important prediction of inflation, similar to asymptotic freedom in QCD.)

$$f_{\rm NL}^{\rm local} = 0.8 \pm 5$$

Agrees with predictions of the simplest inflationary models with accuracy $O(10^{-4})$.

An impressive success of inflationary theory

Can we test inflation even better ?

1) Yet another Planck data release in about a year.

2) **B-modes**: a special polarization pattern which can be produced by gravitational waves generated during inflation. A discovery of the gravitational waves of this type (BICEP2, Keck and other experiments) could provide a strong **additional** evidence in favor of inflation.

   A non-discovery is fine too: many inflationary models predict a very small amplitude of the gravitational waves.

Dreaming about the future

The total cost of finding the Higgs boson ran about **$10 billion**… which seems like a **bargain**… especially when you consider the fact that LHC and its associated experiments are bringing us much closer to understanding the **mysteries of the universe**.

Forbes Magazine
7/05/2012

The total cost of the Planck satellite, which, arguably, brings us much closer to understanding the mysteries of the universe than LHC, is about **$1 billion**.

Long way to go!