DUAL LATTICE BLOCKSPIN TRANSFORMATION AND PERFECT MONOPOLE ACTION FOR SU(2) GAUGE THEORY

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INTRODUCTION

It is a very important problem to understand low-energy hadron physics starting directly from quantum chromodynamics (QCD). Numerical Monte-Carlo simulations of lattice QCD have been done extensively. To obtain the continuum limit reliably, we have to consider much larger lattices than presently available ones.

Recently, considerable efforts have been made to improve a lattice action in various ways in order to get more reliable data on available lattices[1, 2]. Theoretically the best is a perfect action which could be obtained as the renormalized trajectory after renormalization transformations like a block-spin one. However, a block-spin transformation using nonabelian link variables can not be carried out practically in QCD unless we are restricted to too simple a truncated form of lattice action[3].

The fixed point action (the classical perfect action) which is fixed analytically is interesting[4]. It is a perfect action in the ultraviolet limit. However, it is not clear how far the fixed point action is different from the perfect action in the interesting infrared region.

Is it possible to fix a perfect lattice action which is valid in the infrared region? Note that there are various ways of renormalization transformations. It seems important to pay attention to a dynamical variable which plays a dominant role in the infrared region. The usual nonabelian link variables do not look suitable to describe long-range behaviors of QCD.

In this respect, the old conjecture which was proposed by 't Hooft is very interesting[5]. It says that QCD is reduced to abelian $U(1) \times U(1)$ theory with electric and magnetic charges after performing a partial gauge fixing called abelian projection. Quark confinement could be understood by the dual Meissner effect due to condensation of the magnetic monopoles.

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Actually there have been many numerical data supporting the above conjecture when we perform the abelian projection in the maximally abelian (MA) gauge, using so-called abelian and monopole dominance[6, 7, 8].

However, why the MA gauge is so beautiful is not known yet, these facts indicate that the abelian (monopole) component after abelian projection is dominant in the infrared QCD. It is interesting to perform a block-spin transformation paying attention to these abelian components.

The preliminary studies have been done by Shiba and Suzuki[9, 10]. They get a $U(1)$ effective action in terms of monopole currents in $T=0$ $SU(2)$ gauge theory after performing a dual transformation numerically. The effective action is fixed also for extended monopoles[11]. Considering an extended monopole corresponds to making a block spin transformation on the dual lattice. They restricted the forms of monopole current interactions only to quadratic ones. This action seems to show scaling behavior, that is, it depends only on a physical scale $b = na(\beta)$.

However there are some insufficiencies in their works. The biggest problem is that the obtained action violates discrete rotational invariance. Since the renormalized trajectory leads us to the continuum theory, such a violation of the lattice rotational invariance is unnatural. This suggests that the restriction of the action to quadratic forms alone was too naive.

We report here on our recent results in which a discrete rotational invariant action is successfully obtained analyzing longer-range region on larger lattices.

STEPS OF CALCULATIONS

The process of our study is the following (see Fig.1):

1. We first generate the thermalized nonabelian link fields $\{U(s,\mu)\}$ using a simple Wilson action for pure $SU(2)$ gauge theory. The lattices considered are 16^4, 24^4, 32^4 and 48^4 for $\beta = 2.0 \sim 2.8$

2. Next we perform abelian projection in the MA gauge and then separate abelian link variables $\{u_\mu(s)\}$ from gauge-fixed $SU(2)$ link fields $\{U'(s,\mu)\}$. The MA gauge fixing condition is to maximize the following operator:

$$R = Tr \sum_{s,\mu} \left[U(s,\mu)\sigma_3 U^\dagger(s,\mu)\sigma_3 \right]. \tag{1}$$

Then

$$X(s) = \sum_{\mu} \left[U(s,\mu)\sigma_3 U^\dagger(s,\mu) + U^\dagger(s-\hat{\mu},\mu)\sigma_3 U(s-\hat{\mu},\mu) \right] \tag{2}$$

is diagonalized. After the gauge-fixing, the $SU(2)$ link fields are expressed as

$$U'(s,\mu) = \begin{pmatrix} \sqrt{1-|c_\mu(s)|^2} & -c_\mu^*(s) \\ c_\mu(s) & \sqrt{1-|c_\mu(s)|^2} \end{pmatrix} \begin{pmatrix} u_\mu(s) & 0 \\ 0 & u_\mu^*(s) \end{pmatrix}, \tag{3}$$

where

$$u_\mu(s) = e^{i\theta_\mu(s)}, \quad (-\pi \le \theta_\mu(s) < \pi). \tag{4}$$

Here $u_\mu(s)$ transforms as the gauge field of the residual $U(1)$ symmetry and $c_\mu(s)$ as the charged matter field.

