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Monopole Constituents inside SU(n) Calorons

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Abstract: We present a simple result for the action density of the $SU(n)$ charge one periodic instantons - or calorons - with arbitrary non-trivial Polyakov loop $\mathcal{P}_\infty$ at spatial infinity. It is shown explicitly that there are n lumps inside the caloron, each of which represents a BPS monopole, their masses being related to the eigenvalues of $\mathcal{P}_\infty$. A suitable combination of the ADHM construction and the Nahm transformation is used to obtain this result.

1 Introduction

Instantons and BPS monopoles are self-dual finite action solutions to the Yang-Mills equations of motion. Their origin is topological and related to windings in the gauge transformations describing their behaviour at infinity, where these solutions approach vacua, necessary for the action to be finite. The action of these solutions is proportional to these winding numbers or charges, which suggests that they are composed out of elements with unit charge. We will find by computing the action density that even charge one periodic instantons (calorons) for the gauge group $SU(n)$ are further composed out of constituents. The constituents are n basic BPS monopoles [1], whose magnetic charges cancel exactly.

Periodic instantons were first discussed in the context of finite temperature field theory [2, 3]. Motivated by issues of T-duality [4] and D-brane constructions [5] in string theory, periodic instantons with a non-trivial Polyakov loop were recently constructed. The ingredients of the Nahm transformation for $SU(n)$ calorons [6], can be naturally presented in terms of n monopole constituents [7]. This has been the basis of the approach followed by Lee and co-workers [8, 9]. Our work relies on a more direct approach, combining the ADHM construction of multi-instanton solutions [10] with the Nahm construction [6], related by Fourier transformation. The constituent monopoles appear as explicit lumps in the action density [11].

The composite nature of the periodic instantons can also be appreciated by an argument due to Taubes [12] on how to build out of monopoles, configurations with non-trivial topological charge. It has far reaching consequences, which go beyond the existence of exact caloron solutions [11]. Its relevance for QCD has motivated us to extend our work to $SU(3)$.

Finite action solutions on $\mathbb{R}^3 \times S^1$ should approach vacua at spatial infinity. Due to the topology of this base manifold, these vacua can be non-trivial. This endows the caloron with extra parameters - labeling these vacua - which are studied in terms of the eigenvalues of the Polyakov loop around S^1 at spatial infinity. The Polyakov loop is defined in the periodic gauge, $A_\mu(\vec{x}, x_0 + \mathcal{T}) = A_\mu(\vec{x}, x_0)$, as

$$\mathcal{P}(\vec{x}) = P\exp(\int_0^{\mathcal{T}} A_0(\vec{x}, x_0)dx_0), \tag{1}$$

where $\mathcal{T}$ is the period and P denotes path-ordering. At infinity the value of the Polyakov loop does not change under continuous deformations of the loop and its eigenvalues are topological invariants.

In extending our work to the $SU(3)$ case it turned out to be natural to generalise to $SU(n)$ (see also the appendices of ref. [9]). We will present the formula for the action density in section 2. The derivation is outlined in section 3. A detailed description will be given elsewhere. In section 4 we discuss the properties of the solution.

2 The result

We consider the calorons with no net magnetic charge, in which case the Polyakov loop (holonomy) at spatial infinity becomes constant. Its eigenvalues $e^{2\pi i\mu_m}$ will play an important role in the construction,

$$\lim_{|\vec{x}|\to\infty} \mathcal{P}(\vec{x}) = \mathcal{P}_\infty = V\mathcal{P}_\infty^0 V^{-1}, \quad \mathcal{P}_\infty^0 = \exp[2\pi i \mathrm{diag}(\mu_1, \ldots, \mu_n)]. \tag{2}$$

