Lecture 3, part 2

Kallosh, R

Cosmological Attractors and B-mode Targets

From the sky to fundamental physics and back

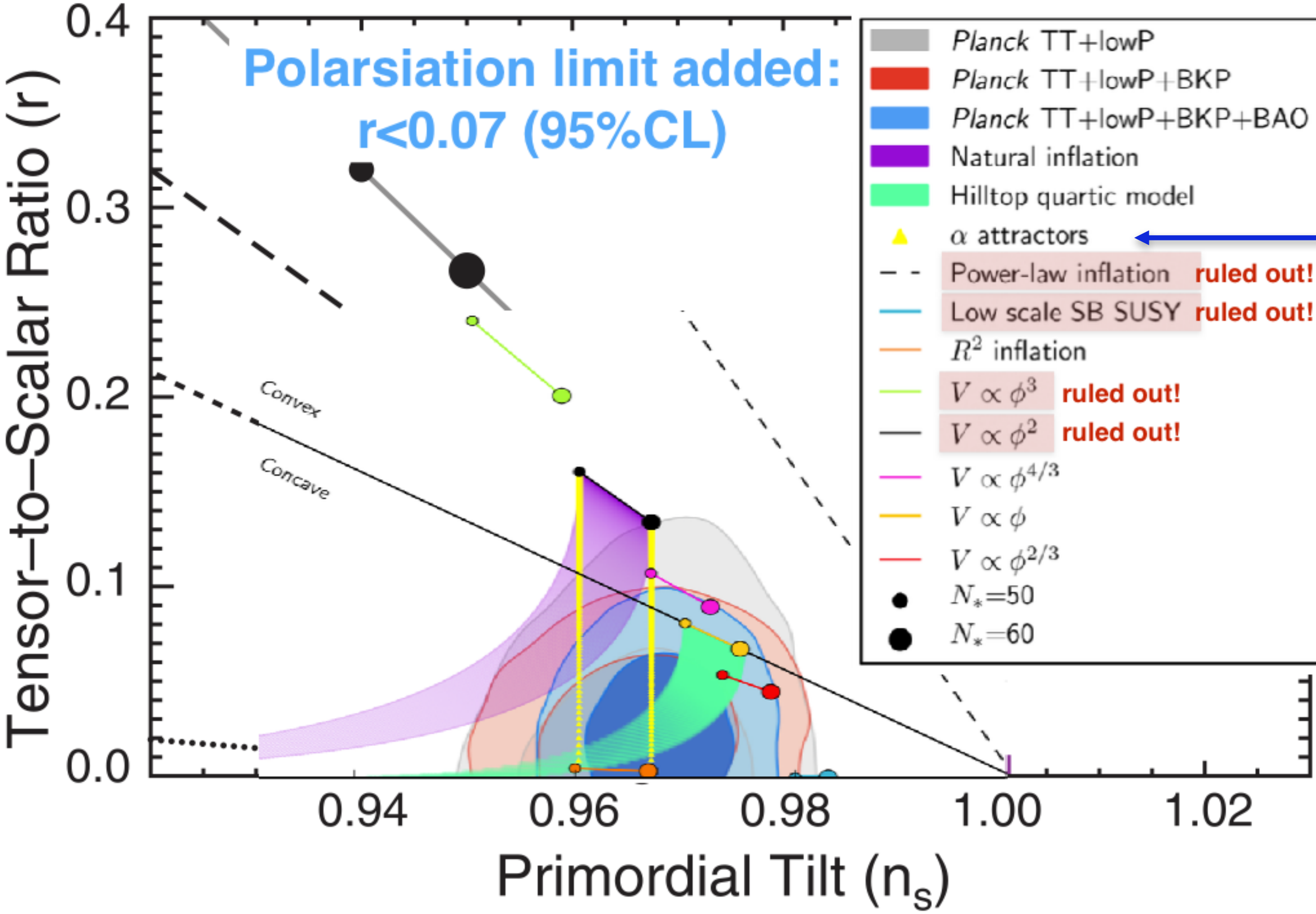
If polarization from GW is found…

Then what? The next step is to nail the specific model of inflation

We are closing in on the first 10^{-35} seconds of the universe, using General Relativity and QFT

Using WMAP's Komatsu talk in 2017

Planck Collaboration (2015); BICEP2/Keck Collaboration (2016)



RK, Linde, Roest, 2013

$$V \sim \varphi^{2n}$$

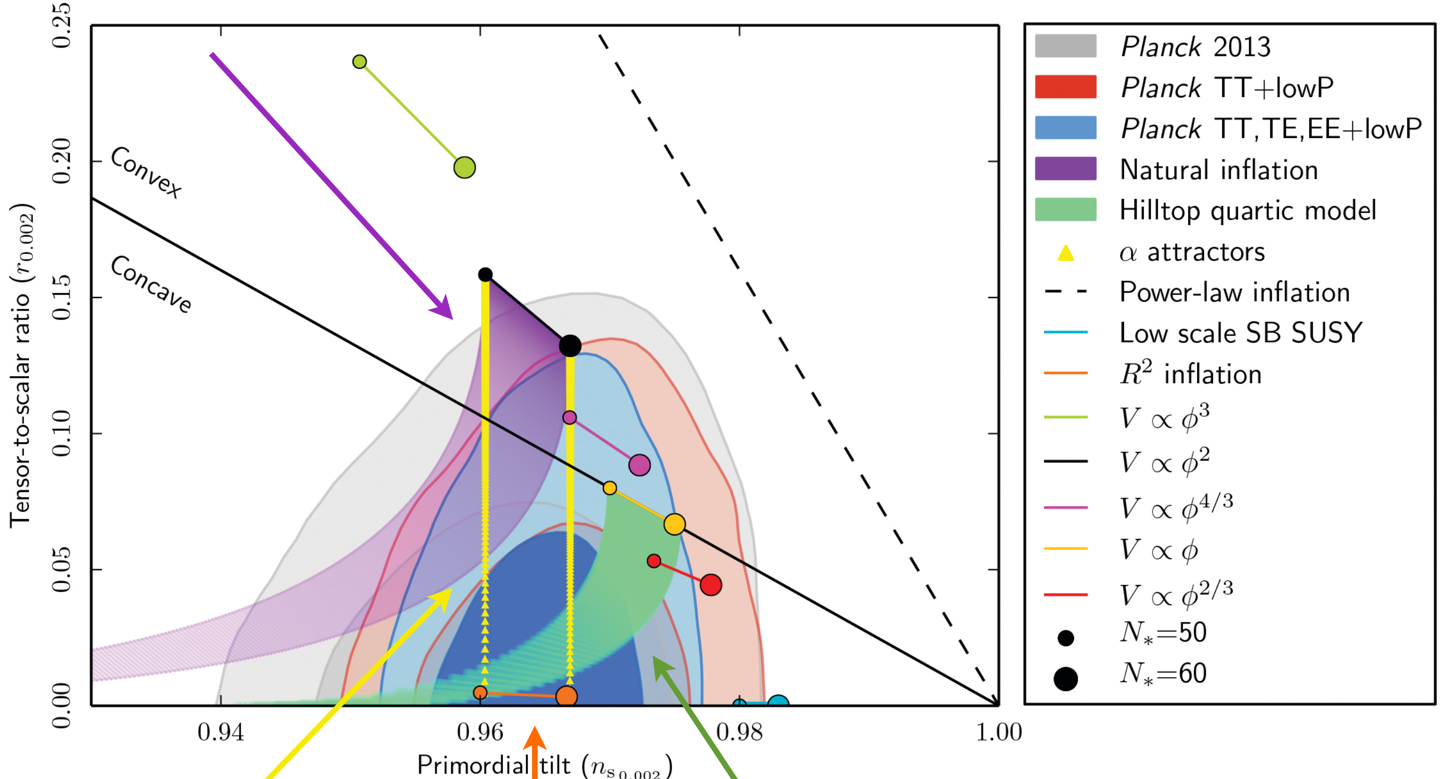
$$n_s \approx 1 - \frac{2}{N}\left(\frac{n+1}{2}\right)$$