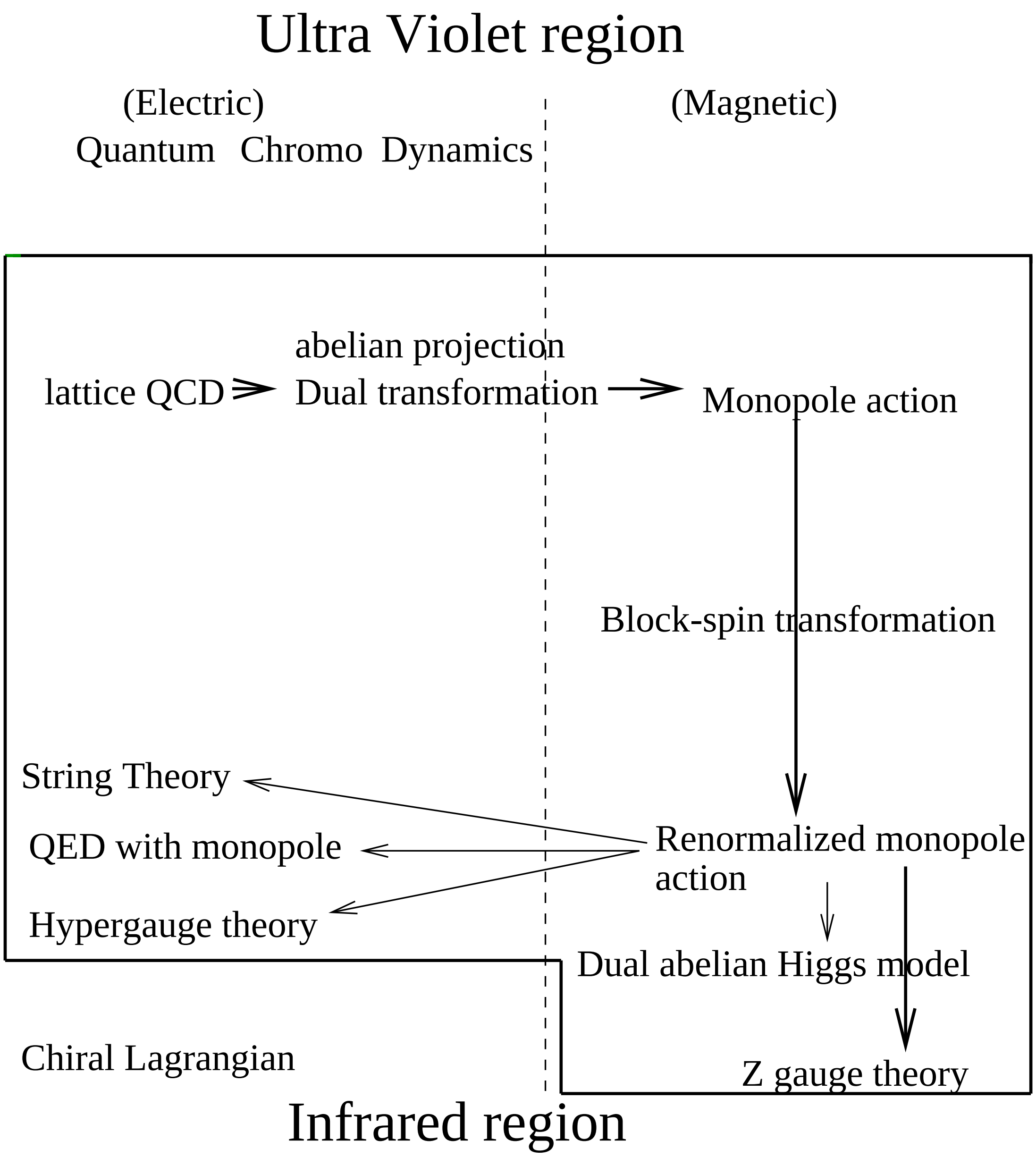

Figure 1. The flowchart of the study. The electric/magnetic means the original/dual lattice. In this report, only the dual abelian Higgs model and string model are discussed explicitly among various equivalent effective theories on the lattice.

3. The abelian dominance suggests that operators $\mathcal{O}(u_\mu(s))$ alone are enough to describe the infrared behavior of QCD. Then a sufficiently local abelian effective action is expected to exist:

$$Z = \int Du[\int Dce^{-S(u,c)}\delta(X^{\pm})\Delta_F(U)] \tag{5}$$

$$= \int Due^{-S_{eff}(u)}, \tag{6}$$

where $X^{\pm}$ is the off-diagonal part of (2) and $\Delta_F(U)$ is the Faddeev-Popov determinant.

4. Monopole currents can be defined from the abelian plaquette variables $\theta_{\mu\nu}(s)$ following DeGrand and Toussaint[12]. The abelian plaquette variables are

$$\theta_{\mu\nu}(s) \equiv \theta_\mu(s) + \theta_\nu(s+\hat{\mu}) - \theta_\mu(s+\hat{\nu}) - \theta_\nu(s), \quad (-4\pi < \theta_{\mu\nu}(s) < 4\pi). \quad (7)$$

It is decomposed into two terms:

$$\theta_{\mu\nu}(s) \equiv \bar{\theta}_{\mu\nu}(s) + 2\pi n_{\mu\nu}(s), \qquad (-\pi \leq \bar{\theta}_{\mu\nu}(s) < \pi). \quad (8)$$

Here, $\bar{\theta}_{\mu\nu}(s)$ is interpreted as the electro-magnetic flux through the plaquette and the integer $n_{\mu\nu}(s)$ corresponds to the number of Dirac strings penetrating the plaquette. One can define the quantized conserved monopole currents $k_\mu(s)$:

$$k_\mu(s) = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\nu n_{\rho\sigma}(s+\hat{\mu}). \quad (9)$$

5. The monopole dominance suggests that operators $\mathcal{O}(k_\mu(s))$ alone describe the infrared behavior of QCD. An effective monopole action is expected to exist:

$$\begin{aligned} Z &= (\prod\sum)\int Du\delta(k,u)e^{-S_{eff}(u)} \\ &= (\prod_{s,\mu}\sum_{k_\mu(s)=-\infty}^{\infty})(\prod_s \delta_{\partial'_\mu k_\mu(s),0})e^{-S[k]}, \end{aligned} \quad (10)$$

where $\delta(k,u)$ stands for the condition (9). Since the monopole currents are attached to the dual lattice, Eq.(10) is a kind of dual transformation.

6. Now we perform a block-spin transformation in terms of the monopole currents on the dual lattice to investigate the perfect action in the infrared region. We study $n = 1, 2, 3, 4, 6, 8, 12$ extended monopole currents[11] as an n blocked operator:

$$k_\mu^{(n)}(s) = \sum_{i,j,l=0}^{n-1} k_\mu(ns + (n-1)\hat{\mu} + i\hat{\nu} + j\hat{\rho} + l\hat{\sigma}). \quad (11)$$

Actually the definition includes the usual block-spin transformation. For example,

$$\begin{aligned} k_\mu^{(2)}(s) &= \sum_{i,j,l=0}^{1} k_\mu(2s + \hat{\mu} + i\hat{\nu} + j\hat{\rho} + l\hat{\sigma}), \quad (12) \\ k_\mu^{(4)}(s) &= \sum_{i,j,l=0}^{3} k_\mu(4s + 3\hat{\mu} + i\hat{\nu} + j\hat{\rho} + l\hat{\sigma}) \\ &= \sum_{i,j,l=0}^{1} k_\mu^{(2)}(2s + \hat{\mu} + i\hat{\nu} + j\hat{\rho} + l\hat{\sigma}) \quad (13) \end{aligned}$$

The renormalized lattice spacing is $b = na(\beta)$ and the continuum limit is taken as the limit $n \to \infty$ for a fixed physical length b. The renormalization flow is determined by the relation

$$e^{-S^{(n)}[k^{(n)}]} = (\prod_{s,\mu}\sum_{k_\mu(s)=-\infty}^{\infty})(\prod_s \delta_{\partial'_\mu k_\mu(s),0})\delta(k^{(n)},k)e^{-S[k]}, \quad (14)$$

where $\delta(k^{(n)},k)$ stands for the definition (11).