Making use of the gauge symmetry, we can choose the eigenvalues such that

$$\mu_1 < \ldots < \mu_n < \mu_{n+1} \equiv \mu_1 + 1, \quad \sum_{m=1}^{n} \mu_m = 0, \tag{3}$$

assuming maximal symmetry breaking for the moment. We define $\nu_m = \mu_{m+1} - \mu_m$, related to the mass of the m^{th} constituent monopole. Standard arguments, summarised below, gives $4n$ instanton parameters for fixed $\mathcal{P}_\infty$, including the global gauge transformations that do not change $\mathcal{P}_\infty$. We will see that $3n$ parameters can be interpreted as the positions $(\vec{y}_m)$ of the constituents. The remaining parameters in this interpretation are the $n-1$ phases related to the unbroken gauge group $U(1)^{n-1}$, on which the action density does not depend and the position of the caloron in time, which we fix to be 0 by translational invariance. Also we will use the scale invariance to set $\mathcal{T} = 1$. Where needed, the proper $\mathcal{T}$ dependence can be reinstated on dimensional grounds. We find the following surprisingly simple formula

$$-\tfrac{1}{2}\mathrm{tr}F_{\mu\nu}^2 = -\tfrac{1}{2}\partial_\mu^2\partial_\nu^2 \log\psi. \tag{4}$$

where the positive scalar potential ψ is defined as

$$\psi(x) = \tfrac{1}{2}\mathrm{tr}\prod_{m=1}^{n}\left\{\begin{pmatrix} r_m & |\vec{y}_m - \vec{y}_{m+1}| \\ 0 & r_{m+1}\end{pmatrix}\begin{pmatrix}\cosh(2\pi\nu_m r_m) & \sinh(2\pi\nu_m r_m) \\ \sinh(2\pi\nu_m r_m) & \cosh(2\pi\nu_m r_m)\end{pmatrix}\frac{1}{r_m}\right\} - \cos(2\pi x_0). \tag{5}$$

Here $r_m = |\vec{x} - \vec{y}_m|$ denotes the center of mass radius of the m^{th} monopole. The order of matrix multiplication is crucial, $\prod_{m=1}^{n} A_m \equiv A_n \ldots A_1$.

3 The construction

In our description of the caloron with non-trivial Polyakov loop, we pick the so-called algebraic gauge,

$$A_\mu(\vec{x},x_0+\mathcal{T})=\mathcal{P}_\infty A_\mu(\vec{x},x_0)\mathcal{P}_\infty^{-1},\tag{6}$$

which is related to the periodic gauge by the non-periodic gauge transformation $g(\vec{x},x_0)=V\exp[2\pi i x_0\mathrm{diag}(\mu_1,\ldots,\mu_n)/\mathcal{T}]V^{-1}$. In the algebraic gauge *all* gauge field components approach zero at infinity. The technique we use is to interpret the ADHM data as the Fourier coefficients of the functions that appear in the Nahm transformation. This is to solve the quadratic ADHM constraint, which is non-trivial for a periodic array of instantons, twisted in colour space going from one time slice to the next.

We summarise the ADHM formalism for $SU(n)$ charge k instantons [10], to fix our notation. It employs a k dimensional vector $\lambda=(\lambda_1,\ldots,\lambda_k)$, where $\lambda_i^\dagger$ is a two-component spinor in the $\bar{n}$ representation of $SU(n)$. Alternatively, λ can be seen as an $n\times 2k$ complex matrix. In addition one has four complex hermitian $k\times k$ matrices B_μ, combined into a $2k\times 2k$ complex matrix $B=\sigma_\mu\otimes B_\mu$, using the unit quaternions $\sigma_\mu=(1_2,i\vec{\tau})$ and $\bar{\sigma}_\mu=(1_2,-i\vec{\tau})$, where τ_i are the Pauli matrices. With abuse of notation, we often write $B=\sigma_\mu B_\mu$. Together λ and B constitute the $(n+2k)\times 2k$ dimensional matrix $\Delta(x)$, to which is associated a complex $(n+2k)\times n$ dimensional normalised zero mode vector $v(x)$,

$$\Delta(x)=\begin{pmatrix}\lambda\\ B(x)\end{pmatrix},\quad B(x)=B-x,\quad \Delta^\dagger(x)v(x)=0,\quad v^\dagger(x)v(x)=1_n.\tag{7}$$

