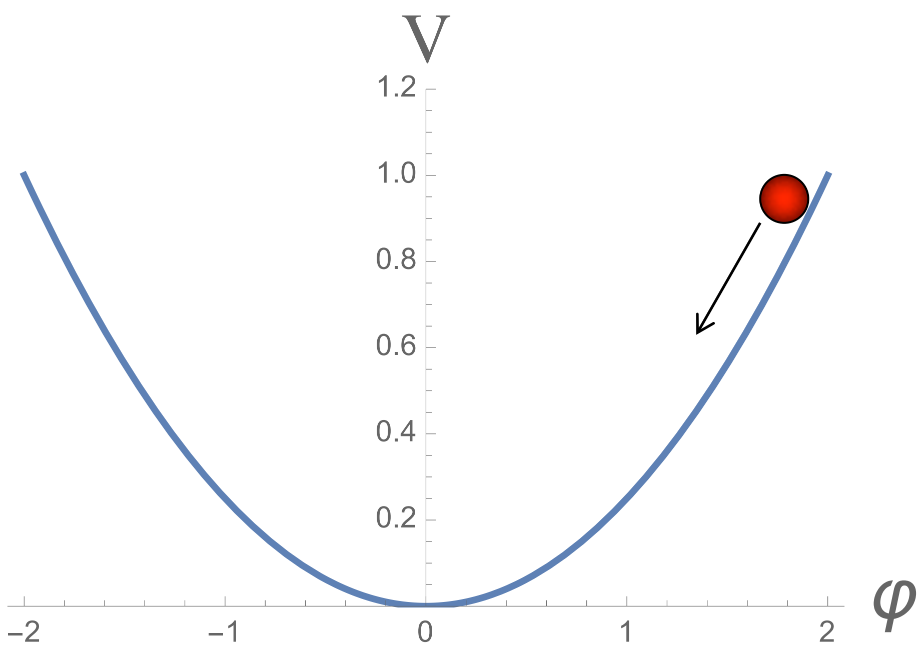
Lecture III, part 1

# Large field inflation and cosmological attractors

## (basic models)

# Simplest inflationary model: $V = \frac{m^2\phi^2}{2}$

Inflation can start at the Planck density if there is **a single Planck size domain** with a potential energy V of the same order as kinetic and gradient density. This is the minimal requirement, compared to standard Big Bang, where initial homogeneity is requires across **$10^{90}$ Planck size domains**.

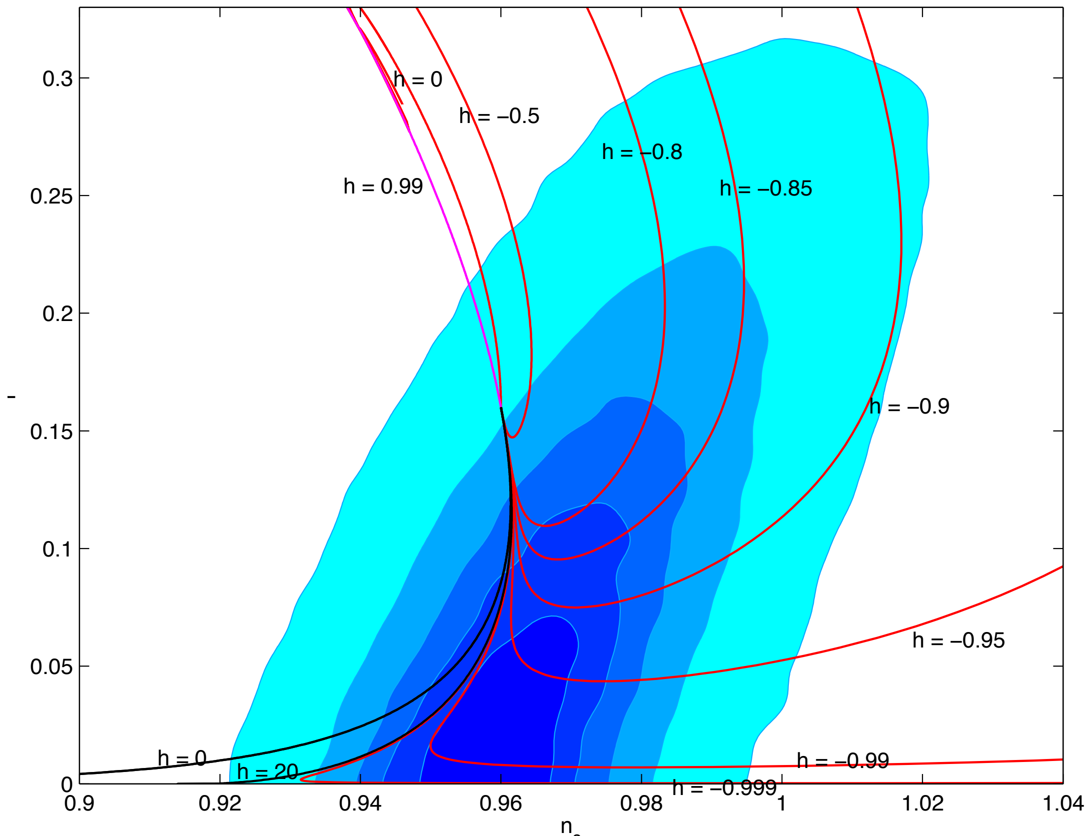


# Polynomial inflation:

Simplest quadratic model predicts too large amount of the gravitational waves. However, it can be trivially generalized to avoid this problem, while still offering the possibility of inflation beginning at the Planck density

$$V = \frac{m^2\phi^2}{2}\left(1 - a\phi + b\phi^2\right)$$

Destri, de Vega, Sanchez, 2007



# One can fit all Planck data by a polynomial, with inflation starting at the Planck density

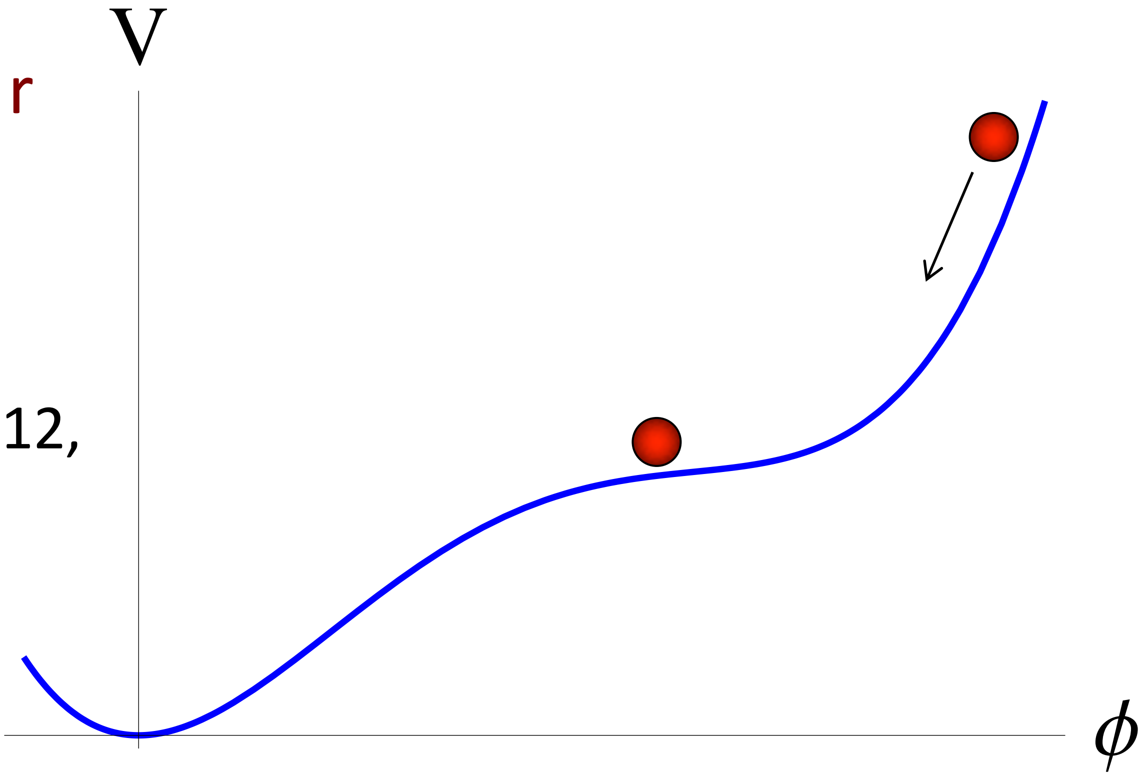
$$V = \frac{m^2\phi^2}{2}\left(1 - a\phi + b\phi^2\right)$$

Destri, de Vega, Sanchez, 2007
Nakayama, Takahashi and Yanagida, 2013
Kallosh, AL, Westphal 2014
Kallosh, AL, Roest, Yamada **1705.09247**