Table 1. The physical scale $a(\beta)$

β	$a(\beta)\sqrt{\kappa}$	3σ
2.0000	.69516	.01504
2.1000	.60641	.01060
2.2000	.48037	.00520
2.3000	.35887	.00361
2.4000	.26784	.00828
2.5000	.18551	.00315
2.6000	.13455	.00386
2.7000	.10155	.00212
2.8000	.07771	.00167

7. Since we study small β regions, we have to fix the physical scale $a(\beta)$ without use of the theoretical asymptotic beta function. In pure $SU(2)$ gauge theory, we have fixed it from evaluating the string tension κ_L from the monopole part of abelian Wilson loops. The lattice spacing $a(\beta)$ is given by the relation $a(\beta) = \sqrt{\kappa/\kappa_L}$, where κ is the physical string tension. Since the monopole contribution to the Wilson loop is known to have only linear and constant terms (no Coulomb term)[13], it is the best operator to fix the string tension for such small β region where Wilson loops are small. To eliminate the finite-size effects, we have used the data of $n = 1$ monopoles on 16^4 lattice for $\beta = 2.0 - 2.3$, $n = 2$ monopoles on 24^4 lattice for $\beta = 2.4 - 2.6$ and $n = 4$ monopoles on 48^4 lattice for $\beta = 2.7 - 2.8$. Although the statistical errors of the original data are small, we adopt 3σ as the errors considering the differences of the string tensions determined by the different extended monopoles. There may be other systematic errors coming from the possible difference between string tensions from nonabelian and abelian Wilson loops. The physical scale in units of $1/\sqrt{\kappa}$ is given in Table 1.

INVERSE MONTE-CARLO METHOD

The renormalization flows are obtained numerically from the ensemble of thermalized (extended) monopole currents $\{k_\mu^{(n)}(s)\}$ with the aid of an inverse Monte-Carlo method first developed by Swendsen[14] and extended to closed monopole currents by Shiba and Suzuki[9, 10].

Consider the expectation value of some operator $\mathcal{O}_i[k]$ assuming the monopole action $S[k] = \sum_i g_i S_i[k]$:

$$\langle \mathcal{O}_i[k] \rangle = \frac{\prod_{s,\mu} \sum_{k_\mu(s)=-\infty}^{\infty} \mathcal{O}_i[k] \exp\left(-\sum_i g_i S_i[k]\right)}{\prod_{s,\mu} \sum_{k_\mu(s)=-\infty}^{\infty} \exp\left(-\sum_i g_i S_i[k]\right)}. \tag{15}$$

Define $\tilde{S}[k]$ as a part of $S[k]$ which contains currents along a special plaquette $(s', \hat{\mu}', \hat{\nu}')$, i.e., $\{k_{\mu'}(s'), k_{\nu'}(s' + \hat{\mu}'), k_{\mu'}(s' + \hat{\nu}'), k_{\nu'}(s')\}$. Using $\tilde{S}[k]$, Eq.(15) is rewritten in the following form:

$$\begin{aligned} \langle \mathcal{O}_i[k] \rangle &= \frac{\prod_{s,\mu} \sum_{k_\mu(s)=-\infty}^{\infty} \overline{\mathcal{O}}_i[k] \exp\left(-\sum_i g_i S_i[k]\right)}{\prod_{s,\mu} \sum_{k_\mu(s)=-\infty}^{\infty} \exp\left(-\sum_i g_i S_i[k]\right)} && (16) \\ &= \langle \overline{\mathcal{O}}_i[k] \rangle, && (17) \end{aligned}$$

where

$$\overline{\mathcal{O}}_i[k] = \frac{\sum_{M=-\infty}^{\infty} \mathcal{O}_i[k'] \exp\left(-\sum_i g_i \tilde{S}_i[k']\right)}{\sum_{M=-\infty}^{\infty} \exp\left(-\sum_i g_i \tilde{S}_i[k']\right)}, \tag{18}$$

$$k'_\mu(s) \equiv k_\mu(s) + M(\delta_{s,s'}\delta_{\mu,\mu'} + \delta_{s,s'+\hat{\mu}'}\delta_{\mu,\nu'} - \delta_{s,s'+\hat{\nu}'}\delta_{\mu,\mu'} - \delta_{s,s'}\delta_{\mu,\nu'}). \tag{19}$$

The identity (17) can be used to fix the monopole action $S[k]$ iteratively. Define $\overline{\mathcal{O}}_i[k,\tilde{g}]$ which is equal to Eq.(18) with trial coupling constants $\{\tilde{g}_i\}$ instead of real ones $\{g_i\}$. When g_i is not equal to $\tilde{g}_i$ for all i, we get the following expansion up to the first order in $g_i - \tilde{g}_i$:

$$\langle \mathcal{O}_i - \overline{\mathcal{O}}_i \rangle = \langle \overline{\mathcal{O}}_i \overline{S}_j - \overline{\mathcal{O}_i S_j} \rangle (g_j - \tilde{g}_j). \tag{20}$$

For details, see Ref.9.

Practically, we first need to restrict the type of interactions. It is natural that short-distant interactions and two-point interactions are more important. Here we adopt discrete rotational invariant terms composed of two-point self and Coulomb interactions and 4- and 6-point interactions as a dominant part. To test the validity of the dominant form, we add general two-point interactions up to lattice distance three. The contribution of the additional terms means insufficiencies of the dominant form of the action adopted.

$$\begin{aligned} S[k] = {} & \frac{g_m^2}{2} \sum_{s,s'} \sum_{\mu=1}^{D} k_\mu(s) \Delta^{-1}_{s,s'} k_\mu(s') + p \sum_s \sum_{\mu=1}^{D} k_\mu^2(s) - q \sum_s \left(\sum_{\mu=-D}^{D} k_\mu^2(s) \right)^2 \\ & + r \sum_s \left(3 \left(\sum_{\mu=-D}^{D} k_\mu^2(s) \right)^3 + \sum_{\mu,\nu,\rho=-D}^{D} k_\mu^2(s) k_\nu^2(s) k_\rho^2(s+\hat{\mu}) \right) + S_2[k]. \end{aligned} \tag{21}$$

Here $\Delta^{-1}_{s,s'}$ is the lattice Coulomb propagator and $S_2[k] = \sum_i f_i S_2^{(i)}[k]$. $S_2^{(i)}[k]$'s are additional two-point interactions. Since the additional term $S_2[k]$ is found to be very small, we neglect it from now on. Other higher-order interactions are assumed to be small. For details, see Ref.15.