Here the quaternion $x=x_\mu\sigma_\mu$ denotes the position (a $k\times k$ unit matrix is implicit) and $v(x)$ can be solved explicitly in terms of the ADHM data by

$$v(x)=\begin{pmatrix}-1_n\\ u(x)\end{pmatrix}\phi^{-\frac{1}{2}},\quad u(x)=(B^\dagger-x^\dagger)^{-1}\lambda^\dagger,\quad \phi(x)=1_n+u^\dagger(x)u(x),\tag{8}$$

As $\phi(x)$ is an $n\times n$ positive hermitian matrix, its square root $\phi^{\frac{1}{2}}(x)$ is well-defined. The gauge field is given by

$$A_\mu(x)=v^\dagger(x)\partial_\mu v(x)=\phi^{-\frac{1}{2}}(x)(u^\dagger(x)\partial_\mu u(x))\phi^{-\frac{1}{2}}(x)+\phi^{\frac{1}{2}}(x)\partial_\mu\phi^{-\frac{1}{2}}(x).\tag{9}$$

For $A_\mu(x)$ to be a self-dual connection, $\Delta(x)$ has to satisfy the quadratic ADHM constraint, which states that $\Delta^\dagger(x)\Delta(x)=B^\dagger(x)B(x)+\lambda^\dagger\lambda$ (considered as $k\times k$ complex quaternionic matrix) has to commute with the quaternions, or equivalently

$$\Delta^\dagger(x)\Delta(x)=\sigma_0\otimes f_x^{-1},\tag{10}$$

defining f_x as a hermitian $k\times k$ Green's function. The self-duality follows by computing the curvature

$$F_{\mu\nu}=2\phi^{-\frac{1}{2}}(x)u^\dagger(x)\eta_{\mu\nu}f_x u(x)\phi^{-\frac{1}{2}}(x),\tag{11}$$

making essential use of the fact that f_x commutes with the quaternions, and $\eta_{\mu\nu}\equiv\sigma_{[\mu}\bar{\sigma}_{\nu]}$ being self-dual ($\bar{\eta}_{\mu\nu}\equiv\bar{\sigma}_{[\mu}\sigma_{\nu]}$ is anti-selfdual). The quadratic constraint can be formulated as $\Im m(\Delta^\dagger(x)\Delta(x))=0$, where $\Im m W\equiv\frac{1}{2}[W-\tau_2W^t\tau_2]$, and one obtains

$$\bar{\eta}_{\mu\nu}\otimes B_\mu B_\nu+\tfrac{1}{2}\tau_a\otimes\mathrm{tr}_2(\tau_a\lambda^\dagger\lambda)=0,\tag{12}$$

where tr_2 is the spinorial trace. Note that this implies that $\mathrm{tr}_2(\tau_a\lambda^\dagger\lambda)$ is traceless for $a = 1,2,3$. To count the number of instanton parameters we observe that the transformation $\lambda \to \lambda T^\dagger$, $B_\mu \to T B_\mu T^\dagger$, with $T \in U(k)$ leaves the gauge field and the ADHM constraint untouched. Taking this symmetry into account, we find the dimension of the instanton moduli space to be $4kn$ dimensional. Global gauge transformations are realised by $\lambda \to g\lambda$, with $g \in SU(n)$. Those that leave $\mathcal{P}_\infty$ invariant reduce the dimension of the gauge invariant parameter space (by $n-1$ for maximal symmetry breaking). Finally, we quote an elegant result [13] for the action density in terms of f_x

$$\mathrm{tr} F_{\mu\nu}^2(x) = -\partial_\mu^2\partial_\nu^2 \log\det f_x. \tag{13}$$