$$r \approx \frac{8n}{N}$$

Inflationary models & Planck 2015

Using Planck's Finelli talk in 2017

$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f}\right)\right]$$



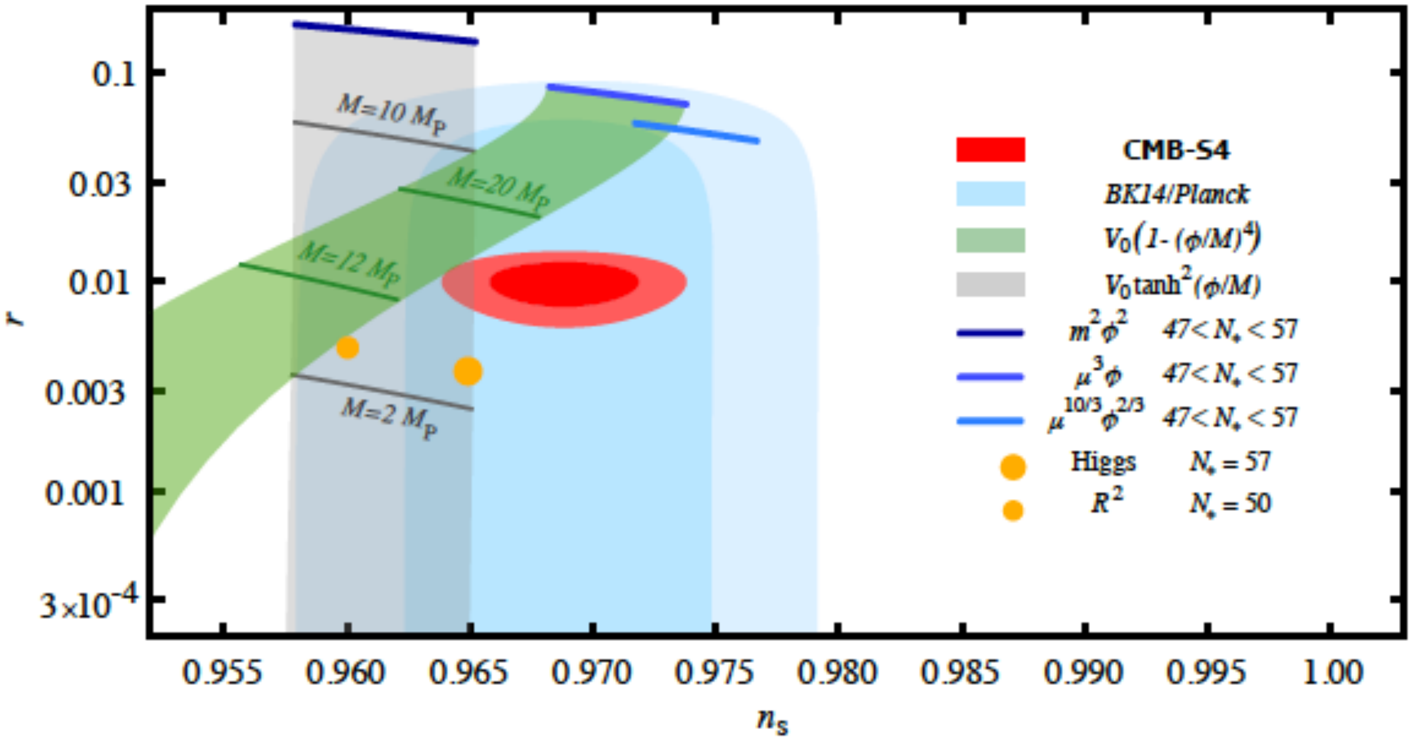
$$V(\phi) = \Lambda^4 \tanh^2\left(\frac{\phi}{\sqrt{6\alpha}M_{\rm pl}}\right)$$

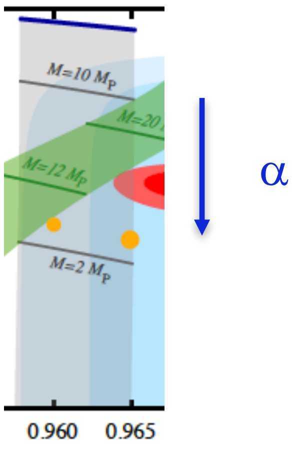
$$V(\phi) = \Lambda^4 \left(1 - \frac{\phi^4}{\mu^4} + \dots\right)$$

$$V(\tilde{\phi}) = \frac{\Lambda^4}{4}\left(1 - e^{-2\tilde{\phi}/\sqrt{6}M_{\rm pl}}\right)^2$$

Looking into future with CMB-S4

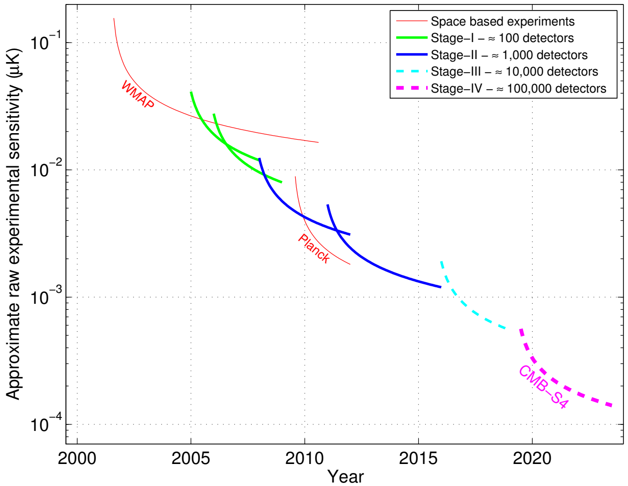
Alpha-Attractors and B-mode Targets





CMB-S4 Science Book

October 2016

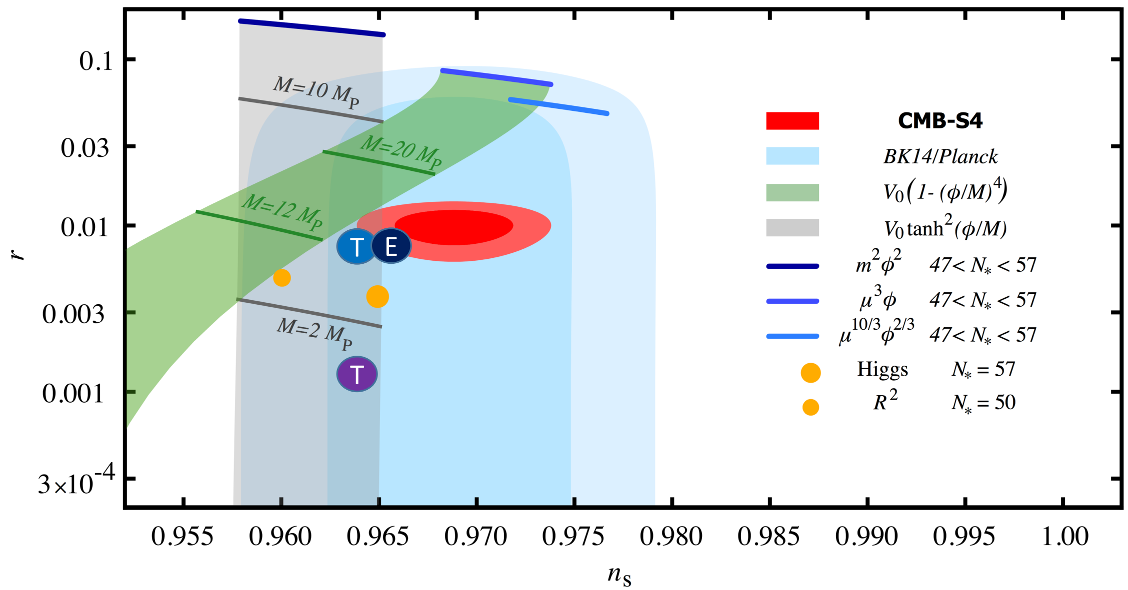


Primordial Gravity Waves

Ferrara, RK, 2016,
RK, Linde, Wrase, Yamada (2017)