RESULTS

The results of our analyses are the following:

1. The inverse Monte-Carlo method works very well and the coupling constants are fixed beautifully in a discrete rotational invariant way, although we need to start with good initial conditions. Since the 4- and the 6-point interactions receive large contriburions from higher charged currents, the choice of the initial conditions is very important to get convergence. This is different from the old approach[9, 10] with 2-point interactions, where the convergence was achieved easily.

2. The first four terms, i.e., self + Coulomb + 4-point + 6-point interactions are found to be enough for the long-range region $b > 1/\sqrt{\kappa}$.

3. Both the limits $V \to \infty$ and $a \to 0$ must be taken to derive the continuum results from our finite-lattice analyses. To get the infinite volume limit, we have determined the actions from the different lattice sizes for each β and monopole size. Especially, the results for 4^3 monopoles show that the volume dependence is

weak (Fig.2). Hence we have used the χ^2 fit assuming the constant behaviors and have fixed the values of the infinite-volume limit for each monopole size. We also have tried the linear fit and the difference of the fits is included in the expected error values of the infinite-volume limit.

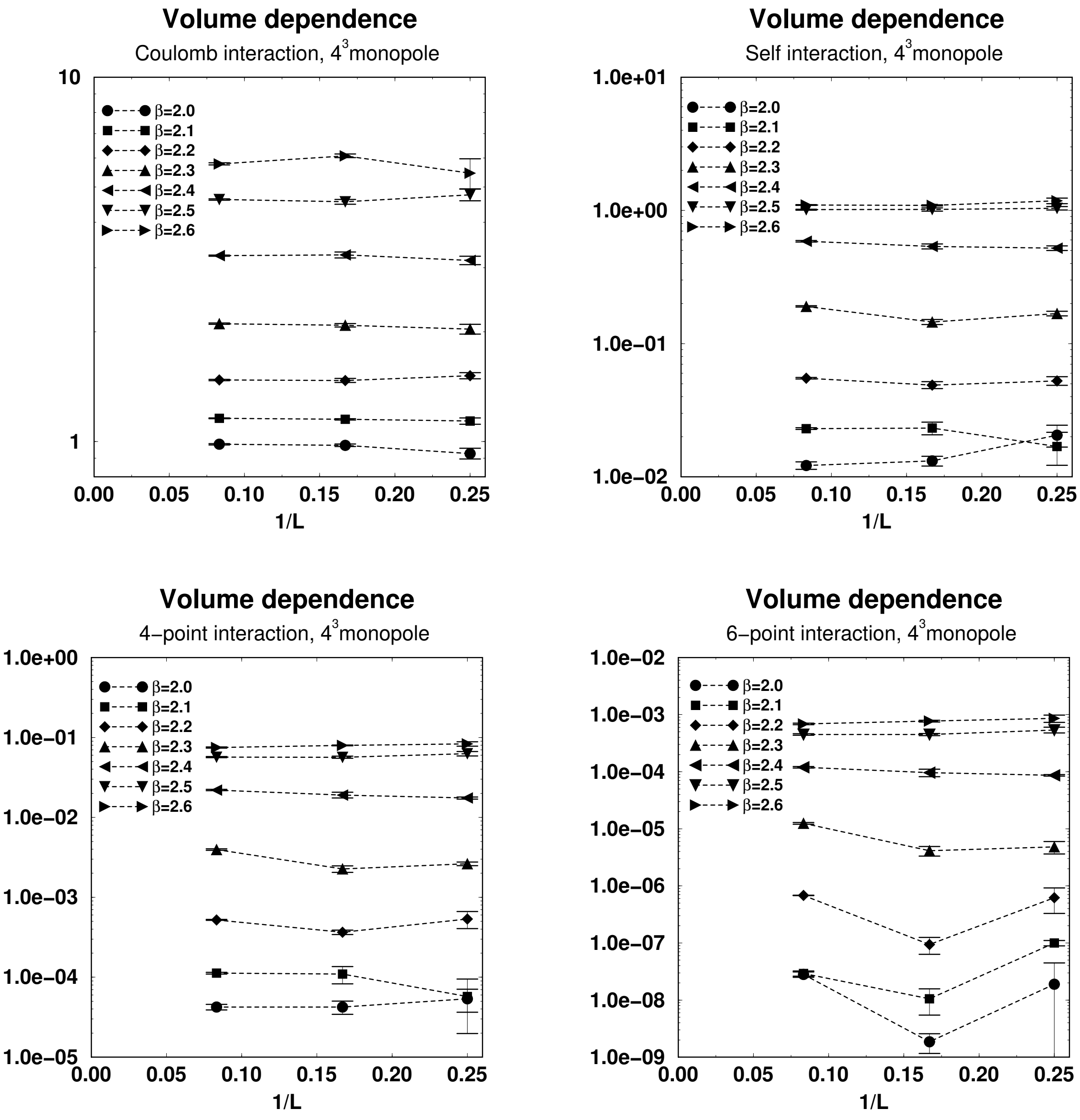

Figure 2. Volume dependence of four-dominant coupling constants.

4. The physical scale is fixed as shown in Table 1. The coupling constants are plotted versus $b = na(\beta)$ for each n extended monopole as shown in Fig.3. For small $b < 1/\sqrt{\kappa}$, all n data seem to be the same. When we go to higher b, the couplings of smaller n extended monopoles begin to deviate from the universal curve. All data with higher extended monopoles ($n \geq 4$) seem to scale in the coupling constants except that of the Coulomb interaction.

5. To study the $n \to \infty$ limit for each b, we have tried to fix the b-dependence for each n data using the χ^2 fit. In this step, we adopt a simple power function of b.

6. The dominant four couplings are plotted versus $1/n$ for each fixed b in Fig.4. As discussed earlier, there seems no n dependence for the small b region. The

Coulomb coupling seems to have small $1/n$ dependence for the large values of b. Other three couplings show almost constant behavior for at least $n \geq 6$ even in the large b region. The χ^2 fit has been performed assuming constant or linear relations in each case and the continuum limit of these couplings have been deduced.