The charge one caloron with Polyakov loop $\mathcal{P}_\infty$ at infinity is built out of a periodic array of instantons, twisted by $\mathcal{P}_\infty$. This is implemented in the ADHM formalism by requiring (suppressing colour and spinor indices)

$$u_{p+1}(x+1) = u_p(x)\mathcal{P}_\infty^{-1} \tag{14}$$

with $p \in \mathbb{Z}$. Using that $\phi^{\pm\frac{1}{2}}(x+1) = \mathcal{P}_\infty \phi^{\pm\frac{1}{2}}(x)\mathcal{P}_\infty^{-1}$, eq. (9) leads to the required result, eq. (6). Demanding

$$\lambda_{p+1} = \mathcal{P}_\infty \lambda_p, \quad B_{p,q} = B_{p-1,q-1} + \sigma_0\delta_{p,q}, \tag{15}$$

suitably implements eq. (14) and is partially solved by imposing

$$\lambda_p = \mathcal{P}_\infty^p \zeta, \quad B_{p,q} = p\sigma_0\delta_{p,q} + \hat{A}_{p-q}, \tag{16}$$

with $\hat{A}$ still to be determined to account for eq. (12), which also constrains the spinor $\zeta^\dagger$ in the $\bar{n}$ representation of $SU(n)$ to

$$\mathrm{tr}_2(\tau_a\zeta^\dagger\zeta) = 0, \quad a = 1,2,3. \tag{17}$$

It is useful to introduce the n projectors $P_m = V P_m^{(0)} V^{-1}$, with $(P_m^{(0)})_{a,b} = \delta_{m,a}\delta_{m,b}$ and $P_m P_{m'} = \delta_{m,m'}P_m$, such that $\mathcal{P}_\infty = \sum_m \exp(2\pi i\mu_m)P_m$ and $\lambda_p = \sum_m \exp(2\pi i p\mu_m)P_m\zeta$.

We now perform the Fourier transformation to the Nahm setting [6], which casts B into a Weyl operator and $\lambda^\dagger\lambda$ into a singularity structure on S^1,

$$\begin{aligned}
&\sum_{p,q} B_{p,q}(x)e^{2\pi i(pz-qz')} = \frac{\delta(z-z')}{2\pi i}\hat{D}_x(z'), \quad \hat{D}_x(z) = \sigma_\mu \hat{D}_x^\mu(z) = \frac{d}{dz} + \hat{A}(z) - 2\pi i x, \\
&\hat{A}(z) = \sigma_\mu \hat{A}^\mu(z), \quad \hat{A}^\mu(z) = 2\pi i \sum_p e^{2\pi i p z}\hat{A}_p^\mu, \\
&\sum_p e^{-2\pi p i z}\lambda_p = \sum_p e^{2\pi i p(\mu_m - z)}P_m\zeta = \hat{\lambda}(z), \quad \hat{\lambda}(z) = \sum_m \delta(z-\mu_m)P_m\zeta, \\
&\sum_{p,q} \lambda_p^\dagger e^{2\pi i(pz-qz')}\lambda_q = \delta(z-z')\hat{\Lambda}(z), \quad \hat{\Lambda}(z) = \sum_m \delta(z-\mu_m)\zeta^\dagger P_m\zeta = \zeta^\dagger\hat{\lambda}(z).
\end{aligned} \tag{18}$$

Introducing the vector $\zeta_{(m)} = P_m\zeta$, it is standard to show

$$\zeta^\dagger P_m \zeta = \zeta_{(m)}^\dagger \zeta_{(m)} = \frac{1}{2\pi}(|\vec{\rho}_m| - \vec{\tau}\cdot\vec{\rho}_m), \tag{19}$$

and the quadratic ADHM constraint, which takes the form

$$\tfrac{1}{2}[\hat{D}_\mu(z),\hat{D}_\nu(z)]\bar{\eta}_{\mu\nu}=4\pi^2\Im m\hat{\Lambda}(z), \tag{20}$$

leads to the (for $k=1$ abelian) Nahm equation

$$\frac{d}{dz}\hat{A}_j(z)=2\pi i\sum_m\delta(z-\mu_m)\rho_m^j. \tag{21}$$

The T symmetry in the ADHM construction translates into a $U(1)$ gauge symmetry on S^1, which leaves $\hat{A}_i$ invariant and allows one to set $\hat{A}_0=2\pi i\xi_0$, ξ_0 being the position in time, which we absorb in x_0. Since $\vec{\rho}_m=-\pi\mathrm{tr}_2(\vec{\tau}\zeta^\dagger P_m\zeta)$, it follows that $\sum_m\vec{\rho}_m=-\pi\mathrm{tr}_2(\vec{\tau}\zeta^\dagger\zeta)=\vec{0}$, see eq. (17), such that we may introduce $\vec{y}_m$ ($\vec{y}_0\equiv\vec{y}_n$), with $\vec{\rho}_m=\vec{y}_m-\vec{y}_{m-1}$. In terms of these we find