3 observables: $A_s$, $n_s$, r

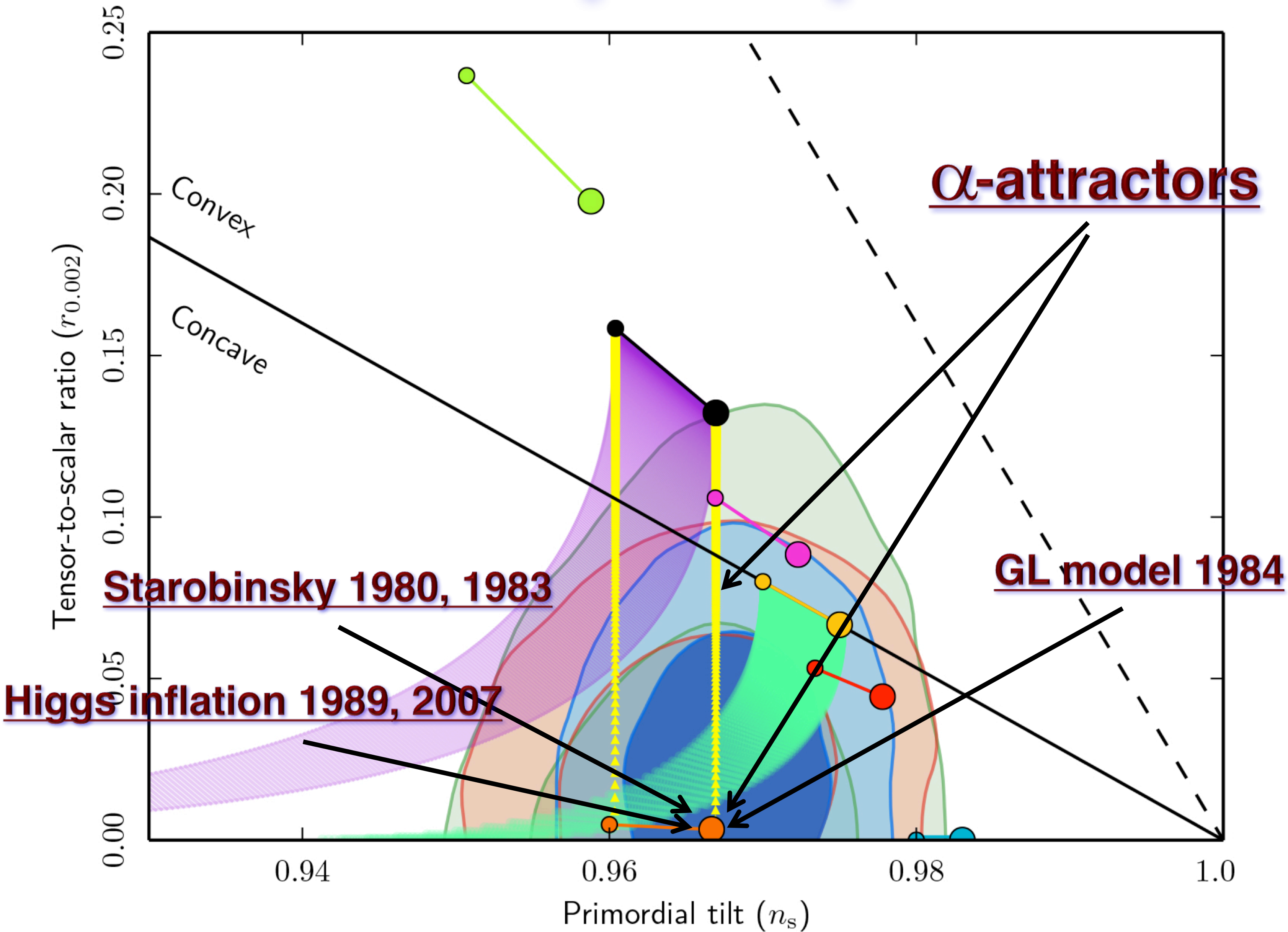
3 parameters: m, a, b

Example: m =$10^{-5}$, a = 0.12, b=0.29



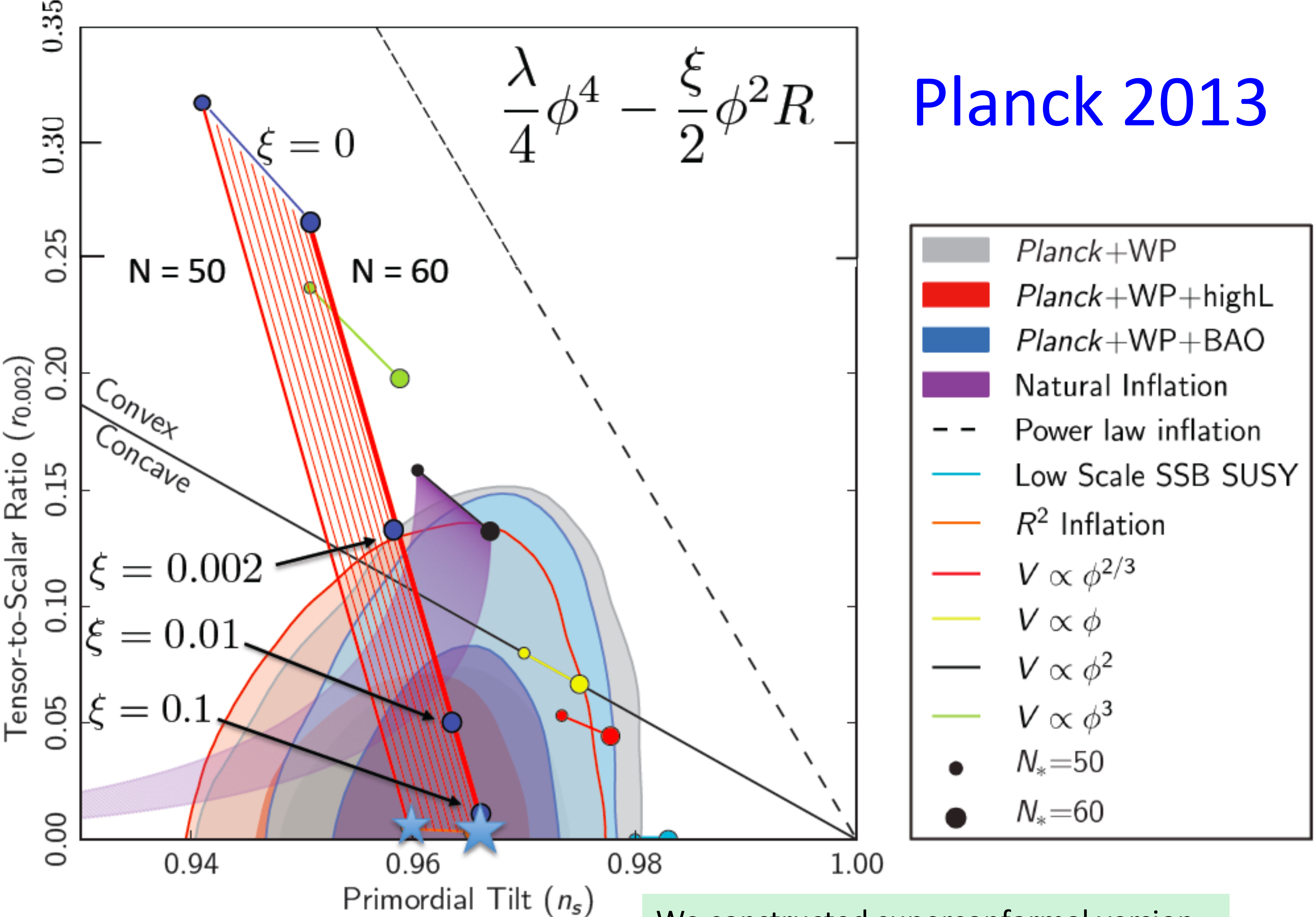

**But the best fit is provided by models with plateau potentials**

# The most natural fit is provided by models with plateau potentials



# First models of cosmological attractors
## and their subsequent generalizations

# Planck 2013




June 2013

We constructed superconformal version of the theory with arbitrary $\xi$

$$\frac{\lambda}{4}\phi^4 - \frac{\xi}{2}\phi^2 R$$

Minimal coupling to gravity $\xi = 0$ r = 0.3, too big

Conformal coupling to gravity $\xi = -\frac{1}{6}$ no inflation

(conformal invariance is broken by the Einstein term R)

Agreement with data for a quartic potential for $\xi \gtrsim 0.002$

When $\xi = 0.1$ the asymptotic values of $n_s$ and r are reached

Standard model Higgs inflation requires $\xi \approx 10^4$ and gives practically the same $n_s$ and r

Is it possible to have local conformal symmetry (Weyl symmetry) for

$$\xi \neq -\frac{1}{6}$$

This is against the intuition of General Relativity, but totally natural in superconformal framework

In General Relativity even with many scalars which live in a real moduli space one would still expect Weyl symmetry only if

$$\xi = -\frac{1}{6}$$

The reason why it is possible to have an effective arbitrary $\xi$ has to do with the feature of the scalar moduli space called Kahler geometry

1306.3211

Note, that for the superconformal symmetry it is necessary to have Kahler geometry

One could have used Kahler geometry in bosonic models, but in supersymmetric ones it was natural

The way to study it is to start with a conformally invariant theory of 2 complex fields, one of which (conformon) can be removed by a gauge transformation. (More about it later.)