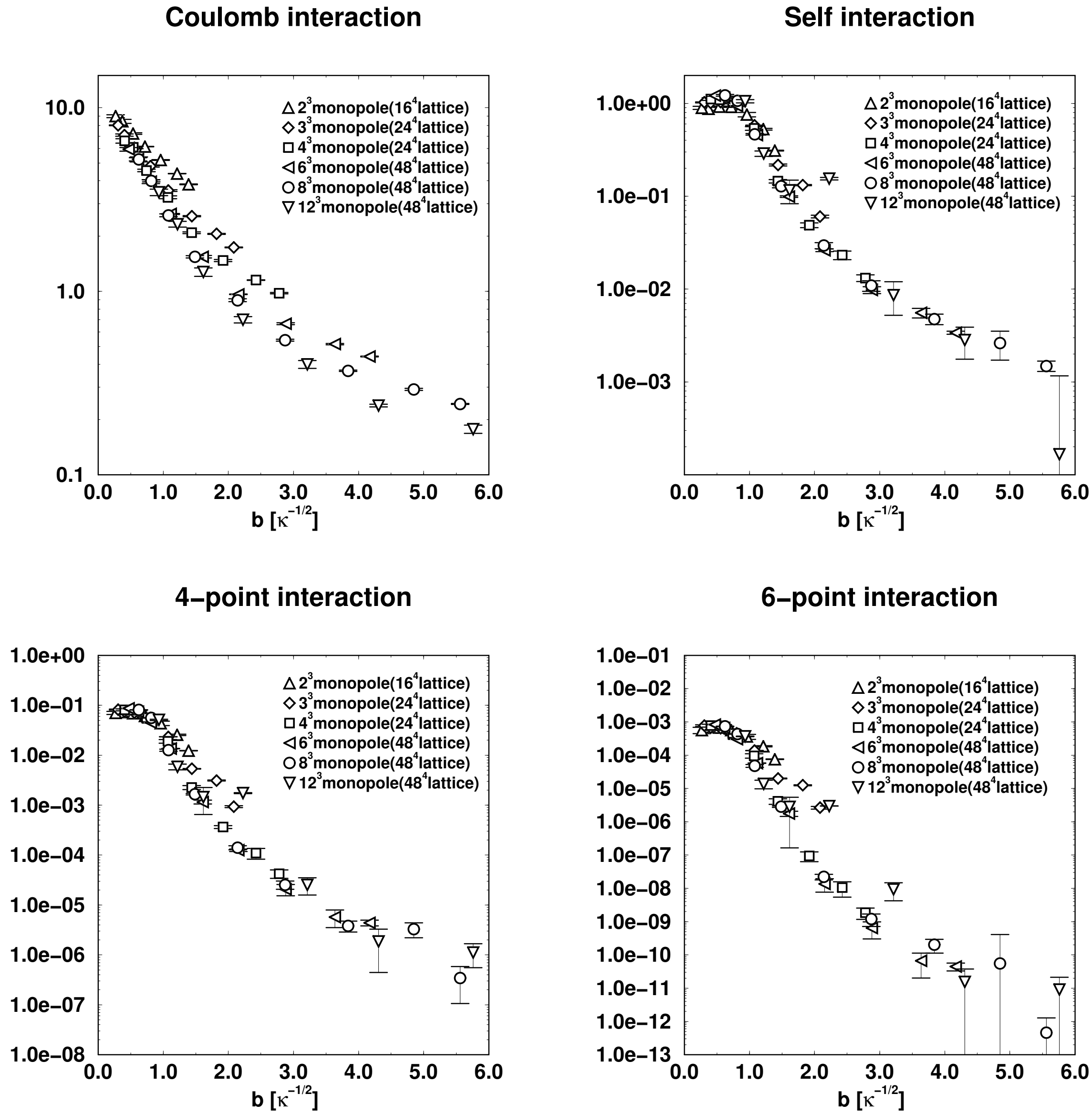

Figure 3. b-dependences for each n.

7. Finally using the $n \to \infty$ fit, we have performed a χ^2 fit using the following functional forms. The Coulomb term is fitted by

$$\frac{g_m^2(b)}{2} = c_1 \left[\frac{22}{3} \ln \left\{ 1 + \frac{1}{(bc_2)^{2c_3}} \right\} + \frac{68}{11} \ln \left\{ 1 + \frac{1}{c_3} \ln \left(1 + \frac{1}{(bc_2)^{2c_3}} \right) \right\} \right]. \quad (22)$$

This function has power behavior in the large b region and takes the form expected from the 2-loop perturbative β-function of pure $SU(2)$ gauge theory in the small b region. The coupling constants of the other three dominant terms p, q and r are fitted by a simple power function of b. Thus the almost perfect monopole action is obtained. The explicit results are shown in Fig.5.