$$\hat{A}_j(z)=2\pi i\sum_m\chi_{[\mu_m,\mu_{m+1}]}(z)y_m^j, \tag{22}$$

where $\chi_{[\mu_m,\mu_{m+1}]}(z)=1$ for $z\in[\mu_m,\mu_{m+1}]$ and 0 elsewhere, taking into account z has period 1. Note that $\vec{y}_m$ is only fixed up to a constant $\vec{\xi}$, related to the freedom of adding a constant to the solution of the Nahm equation, eq. (21). The 4-vector ξ_μ describes the position of the caloron. Also note that the $\vec{\rho}_m$ are independent of the phases of $P_m\zeta$, affected by the residual gauge symmetry, of which $n-1$ are independent due to the gauge group being $SU(n)$ rather than $U(n)$.

The vector $\vec{y}_m$ is interpreted as the position of the $m^{\rm th}$ constituent. On each sub-interval $[\mu_m,\mu_{m+1}]$, $\hat{A}_j(z)=2\pi iy_m^j$ is constant, which is precisely the Nahm datum for a single BPS monopole located at $\vec{y}_m$ [14]. The length of the Nahm interval for the single BPS monopole corresponds to its asymptotic Higgs value, and thereby to its mass. Thus, the $m^{\rm th}$ subinterval $[\mu_m,\mu_{m+1}]$ corresponds to a BPS monopole at $\vec{y}_m$, with mass proportional to $\nu_m=\mu_{m+1}-\mu_m$. Together, the n BPS monopoles form the $SU(n)$ caloron.

The Green's function f_x, central in the ADHM construction, is found after a Fourier transformation of eq. (10), introducing $\hat{f}_x(z,z')\equiv\sum_{p,q}f_x^{p,q}e^{2\pi i(pz-qz')}$ as the solution to the differential equation

$$\left\{\left(\frac{1}{2\pi i}\frac{d}{dz}-x_0\right)^2+\sum_m\chi_{[\mu_m,\mu_{m+1}]}(z)\,r_m^2+\frac{1}{2\pi}\sum_m\delta(z-\mu_m)|\vec{y}_m-\vec{y}_{m-1}|\right\}\hat{f}_x(z,z')=\delta(z-z'), \tag{23}$$

where the radii are given by $r_m=|\vec{x}-\vec{y}_m|$, to be interpreted as the center of mass radii of the constituent monopoles. The solution of a quantum-mechanical problem on the circle with a piecewise constant potential and delta function impurities is obtained by solving it on each sub-interval, where $\hat{f}_x(z,z')$ is of simple exponential form. Starting at $z=z'$ and matching properly at $z=\mu_m$ so as to account for the scattering by the impurity, we can go full circle to return at $z=z'$ where one last matching accounts for the delta function at the rhs. of eq. (23).

With the solution for $f_x(z,z')$ available, we can compute $\mathrm{Tr}\hat{D}_x^\mu f_x$ and show it to be equal to $-\pi i\partial_\mu\log\psi$, ψ being the positive scalar function defined in eq. (5). Using eq. (13), this leads to eq. (4). One retrieves our $SU(2)$ results of refs. [4, 11] by putting $\mu_1=-\omega$, $\mu_2=\omega$ and $\mu_3=1-\omega$, such that $\nu_1=\mu_2-\mu_1=2\omega$ and $\nu_2=\mu_3-\mu_2=1-2\omega\equiv2\bar{\omega}$. Furthermore we identify $r_1=s$, $r_2=r$ and $|\vec{y}_1-\vec{y}_0|=|\vec{y}_2-\vec{y}_1|=\pi\rho^2$.