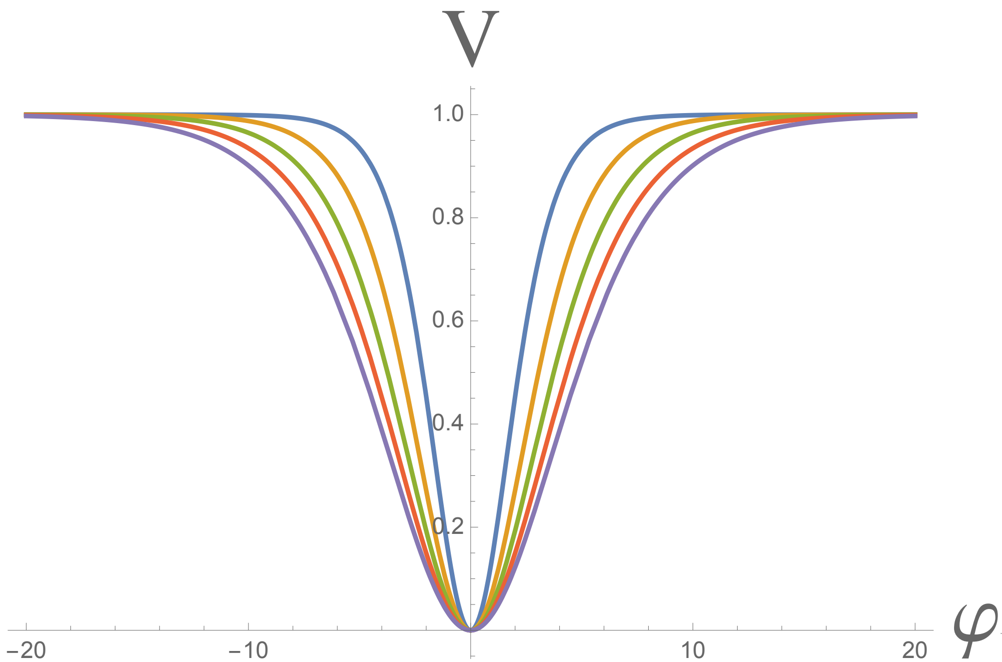
α–attractor models

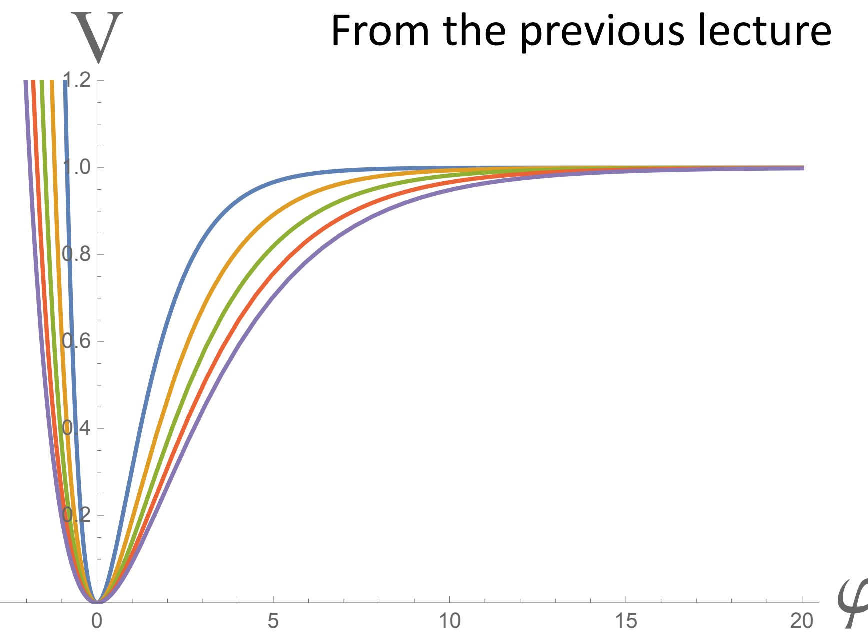
Future B-mode satellite missions



Well motivated new models originating in string theory, M-theory, maximal supergravity

Plateau potentials α-attractors





From the previous lecture

$$\frac{1}{2}R - \frac{1}{2}\partial\varphi^2 \;\; - \alpha\mu^2\left(\tanh\frac{\varphi}{\sqrt{6\alpha}}\right)^2$$

$$\frac{1}{2}R - \frac{1}{2}\partial\varphi^2 \;\; - \alpha\mu^2\left(1 - e^{\sqrt{\frac{2}{3\alpha}}\varphi}\right)^2$$

Simplest T-model in canonical variables Simplest E-model

$$\frac{1}{2}R - 3\alpha\frac{\partial Z\partial\bar{Z}}{(1-Z\bar{Z})^2} - V_0 Z\bar{Z}$$

$$\frac{1}{2}R - 3\alpha\frac{\partial T\partial\bar{T}}{(T+\bar{T})^2} - V_0(T-1)^2$$

In geometric variables

Purely bosonic theory, what is the relation to fundamental theory ?

Complex scalar fields in supergravity and string theory

$$Z(t,\vec{x})\,,\bar{Z}(t,\vec{x})$$

are coordinates of some geometric space: **MODULI SPACE**

$$ds^2 = g_{Z\bar{Z}}\,dZ\,d\bar{Z}$$

The metric of the moduli space is defined by a second derivative of the Kahler potential

$$g_{Z\bar{Z}} = \partial_Z \partial_{\bar{Z}} K(Z,\bar{Z})$$

The curvature of the MODULI SPACE, Kahler curvature for our models is

$$\mathcal{R}_{\text{Kähler}} = -g^{-1}_{Z\bar{Z}} \partial_Z \partial_{\bar{Z}} \log g_{Z\bar{Z}} = -\frac{2}{3\alpha}$$