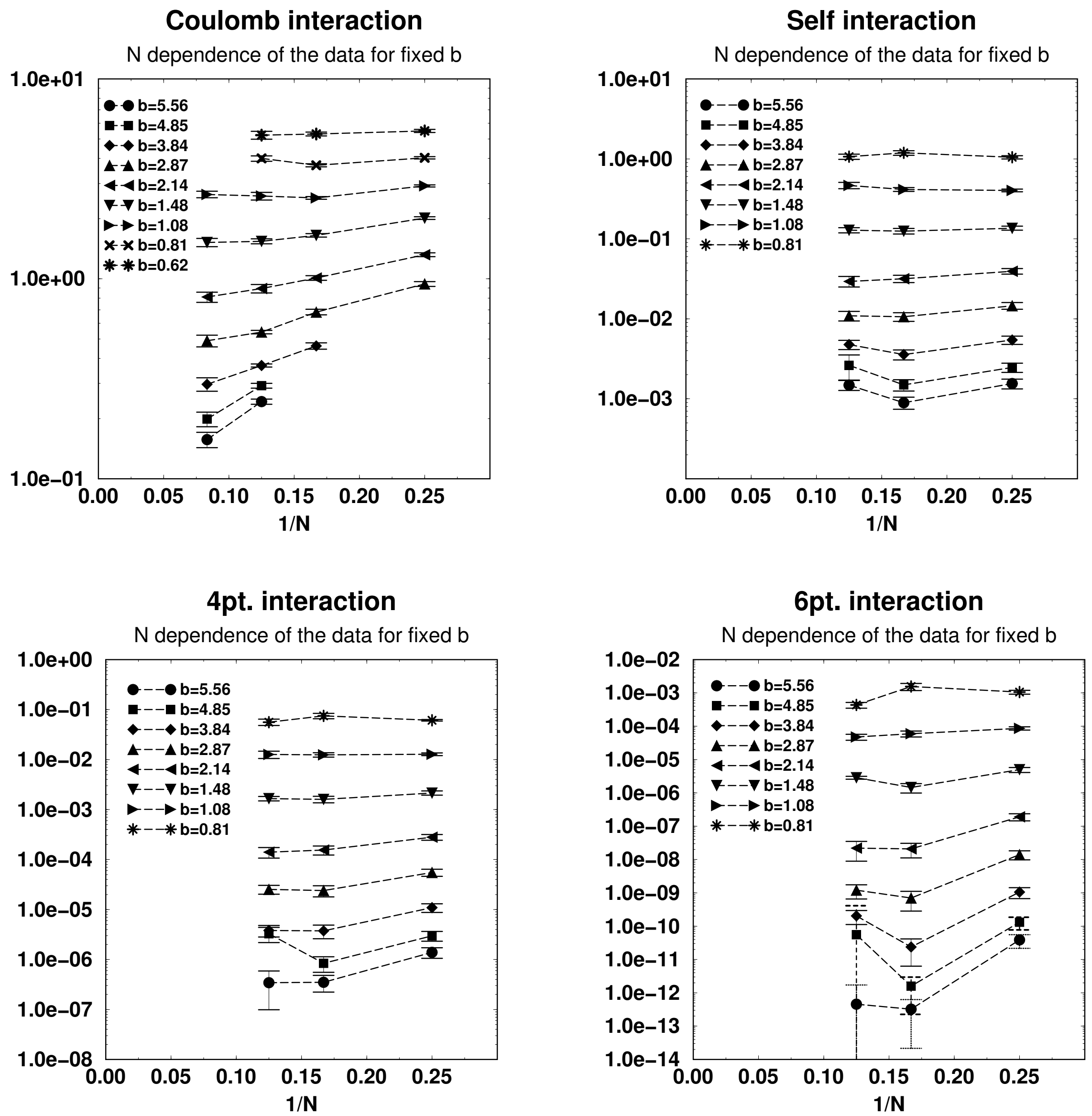

Figure 4. n-dependences for each b.

ABELIAN HIGGS MODEL

As a simple application of the obtained action, let us pay attention to the large b region ($b > 1$). Then the action seems to be well approximated by the Coulomb and the self interactions. If the action is expressed through these two terms only,

$$Z = \sum_{{}^*k \in Z, \delta {}^*k = 0} \exp\left\{ -\frac{g_m^2}{2}({}^*k, \Delta^{-1}{}^*k) + p\|{}^*k\|^2 \right\}, \tag{23}$$

the partition function can be exactly transformed to other representations[16] (here and below we use the lattice differential form formalism).

Using an auxiliary field ${}^*C({}^*c_1)$, the partition function can be written as

$$Z \propto \int_{-\infty}^{\infty} D\,{}^*C({}^*c_1) \sum_{{}^*k({}^*c_1) \in Z, \delta\,{}^*k = 0} \exp\left\{ -\frac{1}{2g_m^2}\|d\,{}^*C\|^2 - i({}^*C, {}^*k) - p\|{}^*k\|^2 \right\}, \tag{24}$$

where ${}^*c_i$ is the i-th dual cell on the lattice. Introducing the phase ${}^*\phi({}^*c_0)$, the current