4 Discussion

We briefly discuss the properties of the $SU(n)$ charge one caloron. From eq. (5) we see that the m^{th} constituent monopole can be located at arbitrary $\vec{y}_m$, with arbitrary mass $8\pi^2\nu_m/\mathcal{T}$, subject only to the constraint $\sum_m \nu_m = 1$, choosing $\mathcal{P}_\infty$, $\vec{\xi}$ and ζ appropriately. As the action density, eq. (4), is expressed as a total derivative, the action is easily found by partial integration, with the expected result of $8\pi^2$. The size of the instanton is related to the differences in position of the constituent monopoles. As we work in units of $\mathcal{T}$, the situation of a small scale (nearby constituents), corresponds to large $\mathcal{T}$, i.e. to an instanton on $\mathbb{R}^4$. At the other extreme one has well separated lumps for small $\mathcal{T}$, i.e. in the static limit. In figure 1 we present a typical $SU(3)$ caloron for decreasing values of $\mathcal{T}$ using eq. (4). We will resist the temptation of showing results for other n, as eq. (4) can be readily implemented.

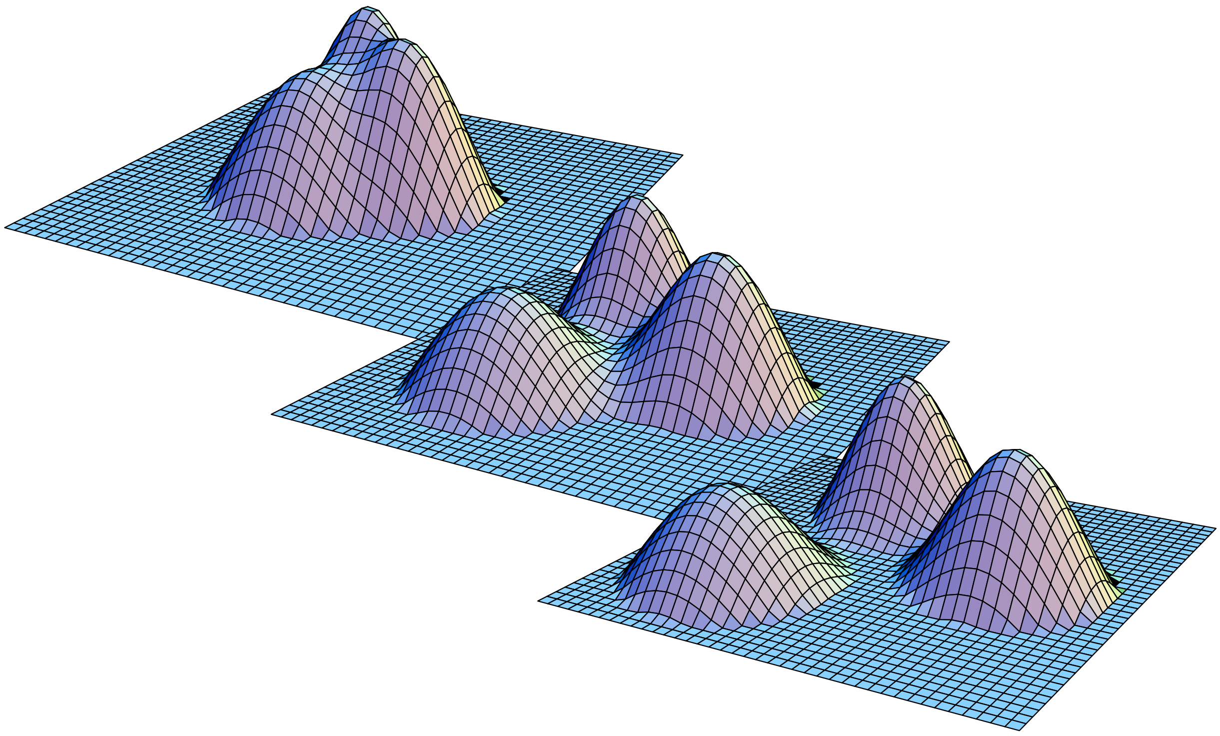

Figure 1: Action densities for the $SU(3)$ caloron at $x_0 = 0$ in the plane defined by the centers of the three constituents for $1/\mathcal{T} = 1.5, 3$ and 4 (increasing temperature from top to bottom). We choose mass parameters $(\nu_1, \nu_2, \nu_3) = (0.25, 0.35, 0.4)$, implemented by $(\mu_1, \mu_2, \mu_3) = (-17/60, -2/60, 19/60)$. The constituents are located at $\vec{y}_1 = (-\frac{1}{2}, -\frac{1}{2}, 0)$, $\vec{y}_2 = (0, \frac{1}{2}, 0)$ and $\vec{y}_3 = (\frac{1}{2}, -\frac{1}{4}, 0)$, in units of $\mathcal{T}$. The profiles are given on equal logarithmic scales, cut-off at an action density below $1/(2e)$.

When the lumps are far apart, they do not deform each other and become spherically symmetric. Since the solution is self-dual, the constituents have to be basic BPS monopoles. This can be proven by carefully analysing eq. (4) for the limit where $r_m \ll r_l$ for all $l \neq m$, in which case the action density approaches $-\frac{1}{2}\partial_\mu^2\partial_\nu^2 \log[\sinh(2\pi\nu_m r_m)/r_m]$. This is precisely the behaviour of the BPS monopole [15]. The other constituents need not be well-separated from each other for the above argument to hold. In particular sending the $m^{\rm th}$ constituent to infinity (i.e. $|\vec{y}_m| \to \infty$) suffices to make the caloron static. What remains are $n-1$ monopole constituents with a combined magnetic charge opposite to the magnetic charge