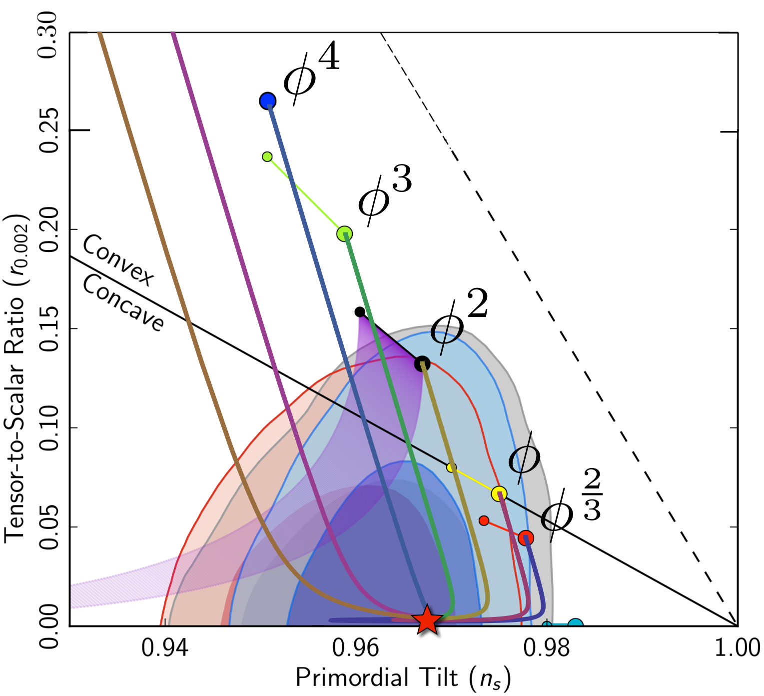
The final result is that the following theory describing two real scalars is (nearly) conformally invariant. (As before, the invariance is broken only by the first (Einstein) term.)

$$\tfrac{1}{2}M_P^2 R - \tfrac{1}{2}\left(\frac{1}{6}-\Delta\right)\varphi_1^2 R - \tfrac{1}{2}\left(\frac{1}{6}+\Delta\right)\varphi_2^2 R - \tfrac{1}{2}\partial^\mu\varphi_1\,\partial_\mu\varphi_1 - \tfrac{1}{2}\partial^\mu\varphi_2\,\partial_\mu\varphi_2 - \tfrac{\lambda}{4}(\varphi_1^2+\varphi_2^2)^2$$

It would be very difficult to guess this form or derive this result without using Kahler geometry (or using supergravity and then ignoring fermions).

# “Combing” Chaotic Inflation: non-minimal coupling to gravity

2013 October




N ~ 50-60: number of e-foldings of inflation

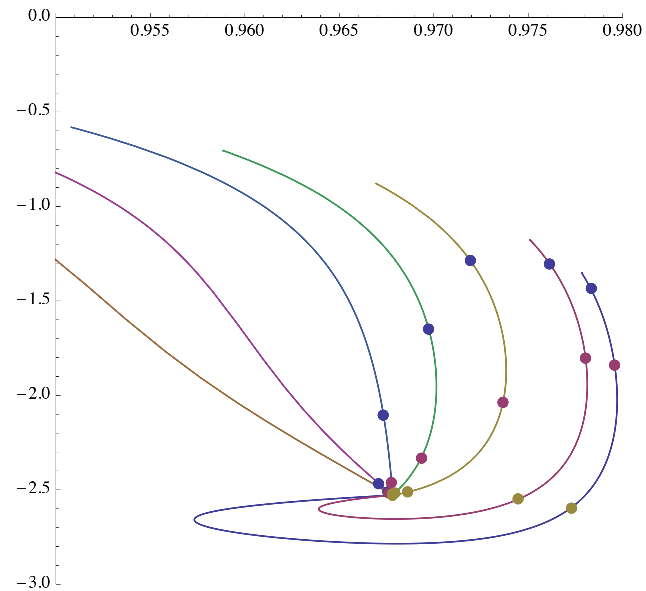
$$\mathcal{L}_{\rm J} = \sqrt{-g}[\tfrac{1}{2}\Omega(\phi)R - \tfrac{1}{2}(\partial\phi)^2 - V_J(\phi)]$$

$$\Omega(\phi) = 1 + \xi f(\phi)\,, \qquad V_J(\phi) = \lambda^2 f^2(\phi)$$

## Approaching the attractor

in the leading approximation in N

$$1 - n_s = 2/N\,, \qquad r = 12/N^2$$




Vicinity of the attractor point

# De Sitter from spontaneously broken conformal symmetry

Kallosh, AL 2013

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\partial_\mu\chi\partial_\nu\chi\, g^{\mu\nu} + \frac{\chi^2}{12}R(g) - \frac{\lambda}{4}\chi^4\right]$$

This theory is locally conformal invariant

$$\tilde{g}_{\mu\nu} = \mathrm{e}^{-2\sigma(\mathrm{x})} g_{\mu\nu}\,, \qquad \tilde{\chi} = \mathrm{e}^{\sigma(\mathrm{x})}\chi$$

The field $\chi$ (x) is referred to as a conformal compensator, which we will call `**conformon**.' It has negative sign kinetic term, but this is not a problem because it can be removed from the theory by fixing the gauge symmetry, for example, using the gauge

$$\chi = \sqrt{6}$$

This gauge fixing can be interpreted as a spontaneous breaking of conformal invariance by the classical field $\chi = \sqrt{6}$

The action in this gauge:
dS or AdS

$$\mathcal{L} = \sqrt{-g}\left[\frac{R(g)}{2} - 9\lambda\right]$$

The simplest conformally invariant two-field model of dS or AdS space and the SO(1,1) invariant conformal gauge

$$\mathcal{L} = \frac{\sqrt{-g}}{2}\left[(\partial_\mu\chi\partial^\mu\chi - \partial_\mu\phi\partial^\mu\phi) + \frac{\chi^2-\phi^2}{6}R(g) - \lambda\frac{(\phi^2-\chi^2)^2}{4}\right]$$

Local conformal symmetry

$$\tilde{g}_{\mu\nu} = \mathrm{e}^{-2\sigma(\mathrm{x})}g_{\mu\nu}\,, \qquad \tilde{\chi} = \mathrm{e}^{\sigma(\mathrm{x})}\chi\,, \qquad \tilde{\phi} = \mathrm{e}^{\sigma(\mathrm{x})}\phi$$

The global SO(1,1) transformation is a boost between these two fields.

SO(1,1) invariant conformal gauge $\chi^2 - \phi^2 = 6$

This gauge condition represents a hyperbola which can be parameterized by a canonically normalized field $\varphi$

$$\chi = \sqrt{6}\cosh\frac{\varphi}{\sqrt{6}}\,, \qquad \phi = \sqrt{6}\sinh\frac{\varphi}{\sqrt{6}}$$

The action in this gauge, dS/AdS

$$L = \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - 9\lambda\right]$$

# Chaotic inflation from conformal theory: T-Model

$$\mathcal{L} = \frac{\sqrt{-g}}{2}\left[(\partial_\mu\chi\partial^\mu\chi - \partial_\mu\phi\partial^\mu\phi) + \frac{\chi^2-\phi^2}{6}R(g) - \frac{(\phi^2-\chi^2)^2}{18}F\left(\phi/\chi\right)\right]$$

Here F is an arbitrary function. When this function is present, it breaks the SO(1,1) symmetry of the de Sitter model. This is the only possibility to keep local conformal symmetry and to deform the SO(1,1) symmetry.

In the gauge $\chi^2 - \phi^2 = 6$, the theory becomes

$$L = \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - F(\tanh\varphi)\right]$$

The attractor behavior near a critical point where SO(1,1) symmetry is restored is the following: start with generic F(tanh), always get

$$n_s \approx 0.967 \qquad r \approx 0.0032$$

**Jordan frame derivation of the same result:** use the gauge $\chi = \sqrt{6}$

$$\mathcal{L}_{\rm total} = \sqrt{-g_J}\left[\frac{R(g_J)}{2}\left(1-\frac{\phi^2}{6}\right) - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - F\left(\phi/\sqrt{6}\right)\left(\frac{\phi^2}{6}-1\right)^2\right]$$

Einstein frame kinetic term $-\frac{1}{2}\frac{\partial_\mu\phi\partial^\mu\phi}{\left(1-\frac{\phi^2}{6}\right)^2}$

Einstein frame potential

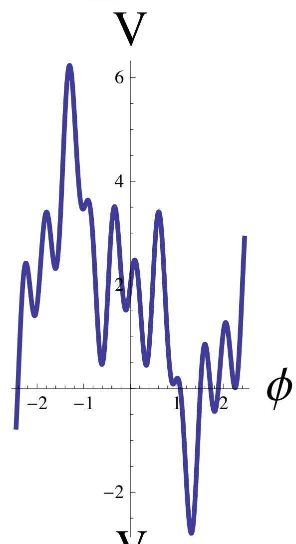
$$V(\phi) = F(\phi/\sqrt{6})$$

Find canonical inflaton $\frac{d\varphi}{d\phi} = \frac{1}{1-\phi^2/6}$

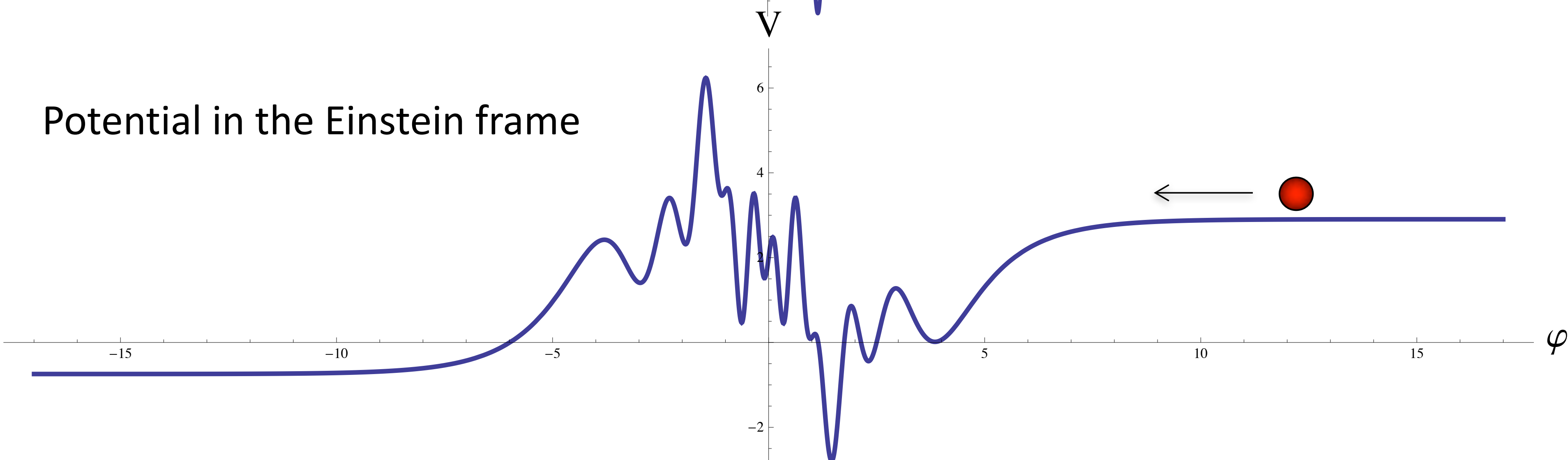
$$V = F(\tanh\frac{\varphi}{\sqrt{6}})$$