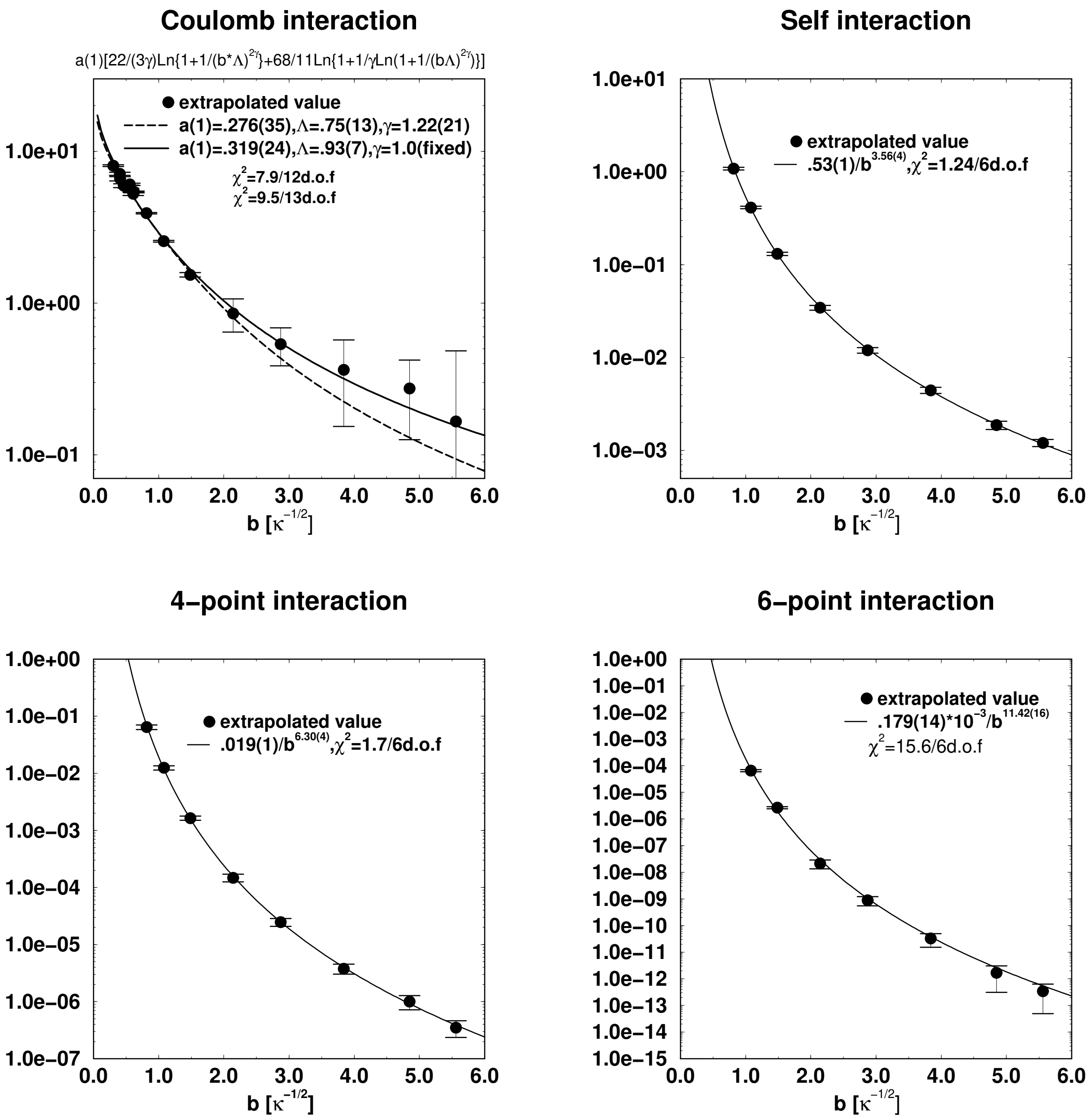

Figure 5. b-dependences of the perfect action. In the case of the Coulomb interactions, $(a(1), \Lambda, \gamma)$ are used instead of (c_1, c_2, c_3) in Eq.22.

conservation law can be rewritten in the following form:

$$\delta(\delta\, {}^*k) = const. \int_{-\pi}^{\pi} D\, {}^*\phi \exp[i({}^*\phi, \delta\, {}^*k)]. \tag{25}$$

Substituting Eq.(25) into Eq.(24), we get

$$Z \propto \int_{-\infty}^{\infty} D\, {}^*C({}^*c_1) \int_{-\pi}^{\pi} D\, {}^*\phi({}^*c_0) \sum_{{}^*k({}^*c_1) \in Z} \exp\left\{ -\frac{1}{2g_m^2} \| d\, {}^*C \|^2 + i(d\, {}^*\phi - {}^*C, {}^*k) - p\| {}^*k \|^2 \right\}. \tag{26}$$

Using the Poisson summation formula

$$\int DF \sum_{l \in Z} \exp[2\pi i(F, l)] f(F) = \sum_{k \in Z} f(k), \tag{27}$$

we get the expression of the partition function:

$$Z \propto \int_{-\infty}^{\infty} D\, {}^*C({}^*c_1) \int_{-\infty}^{\infty} D\, {}^*F({}^*c_1) \int_{-\pi}^{\pi} D\, {}^*\phi({}^*c_0) \sum_{{}^*l({}^*c_1) \in Z}$$

$$\exp\left\{-\frac{1}{2g_m^2}\|d\,{}^*C\|^2 + i(d\,{}^*\phi + 2\pi\,{}^*l - {}^*C,\,{}^*F) - p\|\,{}^*F\|^2\right\}. \tag{28}$$

The Gaussian integral with respect to the auxiliary field ${}^*F({}^*c_1)$ leads to the abelian-Higgs model on the dual lattice (the radius of the Higgs field is fixed):

$$Z^{AH} = \int_{-\infty}^{\infty} D\,{}^*C({}^*c_1) \int_{-\pi}^{\pi} D\,{}^*\phi({}^*c_0) \sum_{{}^*l({}^*c_1)\in Z}$$
$$\exp\left\{-\frac{1}{2g_m^2}\|d\,{}^*C\|^2 - \frac{1}{4p}\|d\,{}^*\phi + 2\pi\,{}^*l - {}^*C\|^2\right\} \tag{29}$$

When we take into account the 4- and the 6-point interactions of the monopole current, we get the dual abelian-Higgs model with the unfixed radius of the Higgs field. Let us start from the partition function of the dual abelian-Higgs model:

$$\mathcal{Z}^{A.H.} = \int_{-\infty}^{+\infty} \mathcal{D}\theta \int_{-\infty}^{+\infty} \mathcal{D}\phi \exp\left\{-\mathcal{S}^{A.H.}\right\}, \tag{30}$$

$$\mathcal{S}^{A.H.}[\theta,\phi] = \sum_P \frac{\beta}{2}\theta_p^2 + \lambda\sum_n \left(\phi^*(n)\phi(n) - 1\right)^2 + \sum_n \phi^*(n)\phi(n) \tag{31}$$

$$- \gamma \sum_{n,\mu} \left(\phi^*(n)U_\mu(n)\phi(n+\mu) + h.c.\right), \tag{32}$$

where $U_\mu = \exp(i\theta_\mu)$ is the dual gauge field, θ_P is the field strength tensor and $\phi(n) = \rho_n \exp(i\varphi_n)$ is the complex Higgs field.

One can rewrite the above integral as the sum over the closed monopole currents ${}^*k$ using the analogue of the BKT transformation[17]. The monopole action calculated in the saddle-point approximation up to $O(\lambda^{-2})$ terms has the form:

$$\begin{aligned} S^{mon}[{}^*k] = {} & \frac{1}{4\beta}\left({}^*k, \Delta^{-1}\,{}^*k\right) + \left\{\frac{1}{4\gamma} + \frac{1}{\lambda}\left(\frac{15}{32\gamma} - 1\right) + \frac{1}{\lambda^2}\left(\frac{865}{2^9\gamma} - \frac{439}{2^6} + 4\gamma\right)\right\}\|{}^*k\|^2 \\ & + \left\{-\frac{1}{2^8\gamma^2\lambda} + \frac{1}{\lambda^2}\left(\frac{7}{2^9\gamma} - \frac{157}{2^{12}\gamma^2}\right)\right\}\sum_x\left(\sum_{\mu=-D}^{D}{}^*k_{x,\mu}^2\right)^2 \\ & - \frac{1}{2^{11}\gamma^2\lambda^2}\sum_x\sum_{\mu=1}^{D}{}^*k_{x,\mu}^4 \\ & + \frac{1}{2^{13}\gamma^3\lambda^2}\sum_x\sum_{\mu,\nu,\delta=-D}^{D}{}^*k_{x,\mu}^2\,{}^*k_{x,\nu}^2\left(3\,{}^*k_{x,\delta}^2 + {}^*k_{x+\nu,\delta}^2\right) + \mathcal{O}\left(\frac{1}{\lambda^3}\right). \end{aligned} \tag{33}$$

When we consider the terms up to $O(\lambda^{-1})$ and determine the b dependence of the parameters β,γ,λ from the monopole action obtained numerically, we can estimate the type of the superconductivity of the QCD vacuum from the Ginzburg-Landau parameter κ_{GL} defined by

$$2\kappa_{GL}^2 = \frac{m_H^2}{m_A^2} = \frac{4\beta\lambda}{\gamma^2}. \tag{34}$$

It is found that the QCD vacuum is a type-II superconductor for $b \geq 1/\sqrt{\kappa}$ (see Fig.6 where the fit with $c_3 = 1$ is used in the Coulomb coupling).