α-attractors in supergravity

$$SL(2,\mathbb{R})$$ symmetry

$$ds^2 = \frac{3\alpha}{(1-Z\bar{Z})^2} dZ d\bar{Z}$$

$$ds^2 = \frac{3\alpha}{(T+\bar{T})^2} dT d\bar{T}$$

$$\mathcal{R}_K = -\frac{2}{3\alpha}$$

Curvature of the moduli space in Kahler geometry

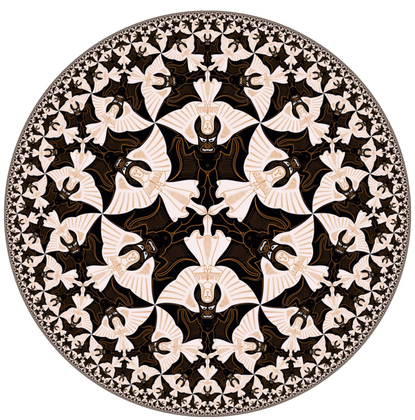
$$Z\bar{Z} < 1$$

Hyperbolic geometry
of a Poincaré disk

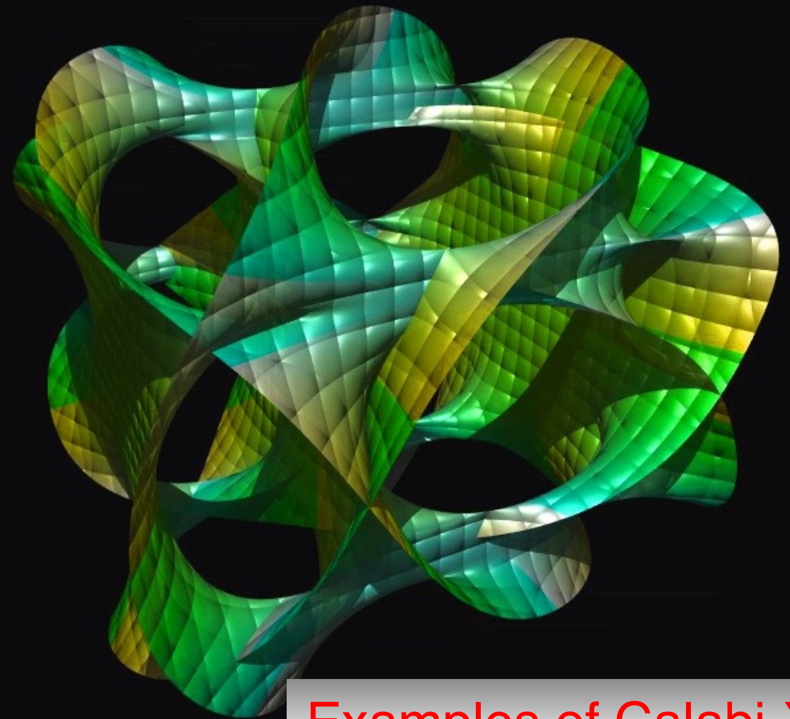
Disk or **half-plane**

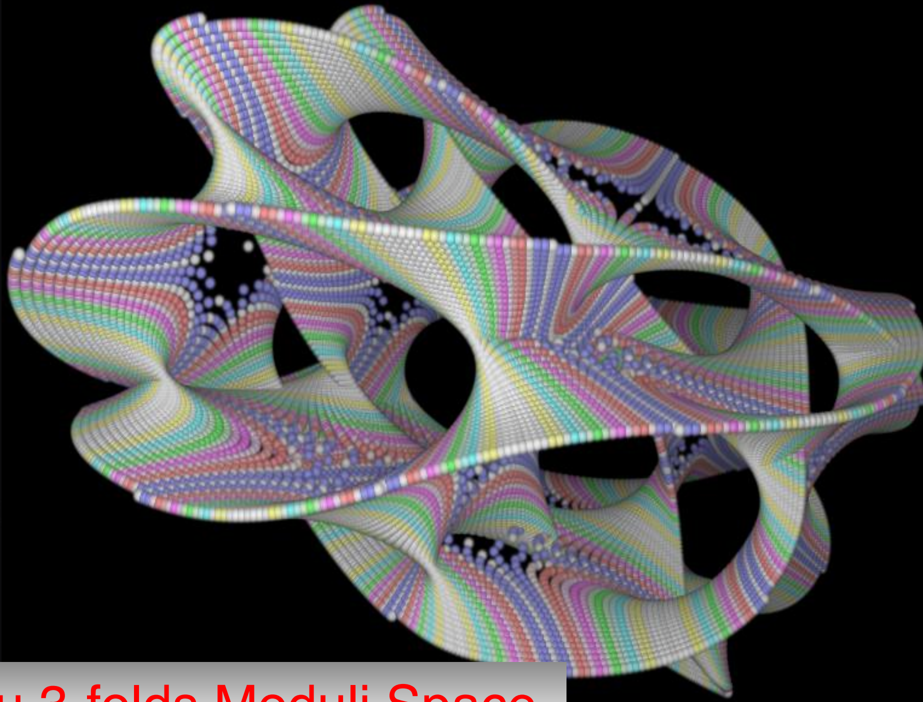
$$T + \bar{T} > 0$$

Escher in the Sky, RK, Linde 2015

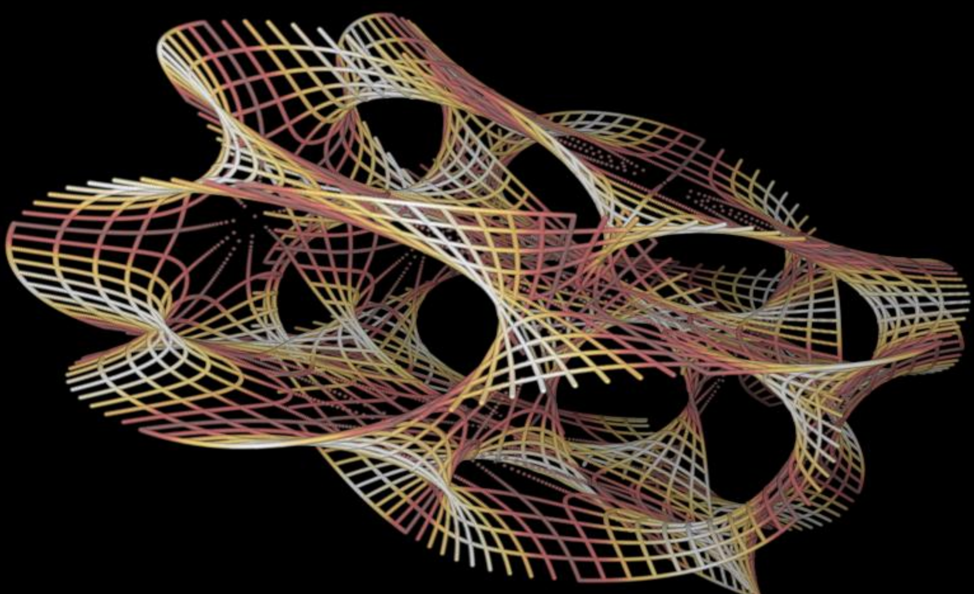


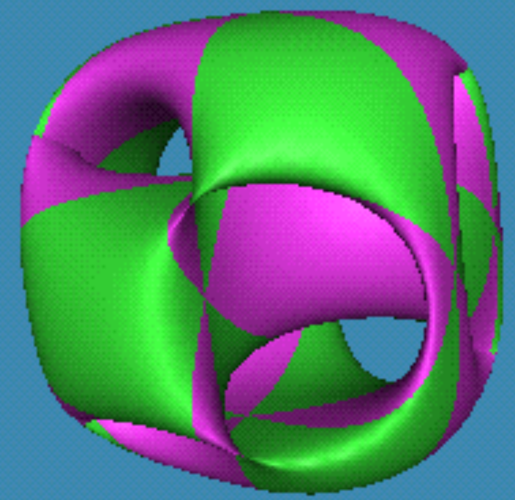
$$3\alpha = R^2_{\text{Escher}} \approx 10^3 r$$





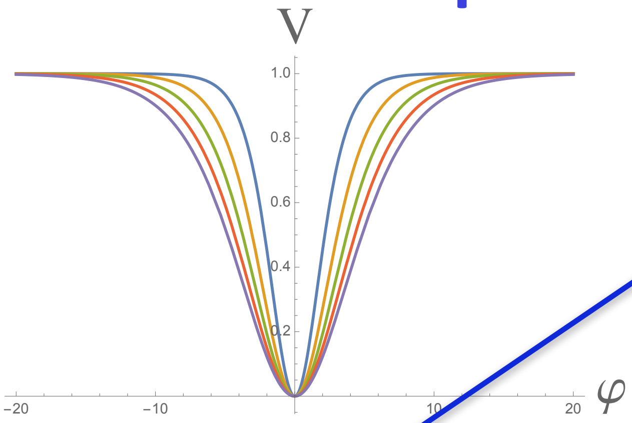
Examples of Calabi-Yau 3-folds Moduli Space



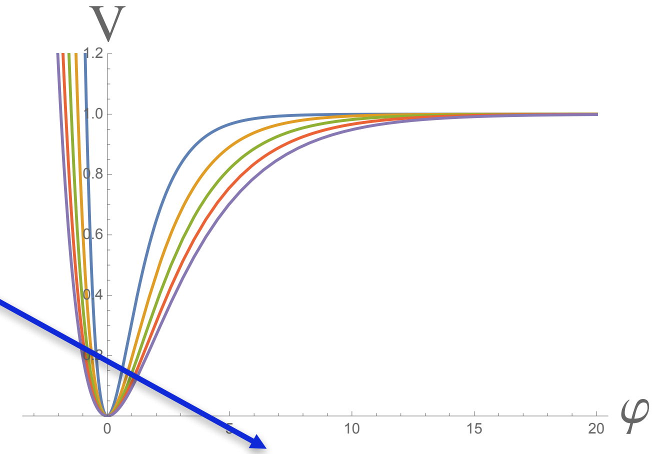


Other moduli: size of cycles

Plateau potentials from maximal superconformal symmetry



What is the origin of geometric kinetic terms



$$SL(2,\mathbb{R})$$

$$\frac{1}{2}R - 3\alpha\frac{\partial Z\partial \bar{Z}}{(1-Z\bar{Z})^2} - V_0 Z\bar{Z}$$

In geometric variables

$$\frac{1}{2}R - 3\alpha\frac{\partial T\partial \bar{T}}{(T+\bar{T})^2} - V_0(T-1)^2$$

Start with **maximal local maximal superconformal $\mathcal{N}$=4 model**, make a gauge-fixing of the local symmetries to derive Poincare supergravity (no potential yet, to get the potential one should break **$\mathcal{N}$=4** to **$\mathcal{N}$=1**)