of the $m^{\rm th}$ constituent monopole that has been removed. As the solution is static in this limit one is left with an $SU(n)$ BPS monopole. Indeed, for $|\vec{y}_m| \to \infty$ we see from the solution of the Nahm equation, eq. (21), that $\hat{A}_i(z)$ lives on an interval, rather than on the circle, as is appropriate for the $SU(n)$ monopole [16]. One readily obtains the energy density of this monopole by taking the limit $|\vec{y}_m| \to \infty$ in eq. (5), verifying that it decays as $1/|\vec{x}|^4$, as opposed to $1/|\vec{x}|^6$ without removing the $m^{\rm th}$ constituent.

Our results have been derived for the case of maximal symmetry breaking, $\mu_m \neq \mu_{m+1}$. The situation of non-maximal symmetry breaking corresponds to a constituent obtaining zero mass, $\nu_m = 0$. In that case its center of mass radius *drops out* of eq. (5), using

$$\begin{pmatrix} r_m & |\vec{y}_m - \vec{y}_{m+1}| \\ 0 & r_{m+1} \end{pmatrix} \frac{1}{r_m} \begin{pmatrix} r_{m-1} & |\vec{y}_m - \vec{y}_{m-1}| \\ 0 & r_m \end{pmatrix} = \begin{pmatrix} r_{m-1} & |\vec{y}_m - \vec{y}_{m+1}| + |\vec{y}_m - \vec{y}_{m-1}| \\ 0 & r_{m+1} \end{pmatrix}. \quad (24)$$

This was also observed for $SU(2)$, in which case non-maximal symmetry breaking corresponds to a trivial Polyakov loop, $\mathcal{P}_\infty = \pm 1$, and the solution becomes that of Harrington and Shepard [2]. Hence our formula for the action density should also be valid for non-maximal symmetry breaking.

Although our formalism can be extended easily to higher topological charges [9], the appropriate Nahm equation (i.e. solving the quadratic ADHM constraint) becomes a non-abelian problem, and finding solutions requires more powerful tools. Nevertheless, it is interesting to note that it is natural to conjecture that k instantons (i.e. an instanton of charge k) can be built from kn monopoles, since each instanton can be considered as being built from n BPS monopoles. The monopole constituents are only well separated when $\mathcal{T}$ is small, where the $4kn$ instanton parameters can be interpreted as $3kn$ positions and kn phases (including $\exp(2\pi i \xi_0/\mathcal{T})$). We will not speculate further on these matters here, but want to emphasise that the monopole constituent picture has some interesting phenomenological implications for the description of the long distance properties of QCD, discussed in detail in ref. [11], and which will be the subject of further investigations.

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