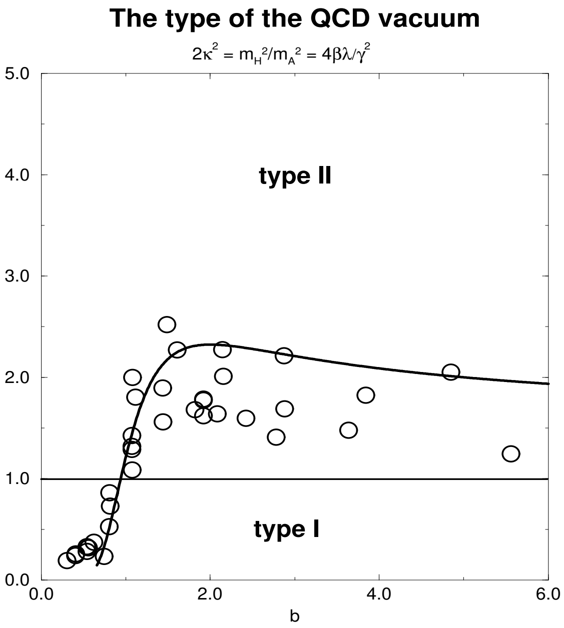

Figure 6. The b dependence of the Ginzburg-Landau parameter for the QCD vacuum. The straight line denotes the critical line between type-I and type-II superconductivity.

STRING TENSION

When the 4- and the 6-point interactions are neglected, we can also get the representation of the string model[16]. Let us perform the BKT transformation for ${}^*l({}^*c_1)$ in the abelian-Higgs model ${}^*l = {}^*s + d\,{}^*r$, $d\,{}^*s = {}^*\sigma$, ${}^*\phi_{nc} = {}^*\phi + 2\pi(\delta\Delta^{-1}\,{}^*s + {}^*r)$, we get the partition function

$$Z \propto \int_{-\infty}^{\infty} D\,{}^*C({}^*c_1) \int_{-\infty}^{\infty} D\,{}^*\phi_{nc}({}^*c_0) \sum_{{}^*\sigma({}^*c_2)\in Z, d\,{}^*\sigma=0}$$
$$\exp\left\{-\frac{1}{2g_m^2}\|d\,{}^*C\|^2 - \frac{1}{4p}\|d\,{}^*\phi_{nc} + 2\pi\delta\Delta^{-1}\,{}^*\sigma - {}^*C\|^2\right\} \tag{35}$$

Choosing the gauge ${}^*\phi_{nc} = 0$ and integrating over ${}^*C({}^*c_1)$, we get the string model on the lattice:

$$Z^{string} = \sum_{\sigma(c_2)\in Z,\delta\sigma=0} \exp\left\{-\frac{\pi^2}{p}\left(\sigma, \left(\Delta + \frac{g_m^2}{2p}\right)^{-1}\sigma\right)\right\}. \tag{36}$$

The condition $\delta\sigma = 0$ means that the world sheets form closed surfaces.

Using such representations, one can evaluate the string tension analytically. The Wilson loop is estimated by the following action of the hadronic string model on the lattice.

$$\langle W\rangle_m = \frac{1}{Z}\sum_{\sigma(c_2)\in Z,\delta\sigma=J} \exp\left[-\frac{\pi^2}{p(b)}\left(\sigma, \left(\Delta + M^2(b)\right)^{-1}\sigma\right)\right].$$

where

$$M^2 = \frac{g_m^2}{2p}. \tag{37}$$

Here we have written only the contribution from monopoles.

Our numerical results show that M^2 is large in the large b region. In this case, a classical picture may be reliable in the string model and the string tension is approximated by the self coupling of the world sheets:

$$\kappa_L = \frac{\pi^2}{p(b)(\Delta + M^2(b))}(0). \tag{38}$$

The physical string tension is obtained as follows:

$$\kappa = \frac{\kappa_L}{b^2} = \frac{\pi^2}{b^2 p(b)(\Delta + M^2(b))}(0), \tag{39}$$

where $p(b) = 0.53(1)/b^{3.56(4)}$ and $g_m^2(b)/2$ is given by Eq.(22).

Since M^2 is large, it can be evaluated by the $1/M^2$ expansion. The results shown in Fig.7 (the fit with $c_3 = 1$ is used) show that for $b > 1/\sqrt{\kappa}$, the obtained action reproduces the experimental string tension rather well. It is very interesting that we can almost reproduce the physical string tension analytically with the action obtained. This means that the monopole action is very near to the perfect action.

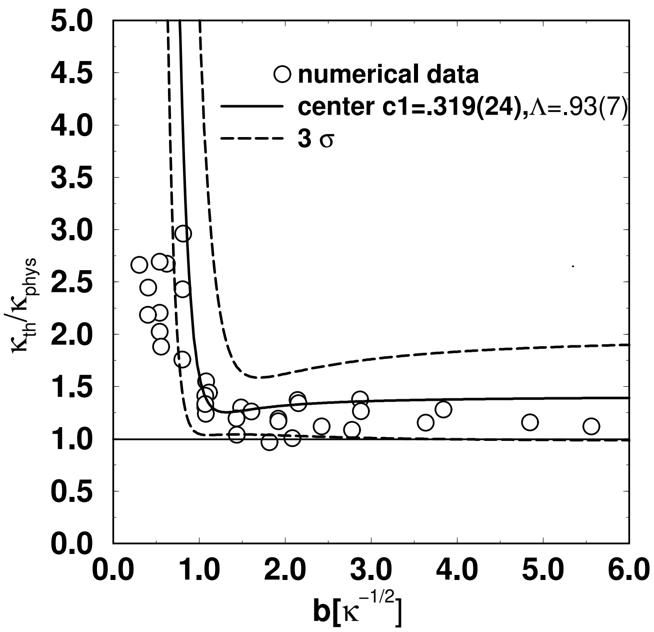

Figure 7. Theoretical string tension.

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