1981-1985 Bergshoeff, de Wit, de Roo

$$\frac{1}{2}R - \frac{\partial Z\partial \bar{Z}}{(1-Z\bar{Z})^2} + \dots$$

2013 Ferrara, RK, Van Proeyen

$$\frac{1}{2}R - \frac{\partial T\partial \bar{T}}{(T+\bar{T})^2} + \dots$$

Unit size disk

3α=1: lowest fundamental B-mode target

$$r \approx 10^{-3}$$

http://mathworld.wolfram.com/PoincareHyperbolicDisk.html

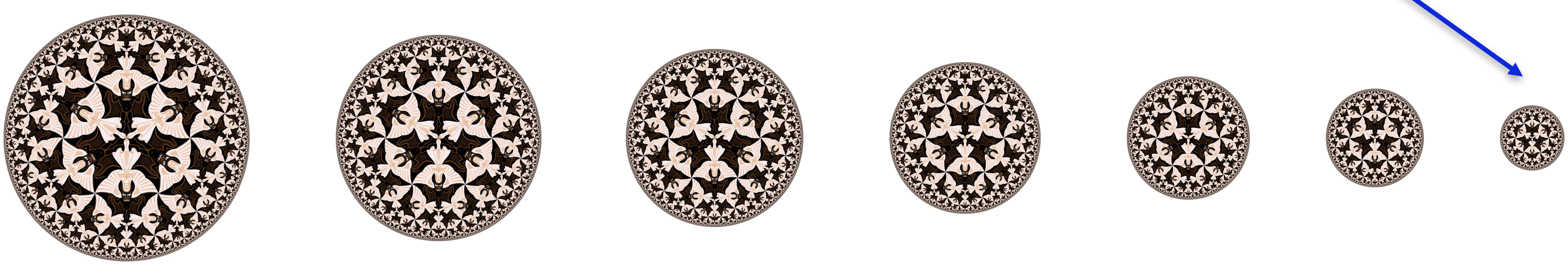
$$ds^2 = \frac{dx^2 + dy^2}{(1 - x^2 - y^2)^2}$$

For a unit size Poincare disk:

$$r \sim 10^{-3} \qquad \alpha = \frac{1}{3}$$

Maximal superconformal theory

Next CMB satellite mission target



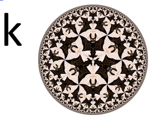
Long waiting time, can we find higher targets from fundamental theory?

Is it possible to realize "r" observable in a relatively near future? Closer to 10^{-2} rather than 10^{-3} ? With clear origin in fundamental theory?

The answer is positive

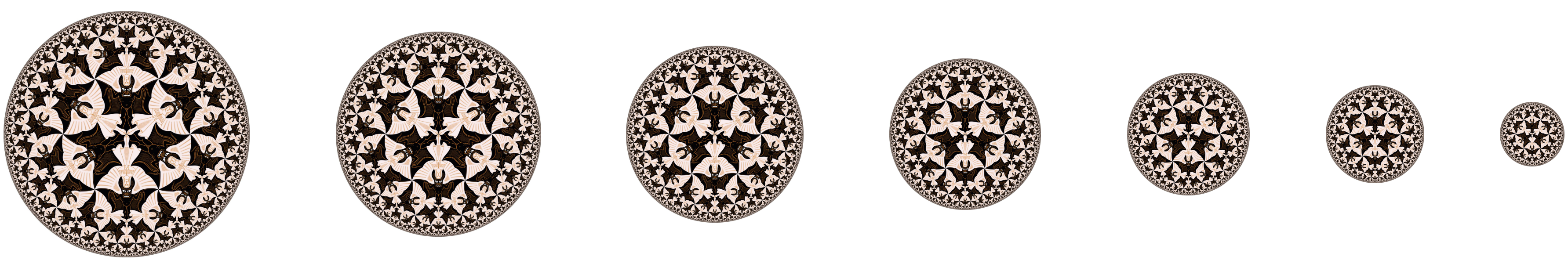
2016, Ferrara and RK

- **Maximal supersymmetry and B-modes**
- M-theory in d=11
- Superstring theory in d=10
- $\mathcal{N}$=8 supergravity in d=4

Based on CMB data on the value of the tilt of the spectrum n_s as a function of N we have deduced that hyperbolic geometry of a Poincaré disk  suggest a way to explain the experimental formula

$$n_s \approx 1 - \frac{2}{N}$$

Using a consistent reduction from maximal $\mathcal{N}=8$ supersymmetry theories: M-theory in d=11, String theory in d=10, maximal supergravity in d=4, to the minimal $\mathcal{N}=1$ supersymmetry we have deduced the favorite models with hyperbolic geometry with $R^2_{\text{Escher}} = 7,6,5,4,3,2,1$



$r \approx 0.9 \times 10^{-2}$ $r \approx 1.3 \times 10^{-3}$

B-mode targets

Details and assumptions underlying the prediction $3\alpha=1,2,3,4,5,6,7$

M-theory compactified on a 7-manifold with G_2 holonomy
special choice of Betti numbers $(b_0, b_1, b_2, b_3) = (1, 0, 0, 7)$
One can obtain d=4 $\mathcal{N}=1$ *supergravity* with rank 7 scalar coset $\left[\frac{SL(2,\mathbb{R})}{SO(2)}\right]^7$

$\mathcal{N}$=8 *supergravity*: consistent reduction to $\mathcal{N}$=1 $E_{7(7)}(\mathbb{R}) \supset [SL(2,\mathbb{R})]^7$

$$K = -\sum_{i=1}^{7} \ln(\tau_i + \bar{\tau}_i)) \Rightarrow -7\ln(\tau + \bar{\tau})$$

String theory compactified on $T_2 \times T_2 \times T_2 \subset T_6$

$$K = -\ln(S + \bar{S}) - 3\ln(U + \bar{U}) - 3\ln(T + \bar{T}) \Rightarrow -7\ln(\tau + \bar{\tau})$$

Larger α from merger

Ferrara, Kallosh (2016)

a simple way to realize larger α

let us consider two-disk model

$$\alpha_i = \frac{1}{3} \qquad\qquad -\frac{(\partial t_1)^2}{4t_1^2} - \frac{(\partial t_2)^2}{4t_2^2}$$

if $t_1 = t_2 = t$: merger of two directions

$$-\frac{2(\partial t)^2}{4t^2} \qquad\qquad \alpha_{\mathrm{eff}} = \frac{2}{3}$$

If we realize the condition dynamically, "r" can be larger

Scalars are coordinates of the coset space in $\mathcal{N}$=8 supergravity in d=4 $\frac{G}{H} = \frac{E_{7(7)}}{SU(8)}$

$$E_{7(7)}(\mathbb{R}) \supset [SL(2,\mathbb{R})]^7$$

The same symmetry which together with supersymmetry is known to protect perturbative *maximal* $\mathcal{N}$=8 *supergravity* in d=4 from UV infinities at least up to 7 loops (or maybe more)

One can argue that geometries with discreet number of unit size Poincaré disks are possible when consistent reduction of supersymmetry is performed. Upon identification of their moduli one finds

$$ds^2 = k\frac{dT d\bar{T}}{(T+\bar{T})^2}, \qquad k = 1,2,3,4,5,6,7 \quad = 3\,\alpha$$

At least one disk and not more than seven

Anti-D3 Brane Induced Geometric Inflation

RK, Linde, Wrase, Yamada, 2017
RK, Linde, Roest, Yamada, 2017
RK, Linde, Roest, Westphal, Yamada, 2017, work in progress

From Geometry to Dynamics

From geometry of anti-D3 brane interacting with CY moduli to effective supergravity models of inflation with the following features

- Fit to data
- Allow an exit to de Sitter vacua
- Models are simple
- Include advanced α-attractor models and new ones
- Hyperbolic disk mergers with discrete $3\alpha = 1,2,3,4,5,6,7$ as B-mode targets
- Simple version of fibre inflation with $3\alpha=2$

Inflationary dynamics including the exit to de Sitter space is fully defined by the geometric Kahler function in the underlying supergravity

In current literature on de Sitter vacua this function is called Kahler function

$$\mathcal{G}$$

Cremmer, Ferrara, Girardello, Julia, Scherk,
van Nieuwenhuizen, Van Proeyen, from 1978
Binetruy, Gaillard, from 1985
Gomez-Reino et al, Achucarro et al, Covi et al, from 2007

We are interested in anti-D3 brane interaction with Calabi-Yau moduli T_i. In supergravity we expect some interaction between the nilpotent superfield S and Calabi-Yau moduli T_i

$$\mathcal{G}(T^i, \bar{T}^i; S, \bar{S})$$

$$\mathcal{G} \equiv K + \log W + \log \bar{W} \,, \qquad \mathbf{V} = e^{\mathcal{G}}(\mathcal{G}^{\alpha\bar{\beta}}\mathcal{G}_\alpha\mathcal{G}_{\bar{\beta}} - 3)$$

Erich Kahler noticed in 1933 and Moroianu suggested in 2004, that once the Hermitian Kahler function is introduced

"a long list of miracles occur then"

May 2017

RK, Linde, Roest, Yamada

Model Building Paradise

now confirmed in the cosmological context

We start with a geometry,

$$\mathcal{G} = \mathcal{G}_0(T_i, \bar{T}_i) + S + \bar{S} + \mathcal{G}_{S\bar{S}}(T_i, \bar{T}_i) S\bar{S}$$

Any phenomenological potential $\mathbf{V}_{pheno}$ can be reconstructed by choosing the metric of a nilpotent superfield S:

$$\mathbf{V}_{pheno} = e^{\mathcal{G}_0}(\mathcal{G}^{S\bar{S}} + \mathcal{G}_i \mathcal{G}^{i\bar{j}} \mathcal{G}_{\bar{j}} - 3)$$

Relation between a general potential and S-geometry

$$\mathcal{G}^{S\bar{S}} = \left(e^{-\mathcal{G}_0} \mathbf{V}_{pheno} - \mathcal{G}_i \mathcal{G}^{i\bar{j}} \mathcal{G}_{\bar{j}} + 3 \right)$$

But the metric becomes quite complicated.