# Stretching and flattening of the potential is similar to stretching of inhomogeneities during inflation

Potential in the original variables of the conformal theory

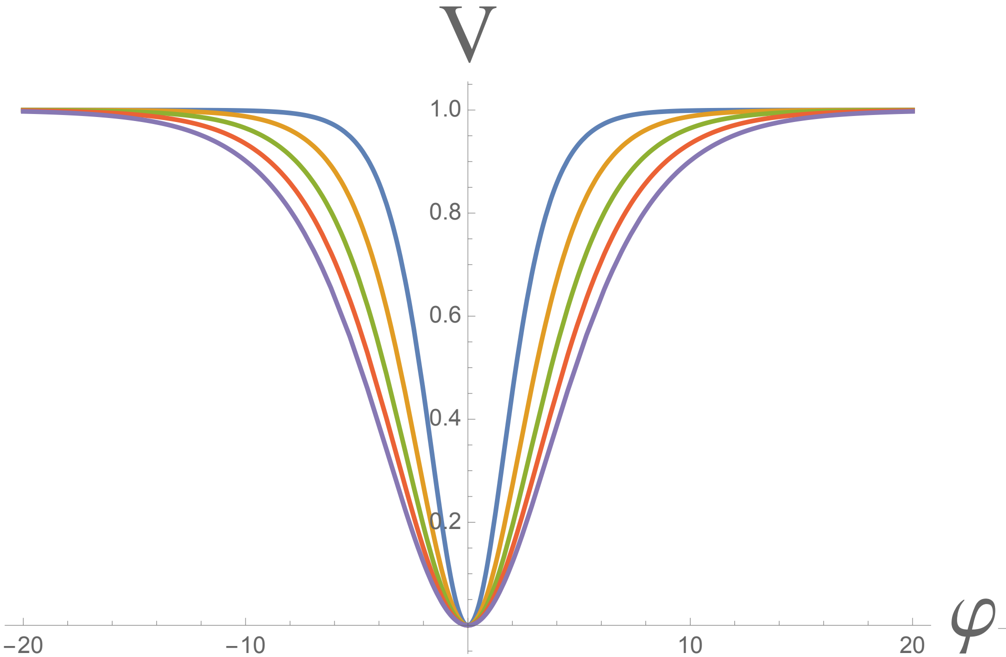


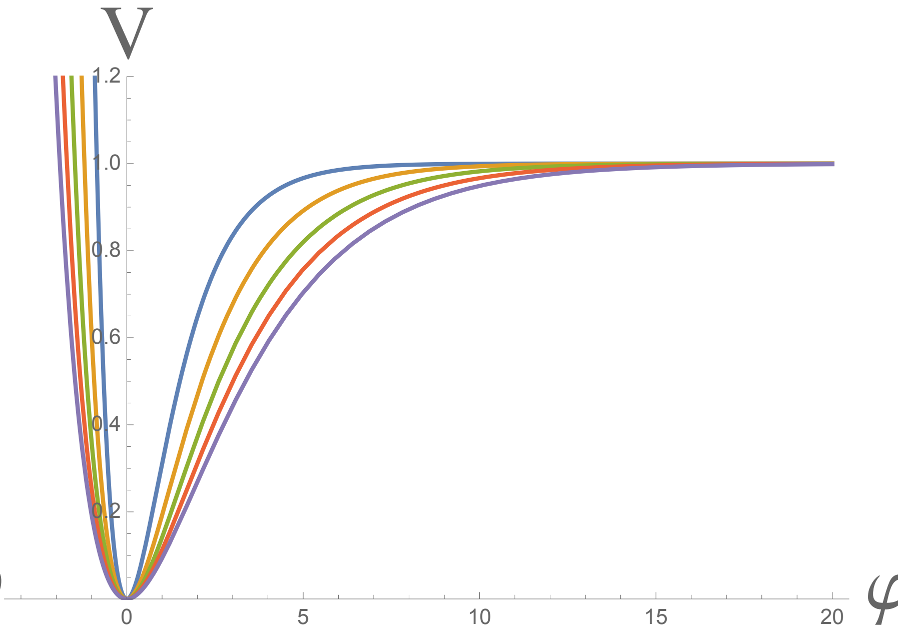

Potential in the Einstein frame




Inflation **in** the landscape is facilitated by inflation **of** the landscape

# Plateau potentials α-attractors




$$\frac{1}{2}R - \frac{1}{2}\partial\varphi^2 \ \ - \alpha\mu^2\left(\tanh\frac{\varphi}{\sqrt{6\alpha}}\right)^2$$

$$\frac{1}{2}R - \frac{1}{2}\partial\varphi^2 \ \ - \alpha\mu^2\left(1 - e^{\sqrt{\frac{2}{3\alpha}}\varphi}\right)^2$$

Simplest T-model in canonical variables Simplest E-model

$$\frac{1}{2}R - 3\alpha\frac{\partial Z\partial\bar{Z}}{(1-Z\bar{Z})^2} - V_0 Z\bar{Z}$$

$$\frac{1}{2}R - 3\alpha\frac{\partial T\partial\bar{T}}{(T+\bar{T})^2} - V_0(T-1)^2$$

In geometric variables

# What is the meaning of α-attractors?

Ferrara, Kallosh, AL, Porrati, 2013;
Kallosh, AL, Roest 2013; Galante, Kallosh, AL, Roest 2014

Start with the simplest chaotic inflation model

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\partial\phi^2 - \frac{1}{2}m^2\phi^2$$

Modify its kinetic term

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{\partial\phi^2}{(1-\underline{\frac{\phi^2}{6\alpha}})^2} - \frac{1}{2}m^2\phi^2$$

Switch to canonical variables $\phi = \sqrt{6\alpha}\,\tanh\frac{\varphi}{\sqrt{6\alpha}}$

The potential becomes

$$V = 3\alpha\, m^2 \tanh^2\frac{\varphi}{\sqrt{6\alpha}}$$

Simplest T-model

General chaotic inflation model

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\partial\phi^2 - V(\phi)$$

Modify its kinetic term

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{\partial\phi^2}{(1-\frac{\phi^2}{6\alpha})^2} - V(\phi)$$

Switch to canonical variables $\phi = \sqrt{6\alpha}\,\tanh\frac{\varphi}{\sqrt{6\alpha}}$

The potential becomes

$$V = V(\tanh\frac{\varphi}{\sqrt{6\alpha}})$$

T-models

This is a **plateau potential** for any nonsingular $V(\phi)$

# The essence of $\alpha$-attractors

Galante, Kallosh, AL, Roest 1412.3797

$$\frac{1}{2}R - \frac{3}{4}\alpha\left(\frac{\partial t}{t}\right)^2 - V(t)$$

Suppose inflation takes place near the pole at t = 0, and V(0) > 0, V'(0) >0, and V has a minimum nearby. Then in canonical variables

$$\frac{1}{2}R - \frac{1}{2}(\partial\varphi)^2 - V_0(1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi} + ...)$$

