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TOPICS FROM 20^{th} CENTURY PHYSICS

An introductory course for students in mathematics

Notes for a course given at the Mathematical Institute of Leiden University in the spring semester of 2000.

Preface

Physics as we know it began in the sixteenth and seventeenth century, when in the study of natural phenomena empirical observation and mathematical modelling were for the first time systematically and successfully combined. This is exemplified in the person of Isaac Newton who laid the foundations of our picture of the physical world. He was equally great as a mathematician and as an empirical investigator and observer.

The interaction between physics and mathematics has remained of great importance. Modern theoretical physics could not exist without advanced mathematics. On the other hand many ideas in mathematics have had their origin in physics – often in a heuristic form.

The importance of the connection between mathematics and physics is no longer reflected in the curriculum of the Dutch universities. Physics students have to learn in their first and second year a great deal of standard mathematics, mainly calculus and linear algebra, but modern mathematics with its more abstract language remains strange in spirit to them, even though they sometimes pick up and learn to use methods based on it. Mathematics, on the other hand, is taught as a self-contained subject, which can be studied for its own sake, without any reference to or knowledge of physics. After a few years of training in rigourously formulated mathematics the average mathematics students will find the loose language of standard physics text books very hard to understand.

There is a strange asymmetry in the situation. Numerous books have been written explaining to physicists advanced topics from mathematics such as functional analysis, differential geometry, Lie groups and Lie algebras. Not much exists in the other direction; books on physics written for mathematicians are very rare, even though one would think that there is a need for such books.

The course for which these notes form the basis is an attempt to teach topics from modern physics to an audience of students in mathematics. It gives an introduction to the two highlights of twentieth century physics, quantum mechanics and relativity, at an advanced undergraduate level – fourth or fifth year students in the present Dutch system. The course should also be suitable for physics students with an interest in the mathematical background of physics.

Appendices A - D are added to – and should be read in parallel with – to the main text of the notes. They supply brief reviews of necessary mathematical material; this should be useful for the sake of establishing conventions and notation and to make the notes self-contained. Mathematics and physics should not be taught along historical lines, but an education in science is incomplete without the acquirement of some knowledge of its history. Appendix E provides short sketches of the lives of some of the interesting and colourful actors that have appeared in it. A selective bibliography is added.

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