In models with Hermitian Kähler function of the form

$$\mathcal{G} = \mathcal{G}_0(T_i, \bar{T}_i) + S + \bar{S} + \mathcal{G}_{S\bar{S}}(T_i, \bar{T}_i) S\bar{S}$$

If during inflation,
as in α-attractor models,
or 7-disk geometries

$$T_i = \bar{T}_i \,, \qquad S = 0$$

$$e^{\mathcal{G}} = m_{3/2}^2 \qquad \mathcal{G}_i = 0$$

one finds the following simple relation between the potential and the nilpotent field geometry

$$\mathcal{G}^{S\bar{S}}(T_i, \bar{T}_i) = \frac{\mathbf{V}(T_i, \bar{T}_i) + 3|m_{3/2}|^2}{|m_{3/2}|^2}$$

From the sky to fundamental physics

Easy to establish stability during and after inflation with the exit into de Sitter vacuum: sectional and bisectional curvature associated with our Kähler function play important role in the stability analysis

Why the conditions

$$T_i = \bar{T}_i \,, \qquad S = 0$$

$$e^{\mathcal{G}} = m_{3/2}^2 \qquad \mathcal{G}_i = 0$$

are universally valid in α-attractor hyperbolic models ?

The choice of the Kahler frame is suggested by the **tessellation** of the hyperbolic geometry

A tessellation is the tiling of a plane or hyperbolic plane, or a hyperbolic disk using one or more geometric shapes, called tiles, with no overlaps and no gaps

It leads to an improved stability of inflationary trajectory, since the moduli-dependent part of the geometry is flat in the inflaton direction due to inversion and scaling symmetry of the Kahler fuction

The models are relatively simple if the T-moduli have hyperbolic geometry of a combination of Poincare disks. In half-flat geometry variables

$$\mathcal{G}\Big|_{S=0} = -\frac{1}{2}\sum_{i=1}\log\Big[\frac{(T_i+\bar{T}_i)^2}{4T_i\bar{T}_i}\frac{1}{m_{3/2}^4}\Big]$$

The Kahler function is invariant under **inversion and scaling part of the Mobius symmetry**

$$T_i \to \frac{1}{T_i}, \qquad T_i \to a^2\, T_i$$

Inflaton shift symmetry is broken only via interaction with the anti-D3 brane, via the S-field geometry

$$\mathcal{G}_{S\bar{S}}(T_i, \bar{T}_i) S\bar{S}$$

Inflaton shift symmetry in a hyperbolic geometry with inversion/scaling symmetry

$$-\frac{1}{2}\log\Big[\frac{(T+\bar{T})^2}{4T\bar{T}}\,\frac{1}{m_{3/2}^4}\Big] \quad =$$

$$-\frac{1}{2}\log\Big[\frac{(\mathrm{Re}T)^2}{(\mathrm{Re}T)^2+(\mathrm{Im}T)^2}\,\frac{1}{m_{3/2}^4}\Big]$$

If during inflation Im T =0 is a minimum, which is valid in our models, we find

$$e^{\mathcal{G}} = m_{3/2}^2 \qquad\qquad \frac{\partial\mathcal{G}}{\partial T} = 0$$

for $\quad S = T - \bar{T} = 0$

Model-building Paradise

Why do we use these words? Let us look again at this equation from the previous page:

$$\mathcal{G}^{S\bar{S}}(T_i,\bar{T}_i) = \frac{\mathbf{V}(T_i,\bar{T}_i) + 3|m_{3/2}|^2}{|m_{3/2}|^2}$$

This equation shows incredible simplicity of inflationary model construction in this approach. One can take any function V(X) of a real inflaton field X, replace X, e.g., by $(T+\overline{T})/2$, and put the resulting function $V(T,\overline{T})$ to the equation above. The resulting potential evaluated by usual SUGRA rules automatically has the desired shape V(X) in the inflaton direction. The only thing remaining to check is stability with respect to the imaginary part of the field T, which is usually not a problem.

By this method we easily reproduced and improved many previously known models, and generalized them in a way allowing to have an arbitrary cosmological constant and SUSY breaking after inflation.

Why so simple?

- One nilpotent multiplet (representing anti-D3 brane)
- Supersymmetry is broken only in the nilpotent goldstino direction due to inversion/scaling symmetry of the Kahler function. It is unbroken in absence of anti-D3 brane
- Only the nilpotent multiplet geometry breaks inversion/scaling symmetry of the moduli geometry

7-disk cosmological model

3α=7 example

Kallosh, AL, Wrase, Yamada 1704.04829, Kallosh, AL, Roest, Yamada 1705.09247

1. Start with M-theory, or String theory, or $\mathcal{N}$=8 supergravity

2. Perform a consistent truncation to $\mathcal{N}$=1 supergravity in d=4 with a 7-disk manifold

$$\mathcal{G} = \log W_0^2 - \frac{1}{2}\sum_{i=1}^{7} \log \frac{(1 - Z_i \overline{Z}_i)^2}{(1 - Z_i^2)(1 - \overline{Z}_i^2)} + S + \overline{S} + \mathcal{G}_{S\overline{S}} S\overline{S},$$

Tessellation

$$\mathcal{G}^{S\overline{S}} = \frac{1}{W_0^2}(3W_0^2 + \mathbf{V}).$$

corresponding to the merger of seven disks of unit size

The scalar potential defining geometry is

$$\mathbf{V} = \Lambda + \frac{m^2}{7}\sum_i |Z_i|^2 + \frac{M^2}{7^2}\sum_{1\leq i\leq j\leq 7}\Big((Z_i + \overline{Z}_i) - (Z_j + \overline{Z}_j)\Big)^2,$$

De Sitter exit

During inflation $\mathbf{V}(\varphi) = \Lambda + m^2 \tanh^2 \frac{\varphi}{\sqrt{14}},$

$r \approx 10^{-2}$

Consider an example of the 2 unit size disks merger

Kallosh, AL, Wrase, Yamada 1704.04829, Kallosh, AL, Roest, Yamada 1705.09247

We will study it in supergravity, using Kahler function

$$\mathcal{G} = \log W_0^2 - \frac{1}{2}\sum_{i=1}^{2}\log\frac{(1-Z_i\overline{Z}_i)^2}{(1-Z_i^2)(1-\overline{Z}_i^2)} + S + \overline{S} + g_{S\overline{S}}S\overline{S},$$

$$g^{S\overline{S}} = \frac{1}{W_0^2}\left(|F_S|^2 + \frac{m^2}{2}(|Z_1|^2+|Z_2|^2) + \frac{M^2}{4}\left((Z_1+\overline{Z}_1)-(Z_2+\overline{Z}_2)\right)^2\right)$$

The scalar potential

$$\mathbf{V} = \Lambda + \frac{m^2}{2}(|Z_1|^2+|Z_2|^2) + \frac{M^2}{4}\left((Z_1+\overline{Z}_1)-(Z_2+\overline{Z}_2)\right)^2$$