Then in the leading approximation in 1/N, for any non-singular V

$$n_s = 1 - \frac{2}{N}, \qquad r = \alpha\frac{12}{N^2}$$

# The essence of $\alpha$-attractors

Galante, Kallosh, AL, Roest 1412.3797

## THE BASIC RULE:

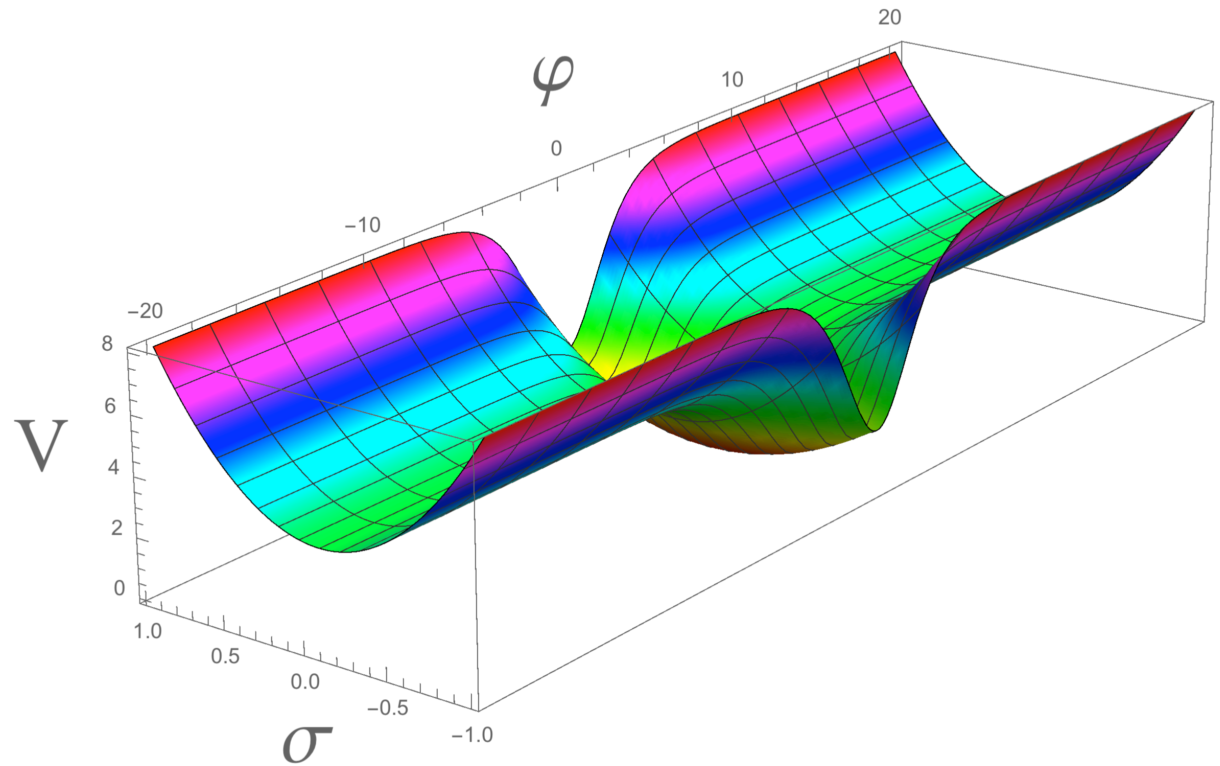
For a broad class of cosmological attractors, the spectral index $\mathbf{n_s}$ depends mostly on the order of the pole in the kinetic term, while the tensor-to-scalar ratio **r** depends on the residue. **Choice of the potential almost does not matter**, as long as it is non-singular at the pole of the kinetic term. Geometry of the moduli space, not the potential, determines much of the answer.

**An often discussed concern about higher order corrections to the potential for large field inflation does not apply to these models.**

# What happens if we add other fields?

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{(\partial\phi)^2}{(1-\frac{\phi^2}{6\alpha})^2} - \frac{1}{2}m^2\phi^2 - \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}M^2\sigma^2 - \frac{g^2}{2}\phi^2\sigma^2$$

Potential in canonical variables has a plateau at large values of the inflaton field, and it is quadratic with respect to σ.



# Asymptotic freedom of the inflaton

Kallosh, AL, 1604.00444

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{(\partial\phi)^2}{(1-\frac{\phi^2}{6\alpha})^2} - \frac{1}{2}(\partial\sigma)^2 - V(\phi,\sigma)$$

Couplings of the canonically normalized fields are determined by derivatives such as

$$\lambda_{\varphi,\sigma,\sigma} = \partial_\varphi \partial_\sigma^2 V(\phi,\sigma) = 2\sqrt{\frac{2}{3\alpha}}\, \underline{e^{-\sqrt{\frac{2}{3\alpha}}\varphi}}\, \partial_\phi \partial_\sigma^2 V(\phi,\sigma)|_{\phi\to\sqrt{6\alpha}}$$

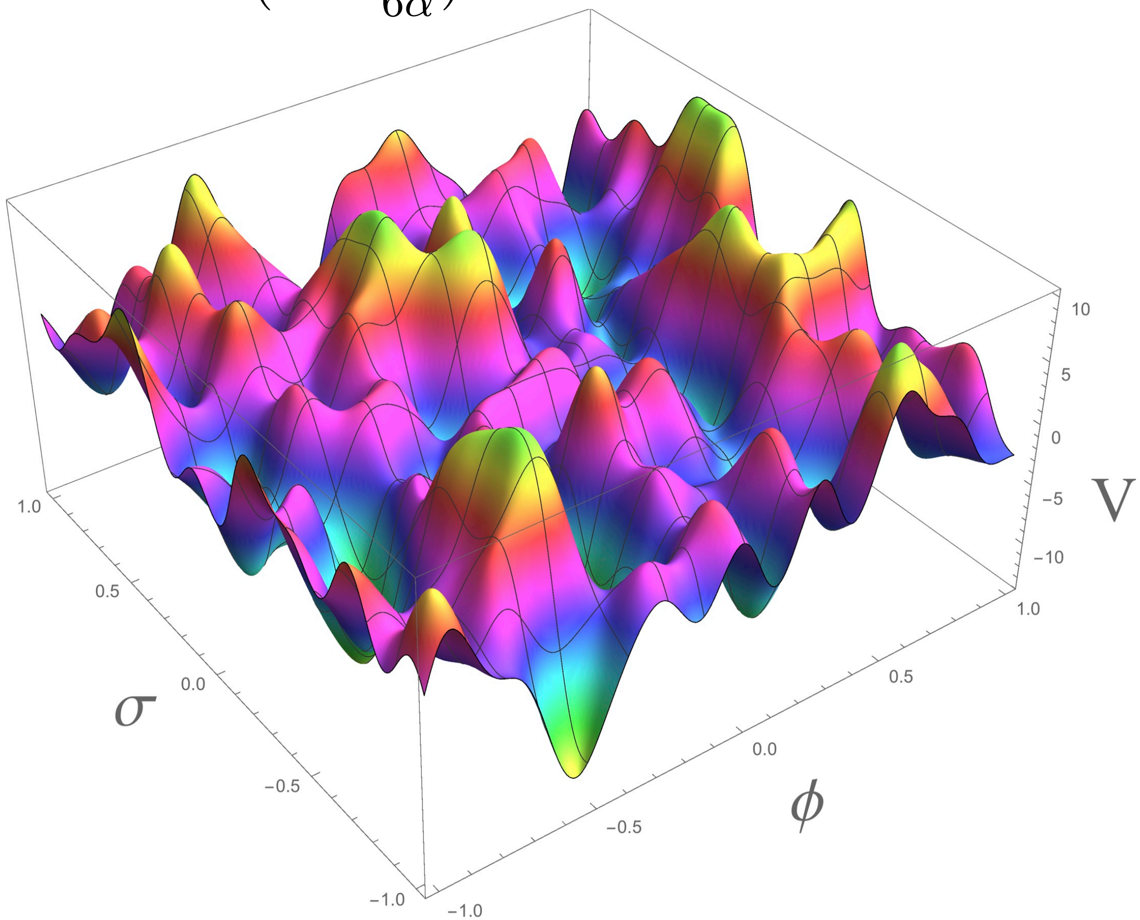
As a result, **couplings of the inflaton field to all other fields are exponentially suppressed during inflation**. The asymptotic shape of the plateau potential of the inflaton is **not** affected by quantum corrections.

# Inflation in Random Potentials and Cosmological Attractors

AL 1612.04505

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{R}{2} - \frac{(\partial_\mu \phi)^2}{2(1-\frac{\phi^2}{6\alpha})^2} - \frac{(\partial_\mu \sigma)^2}{2} - V(\phi,\sigma)$$

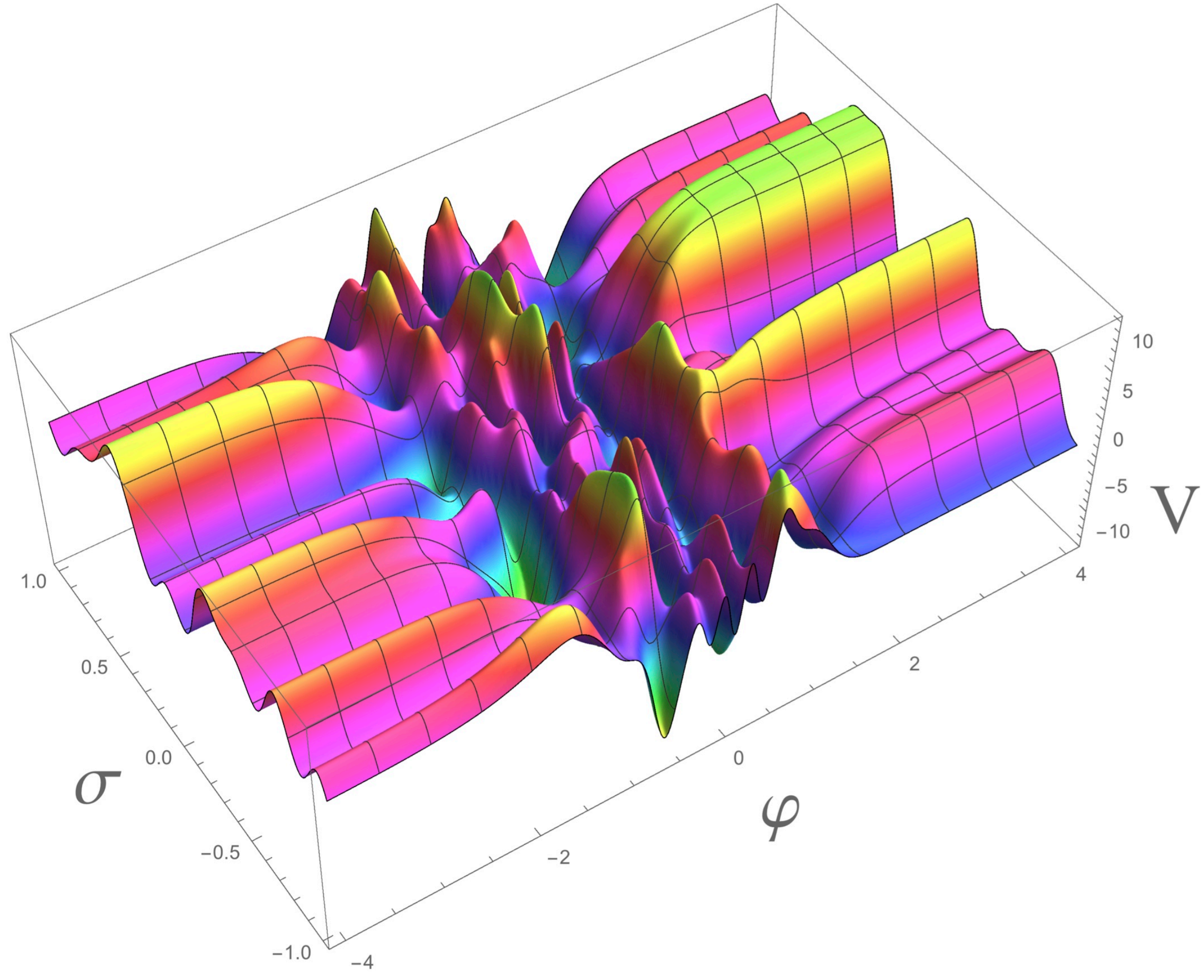
Can we have inflation in such potentials?