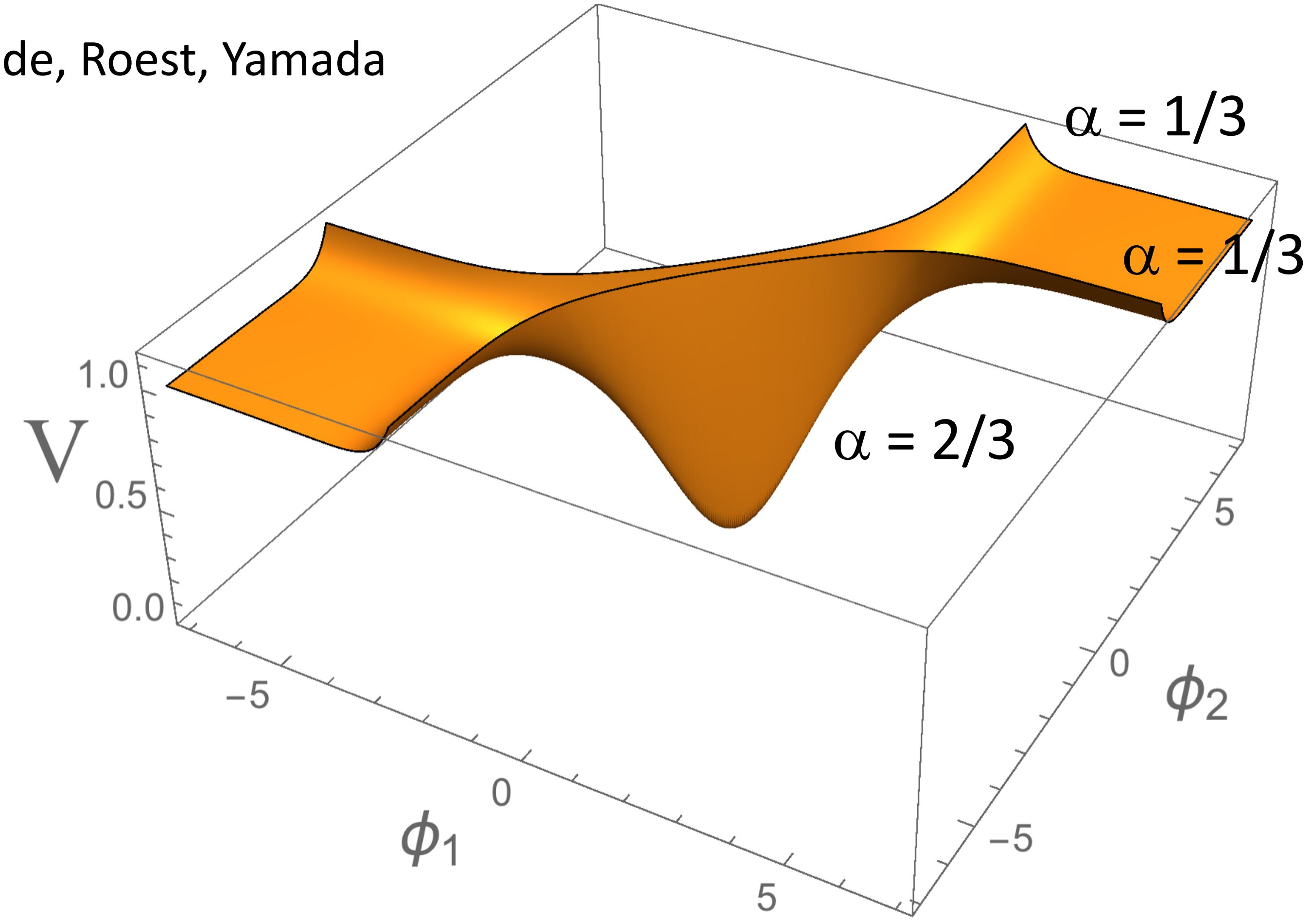
$$Z_i = \tanh\frac{\phi_i + \mathrm{i}\theta_i}{\sqrt{2}} \qquad\qquad \Lambda = |F_S|^2 - 3|W_0|^2$$

For M >> m, the last term in the potential forces the inflaton fields to coincide,

$$\phi_1 = \phi_2$$

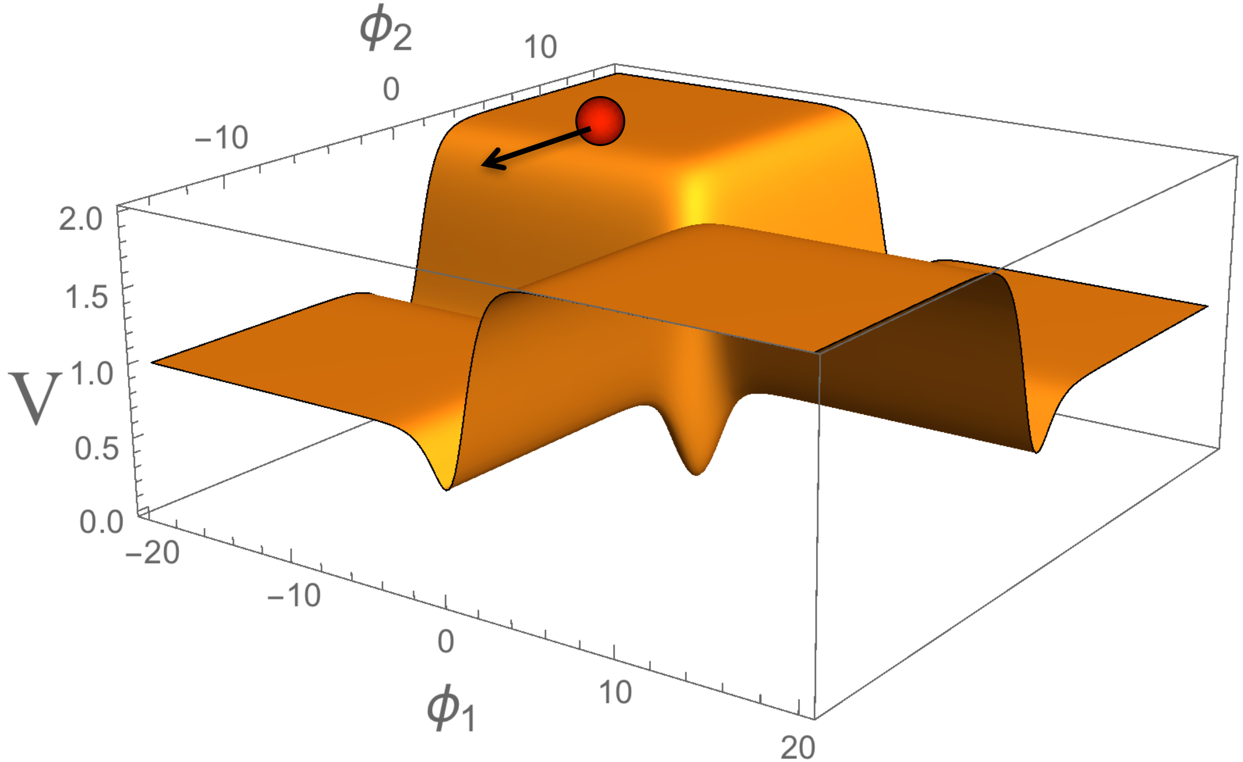
Two strongly interacting attractors with α = 1/3 merge into one attractor with α = 2/3.