In terms of canonical fields $\varphi$ with the kinetic term $\frac{(\partial_\mu \varphi)^2}{2}$, the potential is

$$V(\varphi, \sigma) = V(\sqrt{6\alpha} \tanh \frac{\varphi}{\sqrt{6\alpha}}, \sigma)$$

Many inflationary valleys representing alpha-attractors



# Double Attractors

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{R}{2} - \frac{(\partial_\mu \phi)^2}{2(1-\frac{\phi^2}{6\alpha})^2} - \frac{(\partial_\mu \sigma)^2}{2(1-\frac{\sigma^2}{6\beta})^2} - V(\phi, \sigma)$$

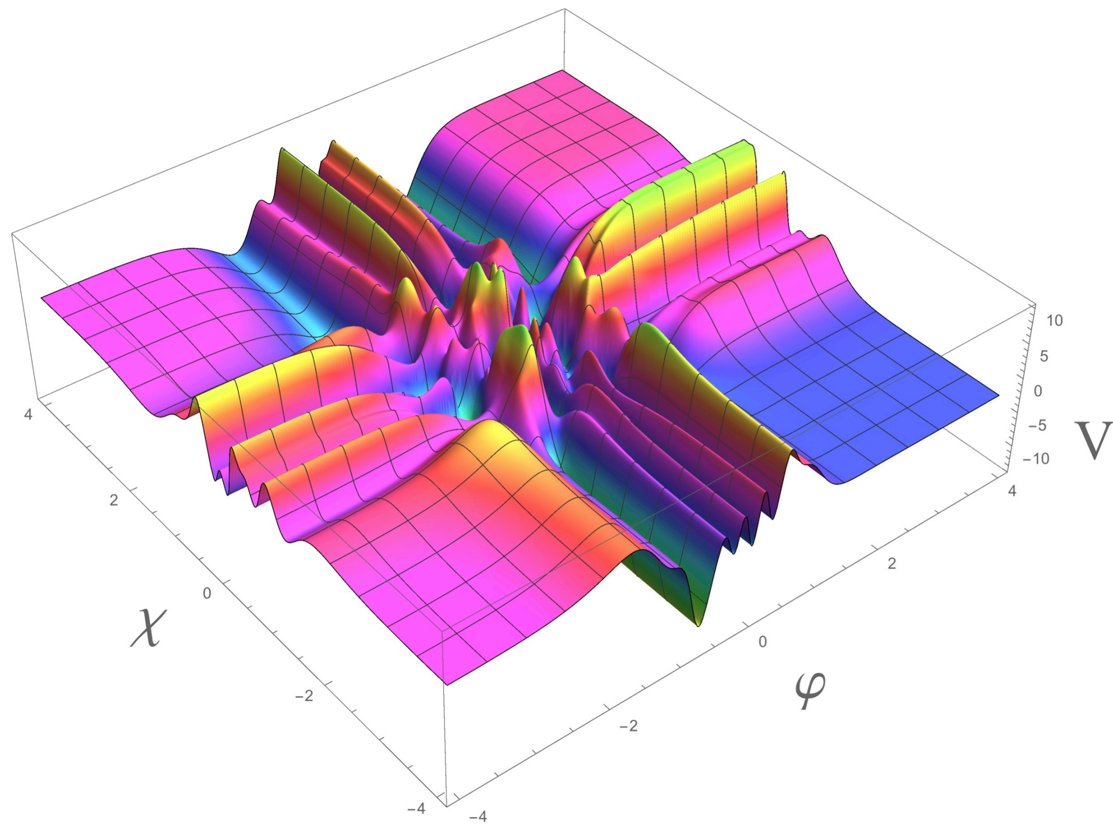
In terms of canonical fields $V(\varphi, \chi) = V(\sqrt{6\alpha}\, \tanh \frac{\varphi}{\sqrt{6\alpha}}, \sqrt{6\beta}\, \tanh \frac{\chi}{\sqrt{6\beta}})$

Two families of attractors, related to the valleys along the two different inflaton directions:

$$1 - n_s \approx \frac{2}{N}\,, \qquad r \approx \frac{12\alpha}{N^2}\ .$$

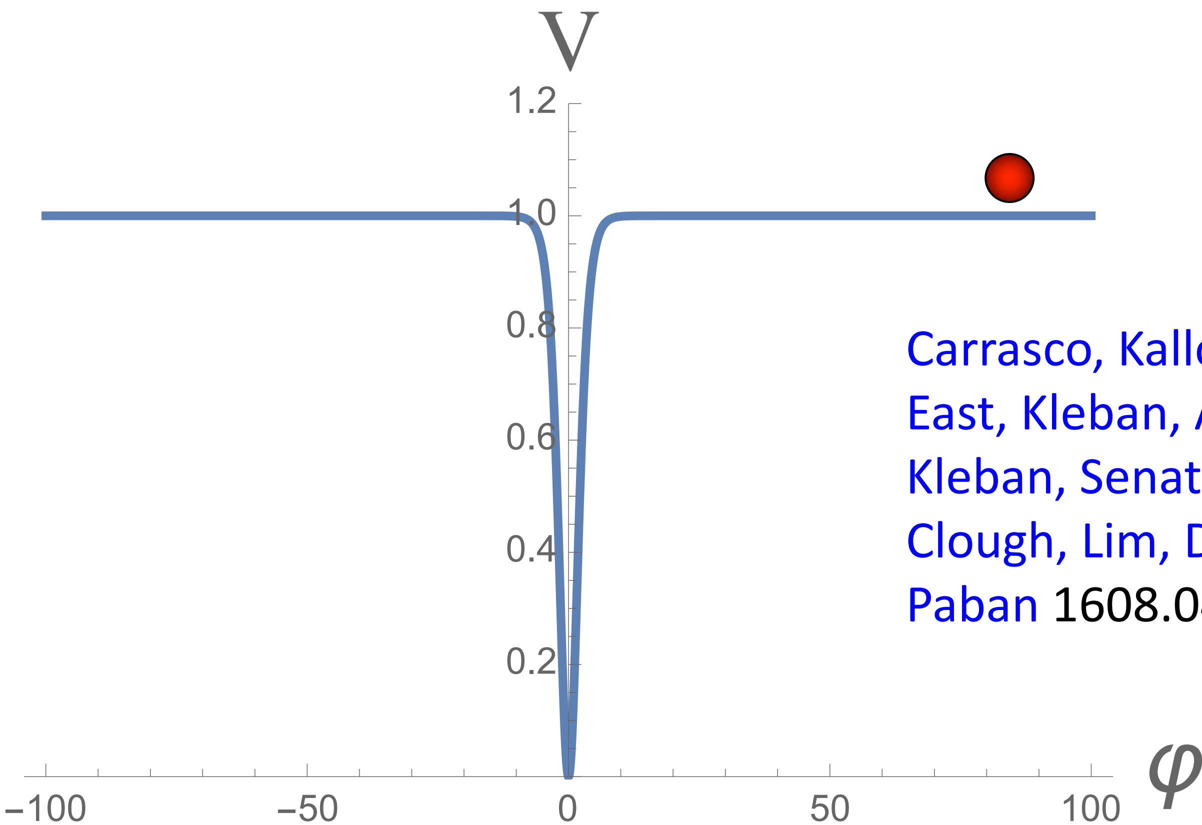
or

$$1 - n_s \approx \frac{2}{N}\,, \qquad r \approx \frac{12\beta}{N^2}\ .$$



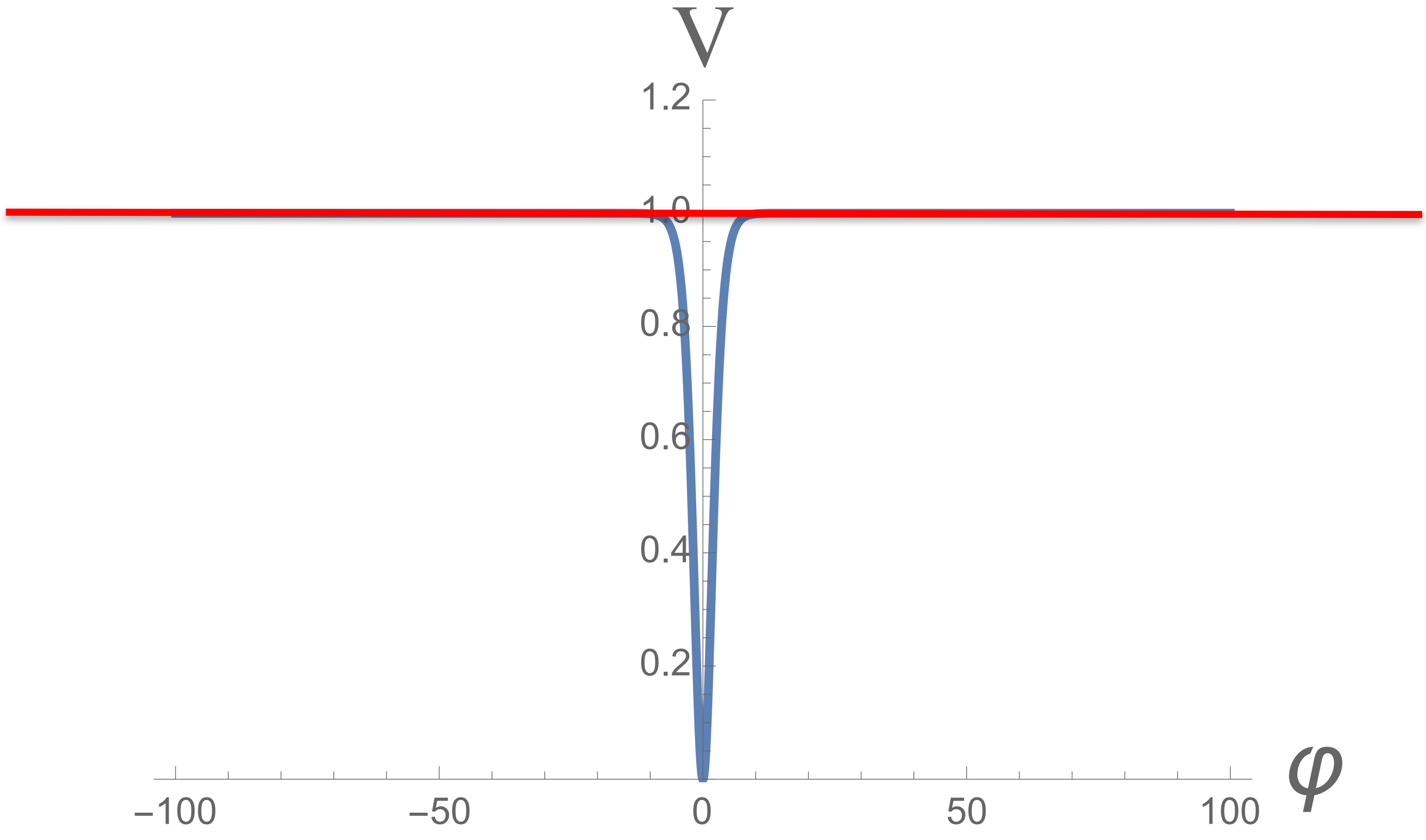
# $\alpha$-attractors: Initial conditions for inflation

At large fields, the $\alpha$-attractor potential remains 10 orders of magnitude below Planck density. Can we have inflation with natural initial conditions here? The same question applies for the Starobinsky model and Higgs inflation.




Carrasco, Kallosh, AL 1506.00936
East, Kleban, AL, Senatore 1511.05143
Kleban, Senatore 1602.03520
Clough, Lim, DiNunno, Fischler, Flauger, Paban 1608.04408

To explain the main idea, note that this potential coincides with the cosmological constant <u>almost everywhere</u>.



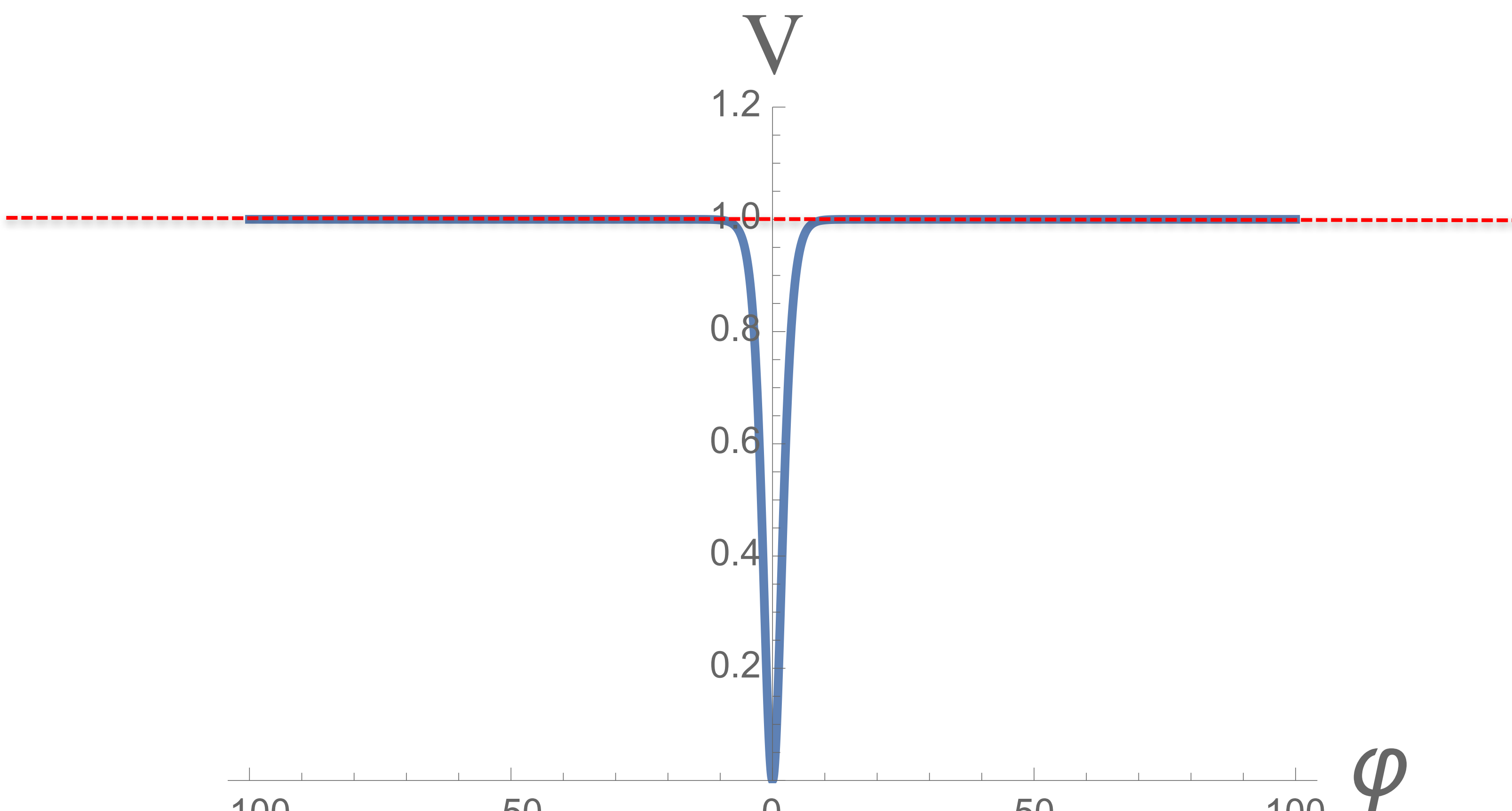
**For the universe with a cosmological constant, the problem of initial conditions is nearly trivial.**

Start at the Planck density, in an expanding universe dominated by inhomogeneities. The energy density of matter is diluted by the cosmological expansion as $1/t^2$. **What could prevent the exponential expansion of the universe which becomes dominated by the cosmological constant $\Lambda$ after the time $t = \Lambda^{-1/2}$ ?**

Inflation does NOT happen in the universe with the cosmological constant $\Lambda = 10^{-10}$ only if the whole universe collapses within $10^{-28}$ seconds after its birth.

**In other words, only instant global collapse could allow the universe to avoid exponential expansion dominated by the cosmological constant. If the universe does not instantly collapse, it inflates.**

This optimistic conclusion related to the cosmological constant applies to α-attractors as well, because their potential coincides with the cosmological constant almost everywhere.



These arguments are valid for general large field inflationary models as well. Recently they have been confirmed by the same methods of numerical GR as the ones used in simulations of BH evolution and merger. The simulations show how BHs are produced from large super-horizon initial inhomogeneities, while the rest of the universe enters the stage of inflation.

East, Kleban, AL, Senatore 1511.05143

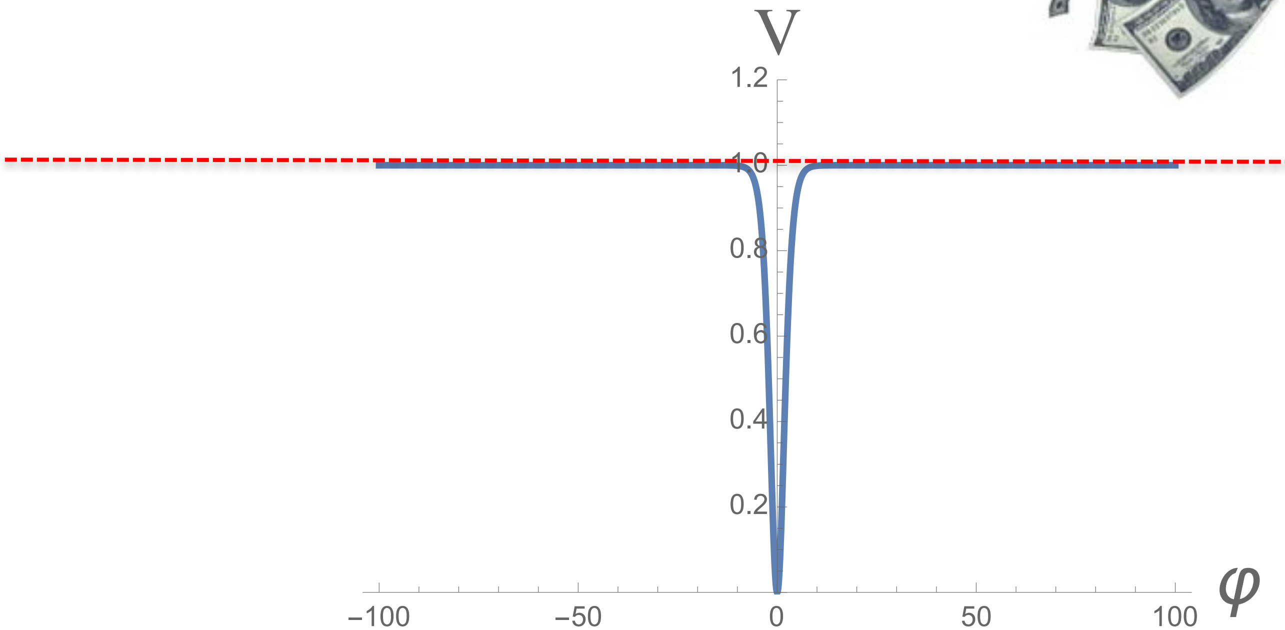
**These results obtained by sophisticated calculations have a very simple interpretation in terms of inflation in economy.**

# It is well known that dropping money from a helicopter may lead to inflation, unless all money miss the target

# A simple interpretation of our results

suggested by Starobinsky

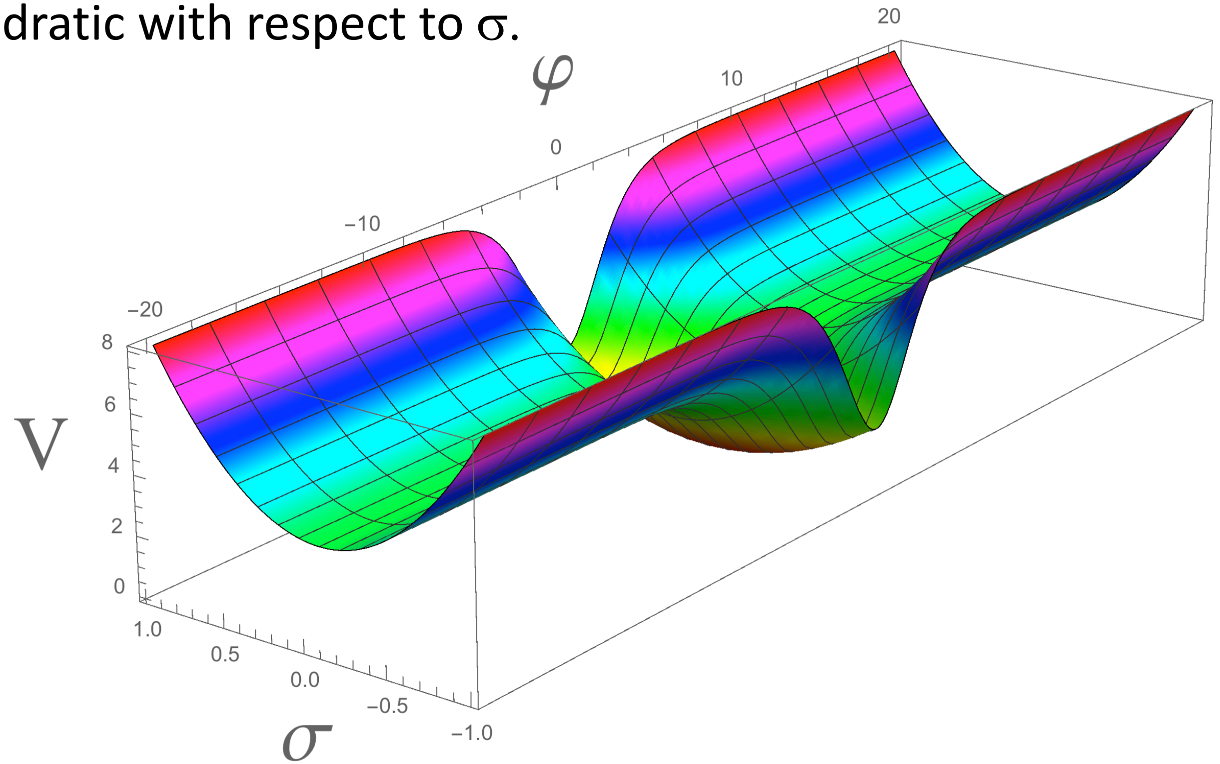
Money dropped from a helicopter have no choice but lend on an infinitely long plateau. This inevitably leads to inflation



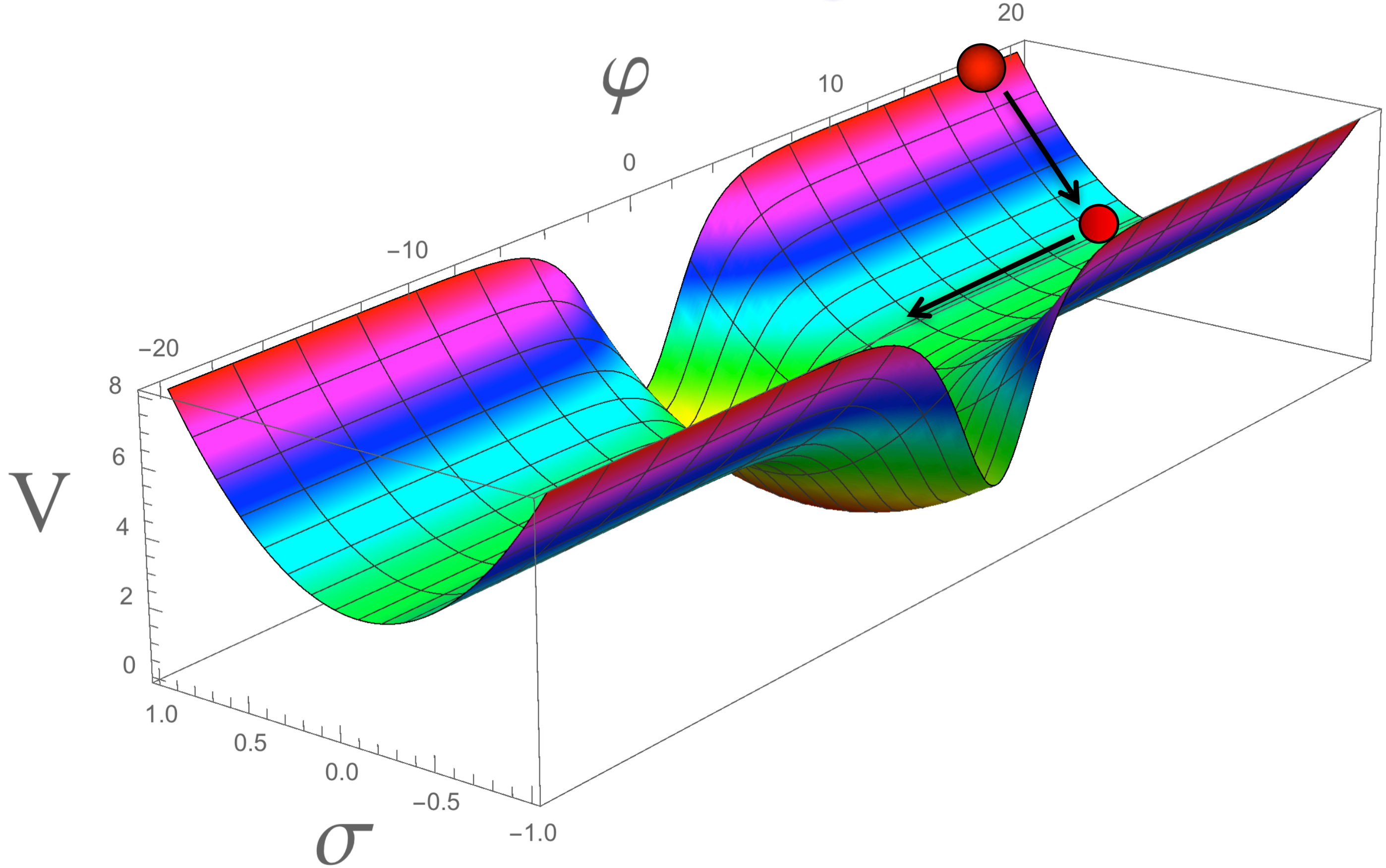
# Adding other fields simplifies it even further

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{(\partial\phi)^2}{(1-\frac{\phi^2}{6\alpha})^2} - \frac{1}{2}m^2\phi^2 - \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}M^2\sigma^2 - \frac{g^2}{2}\phi^2\sigma^2$$

Potential in canonical variables has a plateau at large values of the inflaton field, and it is quadratic with respect to σ.



# Initial conditions for plateau inflation




**Chaotic inflation with a parabolic potential goes first**, **starting at nearly Planckian density**. When the field down, the plateau inflation begins.

**No problem with initial conditions**

# Conclusions:

Cosmological attractors allow to reconsider many usual assumptions with respect to the large field models, resolving some of their often discussed problems, offering new solutions to the problem of initial conditions in inflationary cosmology.

Using the latest developments in supergravity and string theory to be described in the lecture by Renata Kallosh, one can develop inflationary models describing not only inflation but also dark energy and supersymmetry breaking.