Kallosh, Linde, Roest, Yamada
1705.09247



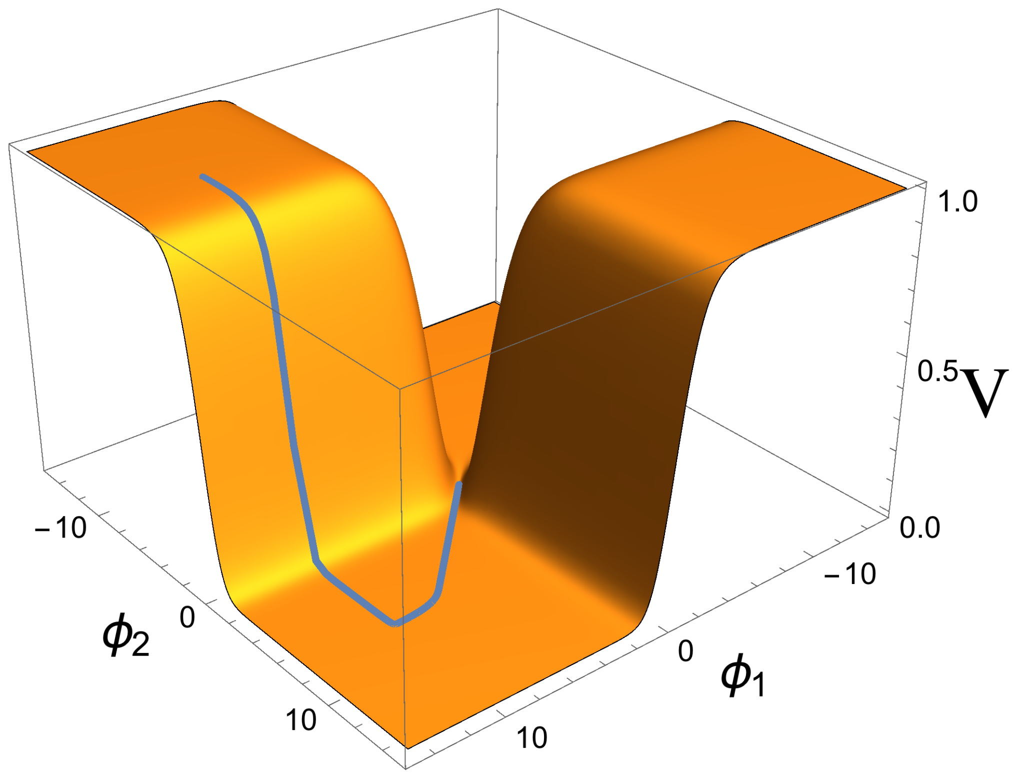
Here we wanted to show the potential at its low values, at the end of inflation, so we cut out its upper part. Now let us restore it in the next slide

Cascade Inflation



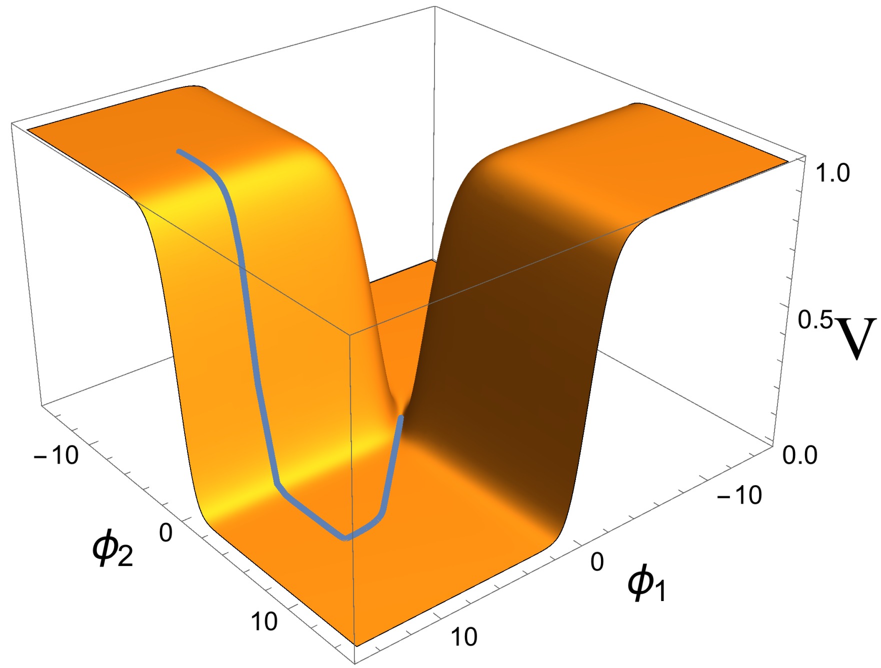
The minimum corresponds to the attractor merger shown at the previous slide. This is where inflation ends. But it begins at the infinitely long upper plateau of height $O(M^2)$. For natural values of M = O(1), this plateau can have nearly Planckian height – no problem to start inflation. After that, the fields cascade down to the inflationary valleys, which later merge. Simple beginning, and last stages matching Planck data.

Cascade Inflation



Inflation begins at the upper plateau of the height M^2, then the field waterfalls to the lower plateau of the height m^2, and gets captured by the gorge along the direction $\phi_1 = \phi_2$ until inflation ends. The original waterfall is described by α-attractor with $\alpha=1/3$. The last stage of inflation corresponds to $\alpha=2/3$. The figure shows the process for M = O(1), m<< M.

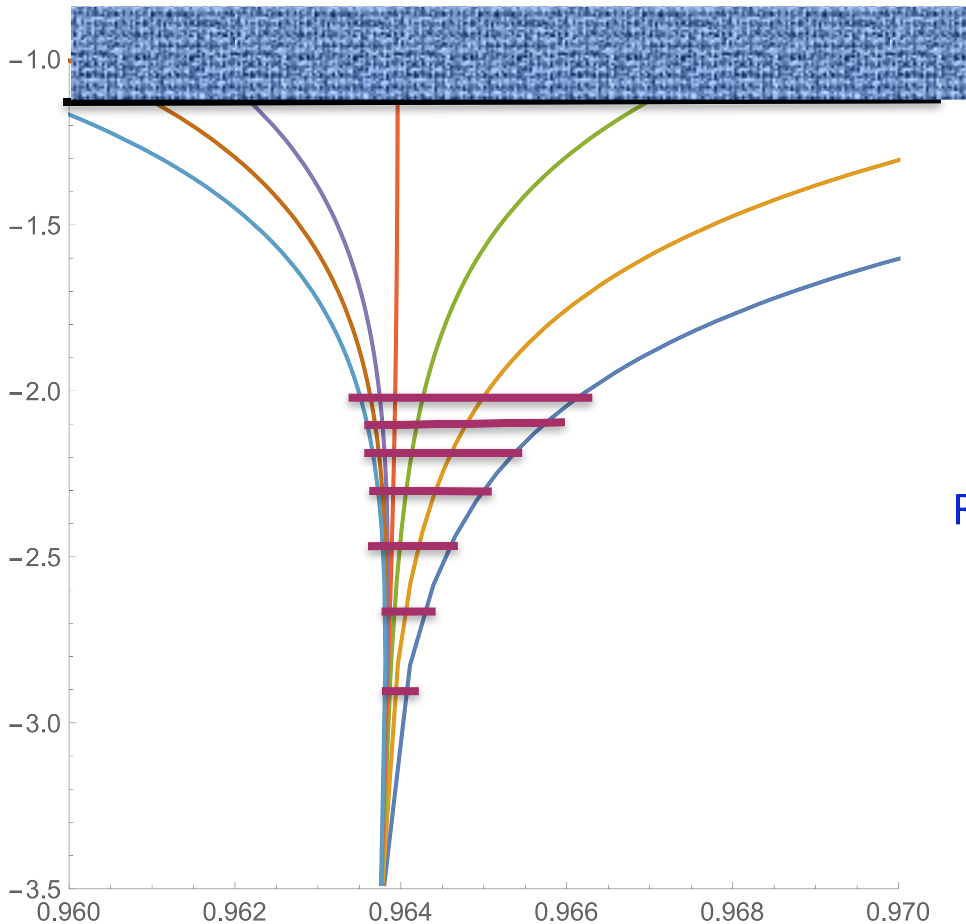
Cascade Inflation and Dark Energy



If inflation begins at asymptotically large values of the fields, then after falling to the lower plateau, the fields move exponentially slowly. If we arrange the parameters of the model to make the height of the lower plateau 10^{-120}, we can describe both inflation and dark energy: The field on the lower plateau will roll down exponentially slowly, and its energy will be dominated by the tiny height of this plateau.

B-mode targets: from maximal supersymmetry to minimal supersymmetry

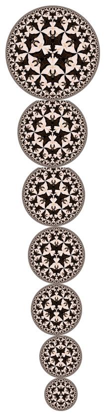
α-attractors $\log_{10} r$- n_s plane



n_s

$r < 0.07$

R^2_{Escher} $=3\alpha$ = 7,6,5,4,3,2,1



Conclusions:

We studied inflationary cosmological attractor models based on hyperbolic geometry of the moduli space of scalars fields. They fit the current cosmological data.

Starting with fundamental theories such as M-theory in d=11, superstring theory in d=10, as well as *$\mathcal{N}$=8 maximal supergravity and $\mathcal{N}$=4 superconformal theory*, we derived these α-attractor models with specific values of Escher's disk size, $3\alpha=7,6,5,4,3,2,1$.

These inflationary models describe inflation and dark energy and supersymmetry breaking. They provide B-mode targets for future B-mode detectors, with r between 10^{-2} and 10^{-3}